Hierarchical Bayesian Nonparametric Models of Language and Text

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Overview

- Probabilistic Models for Language and Text Sequences
- The Sequence Memoizer
  - Hierarchical Bayesian Modelling on Context Trees
  - Modelling Power Laws with Pitman-Yor Processes
  - Non-Markov Models
  - Efficient Inference and Computation
- Conclusions

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Sequence Models for Language and Text

- Probabilistic models for sequences of words and characters, e.g.

toad, in, a, hole

t, o, a, d,_ ,i ,n ,_ ,a ,_ , h, o, l, e

- Uses:
  - Natural language processing: speech recognition, OCR, machine translation.
  - Compression.
  - Cognitive models of language acquisition.
  - Sequence data arises in many other domains.

Markov Models for Language and Text

- Probabilistic models for sequences of words and characters.

P(toad in a hole) =
P(toad)*
P(in | toad)*
P(a | toad in)*
P(hole | toad in a)

Andrey Markov

- Usually makes a Markov assumption:

P(toad in a hole) ~
P(toad)*
P(in | toad)*
P(a | in)*
P(hole | a)

George E. P. Box

- Order of Markov model typically ranges from ~2 to > 10.

Sparsity in Markov Models

- Consider a high order Markov models:

$$P(\text{sentence}) = \prod_i P(\text{word}_i | \text{word}_{i-N+1} \dots \text{word}_{i-1})$$

- Large vocabulary size means naïvely estimating parameters of this model from data counts is problematic for N>2.

$$P^{\text{ML}}(\text{word}_i | \text{word}_{i-N+1} \dots \text{word}_{i-1}) = \frac{C(\text{word}_{i-N+1} \dots \text{word}_i)}{C(\text{word}_{i-N+1} \dots \text{word}_{i-1})}$$

- Naïve priors assuming independent parameters fail as well--- *most* parameters will have *no* associated data.
  - Smoothing.
  - Hierarchical Bayesian models.

Smoothing in Language Models

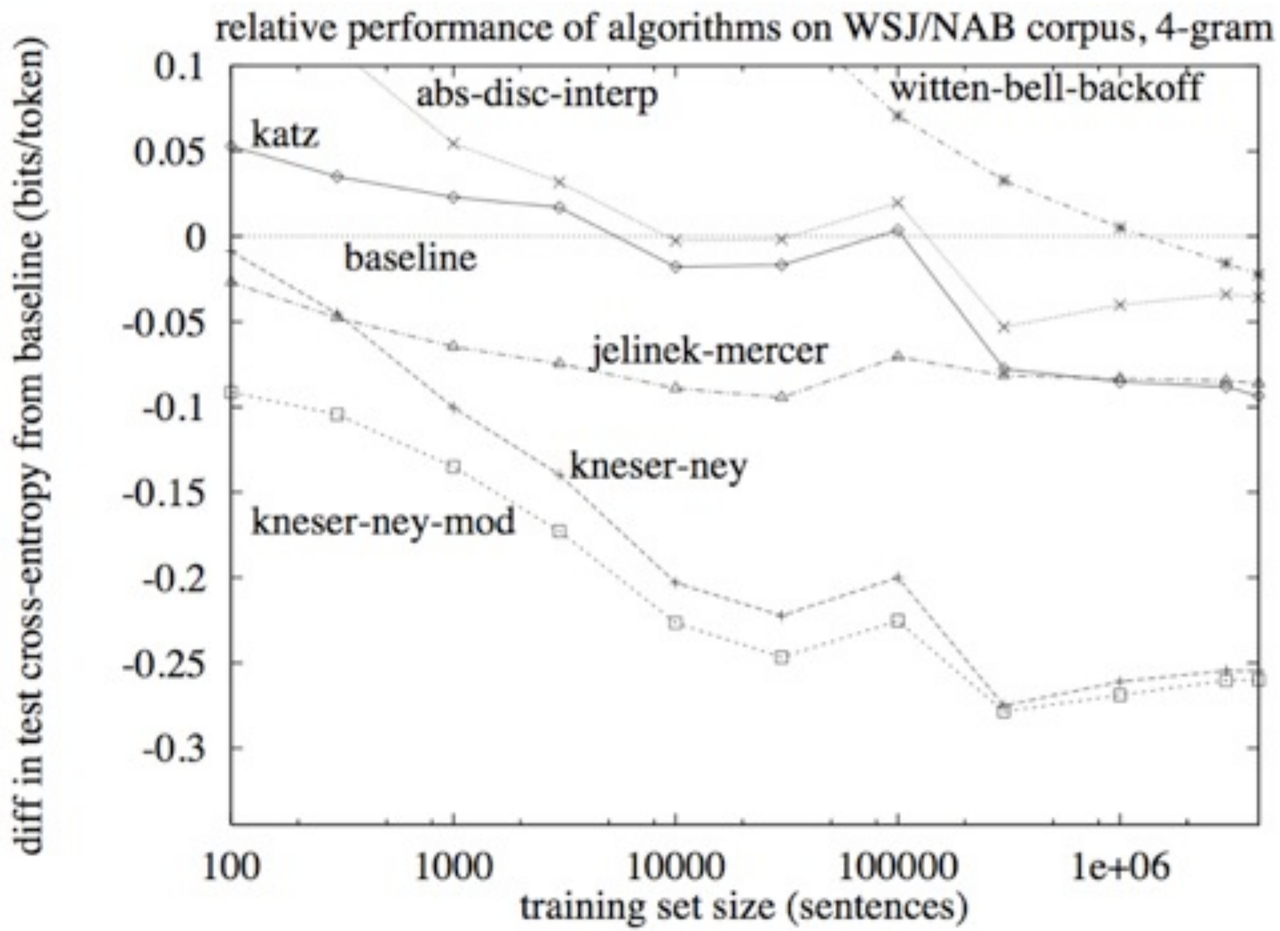
- Smoothing is a way of dealing with data sparsity by combining large and small models together.

$$P^{\text{smooth}}(\text{word}_i|\text{word}_{i-N+1}^{i-1}) = \sum_{n=1}^{N} \lambda(n) Q_n(\text{word}_i|\text{word}_{i-n+1}^{i-1})$$

- Combines expressive power of large models with better estimation of small models (cf bias-variance trade-off).

$$\begin{aligned}
& P^{\text{smooth}}(\text{hole}|\text{toad in a}) \\
= \; & \lambda(4)Q_4(\text{hole}|\text{toad in a}) + \\
& \lambda(3)Q_3(\text{hole}|\text{in a}) + \\
& \lambda(2)Q_2(\text{hole}|\text{a}) + \\
& \lambda(1)Q_1(\text{hole}|\emptyset)
\end{aligned}$$

Smoothing in Language Models



- [Chen and Goodman 1998] found that Interpolated and modified Kneser-Ney are best under virtually all circumstances.

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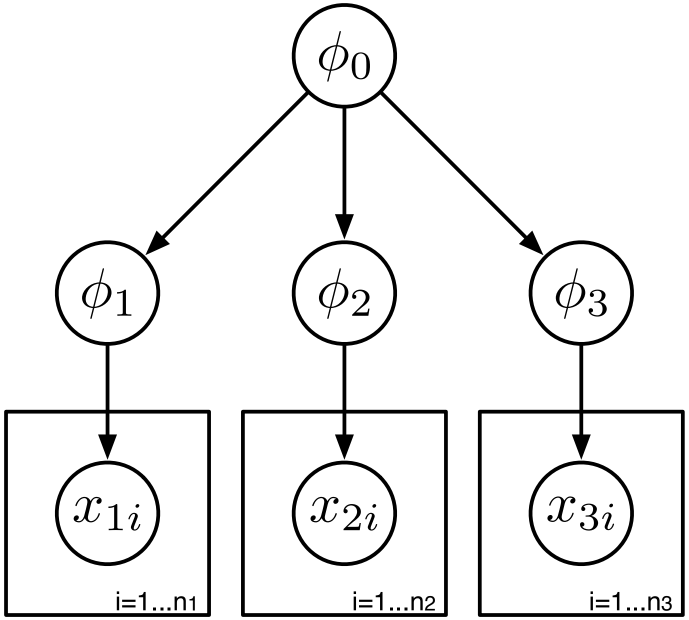
Hierarchical Bayesian Models

- Hierarchical modelling an important overarching theme in modern statistics [Gelman et al, 1995, James & Stein 1961].

- In machine learning, have been used for multitask learning, transfer learning, learning-to-learn and domain adaptation.

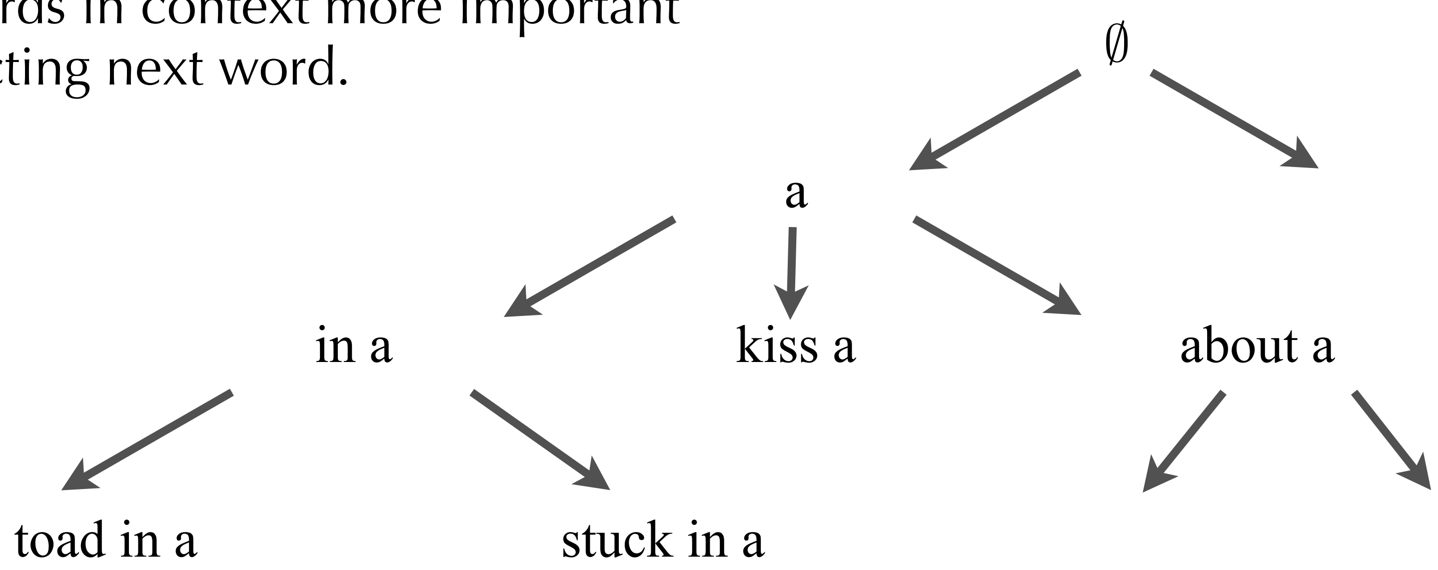


Context Tree

- *Context* of conditional probabilities naturally organized using a *suffix tree*.

$$P(\text{word}_i | \text{word}_{i-N+1}^{i-1})$$

- Smoothing makes conditional probabilities of neighbouring contexts more similar.
- Later words in context more important in predicting next word.



Hierarchical Bayesian Models on Context Tree

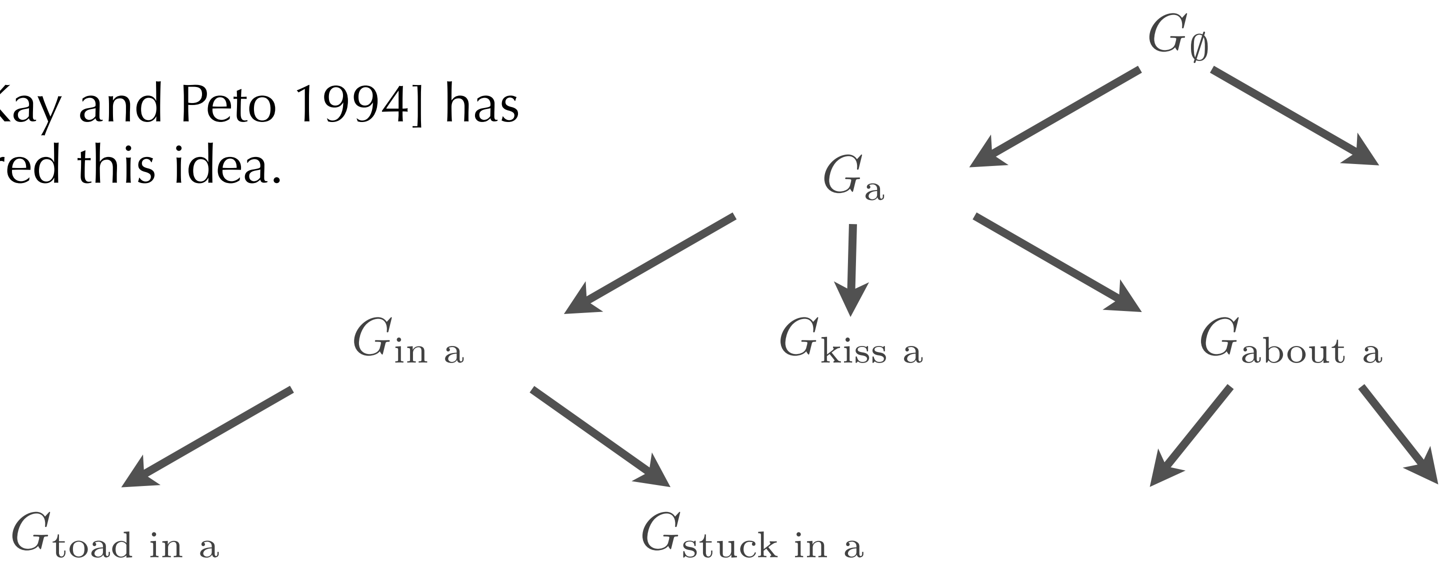
- Parametrize the conditional probabilities of Markov model:

$$P(\text{word}_i = w | \text{word}_{i-N+1}^{i-1} = u) = G_u(w)$$

$$G_u = [G_u(w)]_{w \in \text{vocabulary}}$$

- G_u is a probability vector associated with context u.

- [MacKay and Peto 1994] has explored this idea.



Hierarchical Dirichlet Language Models

- What is $P(G_u|G_{\mathrm{pa}(u)})$?
- Standard Dirichlet distribution over probability vectors---bad idea...

T	N-1	IKN	MKN	HDLM
2×10^6	2	148.8	144.1	191.2
4×10^6	2	137.1	132.7	172.7
6×10^6	2	130.6	126.7	162.3
8×10^6	2	125.9	122.3	154.7
10×10^6	2	122.0	118.6	148.7
12×10^6	2	119.0	115.8	144.0
14×10^6	2	116.7	113.6	140.5
14×10^6	1	169.9	169.2	180.6
14×10^6	3	106.1	102.4	136.6

- We will use Pitman-Yor processes instead [Perman, Pitman and Yor 1992], [Pitman and Yor 1997], [Ishwaran and James 2001].

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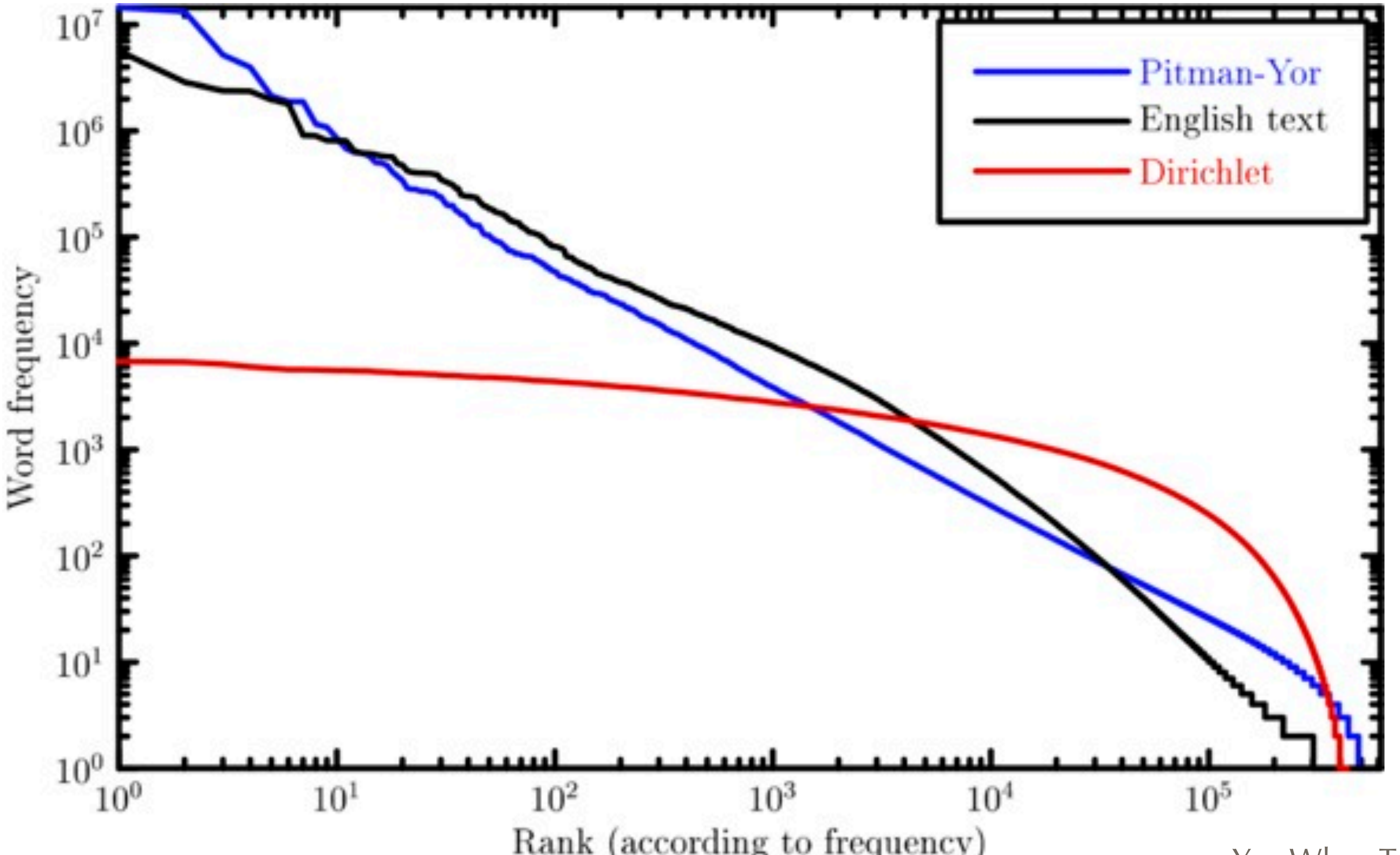
Pitman-Yor Processes

- Produce power-law distributions more suitable for languages.



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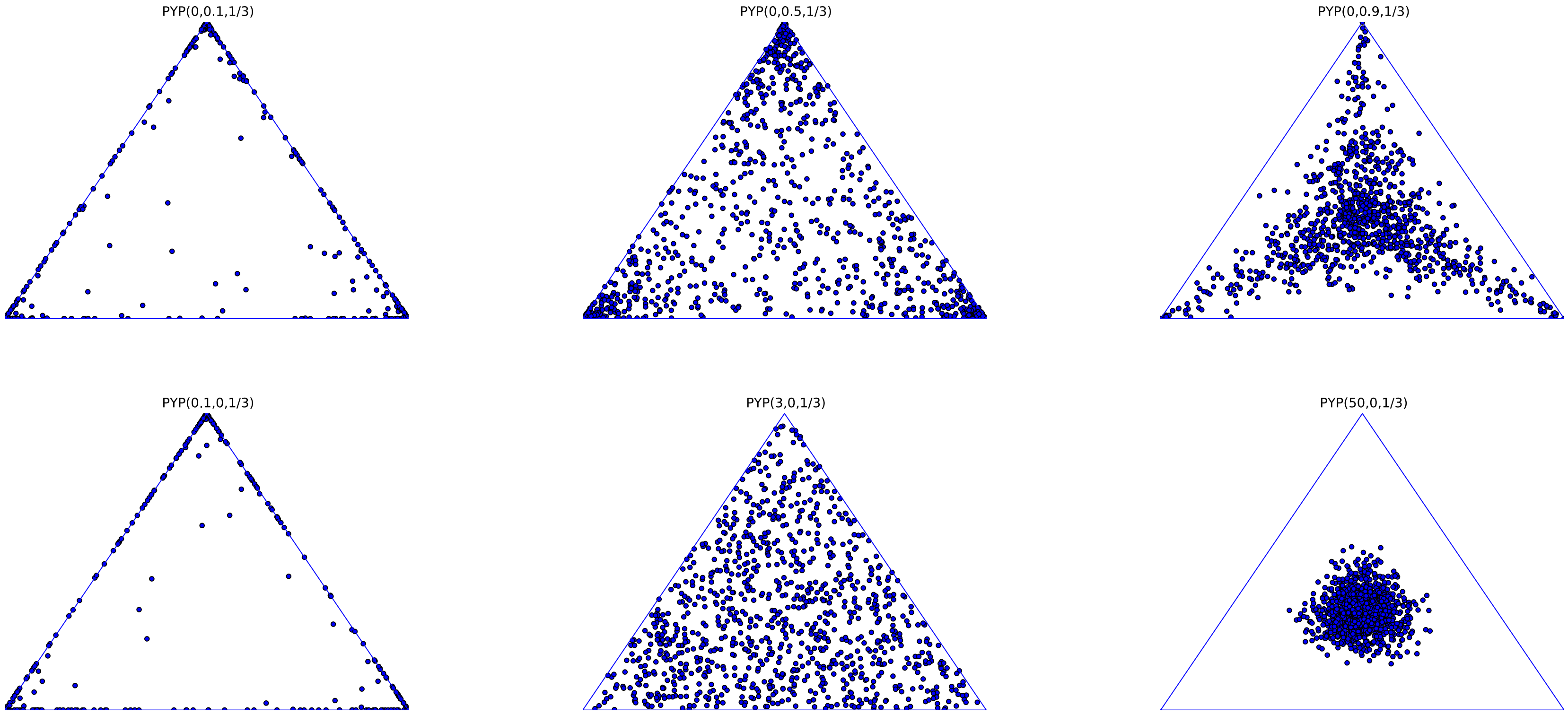
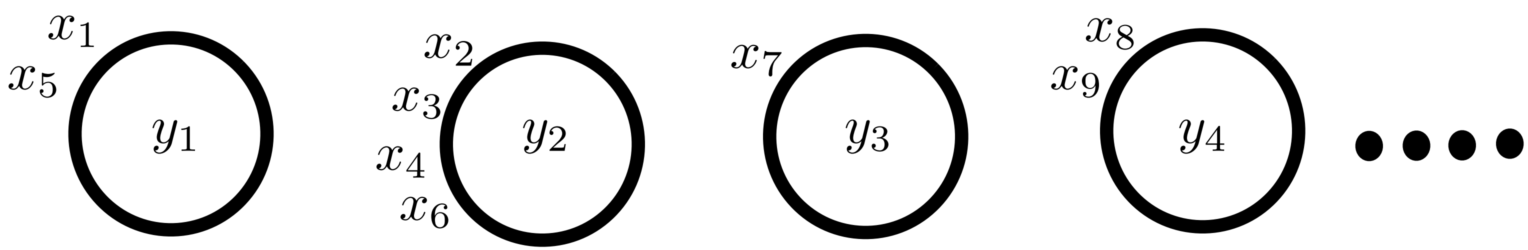
Pitman-Yor Processes



Chinese Restaurant Processes

- Easiest to understand them using Chinese restaurant processes.



$$
\begin{aligned}
p(\text{sit at table } k) &\propto c_k - d \\
p(\text{sit at new table}) &\propto \theta + dK \\
p(\text{table serves dish } y) &= H(y)
\end{aligned}
$$

- Defines an exchangeable stochastic process over sequences $x_1, x_2, \ldots$

- The de Finetti measure is the Pitman-Yor process,

$$
\begin{aligned}
G &\sim \mathrm{PY}(\theta, d, H) \\
x_i &\sim G \qquad i = 1, 2, \ldots
\end{aligned}
$$

[Perman, Pitman & Yor 1992, Pitman & Yor 1997]

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Power Law Properties of Pitman-Yor Processes

- Chinese restaurant process:

$$\begin{aligned} p(\text{sit at table } k) &\propto c_k - d \\ p(\text{sit at new table}) &\propto \theta + dK \end{aligned}$$

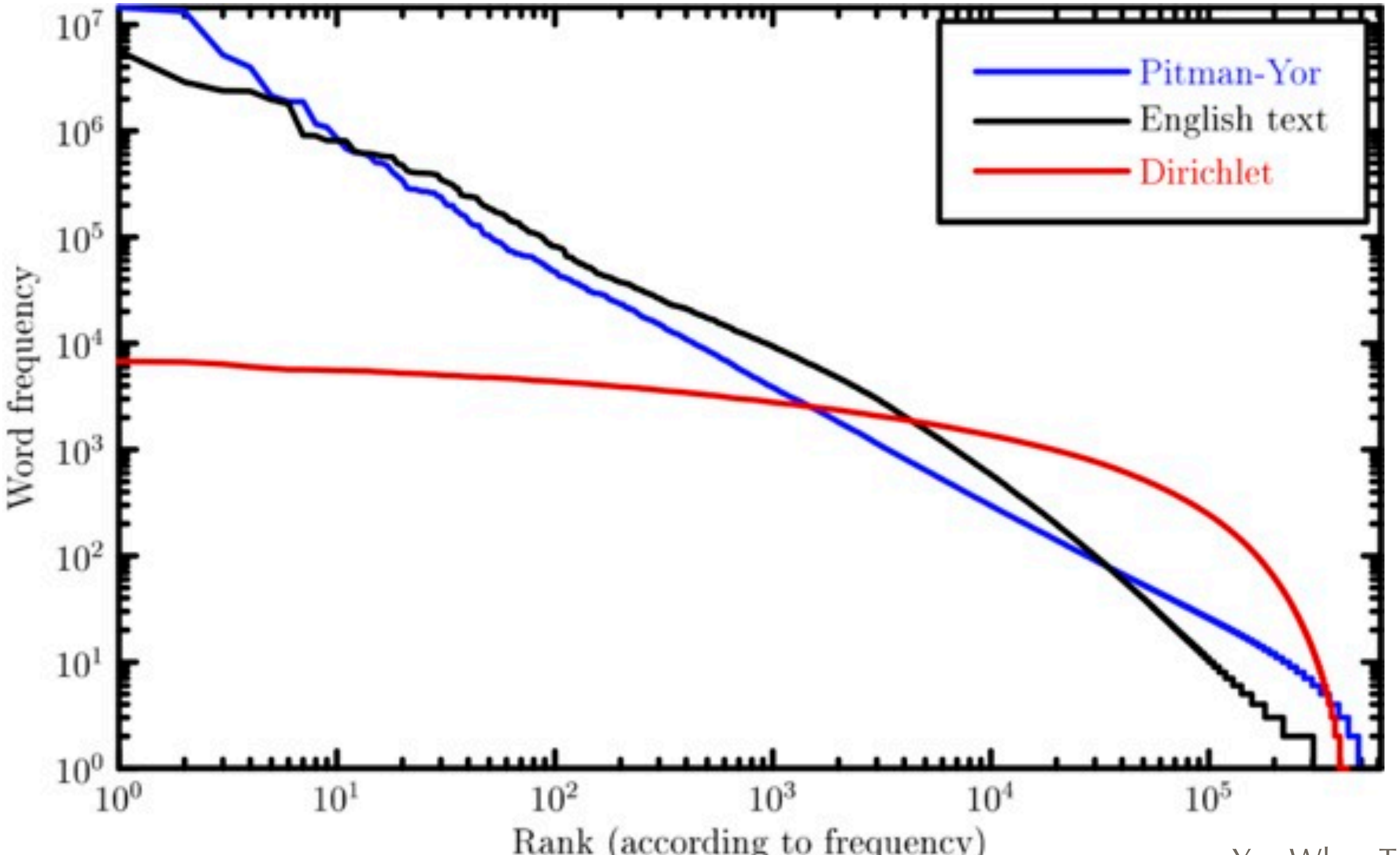
- Pitman-Yor processes produce distributions over words given by a power-law distribution with index $1 + d$.
  - Customers = word instances, tables = dictionary look-up;
  - Small number of common word types;
  - Large number of rare word types.

- This is more suitable for languages than Dirichlet distributions.

- [Goldwater, Griffiths and Johnson 2005] investigated the Pitman-Yor process from this perspective.

Power Law Properties of Pitman-Yor Processes

- Produce power-law distributions more suitable for languages.



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Power Law Properties of Pitman-Yor Processes

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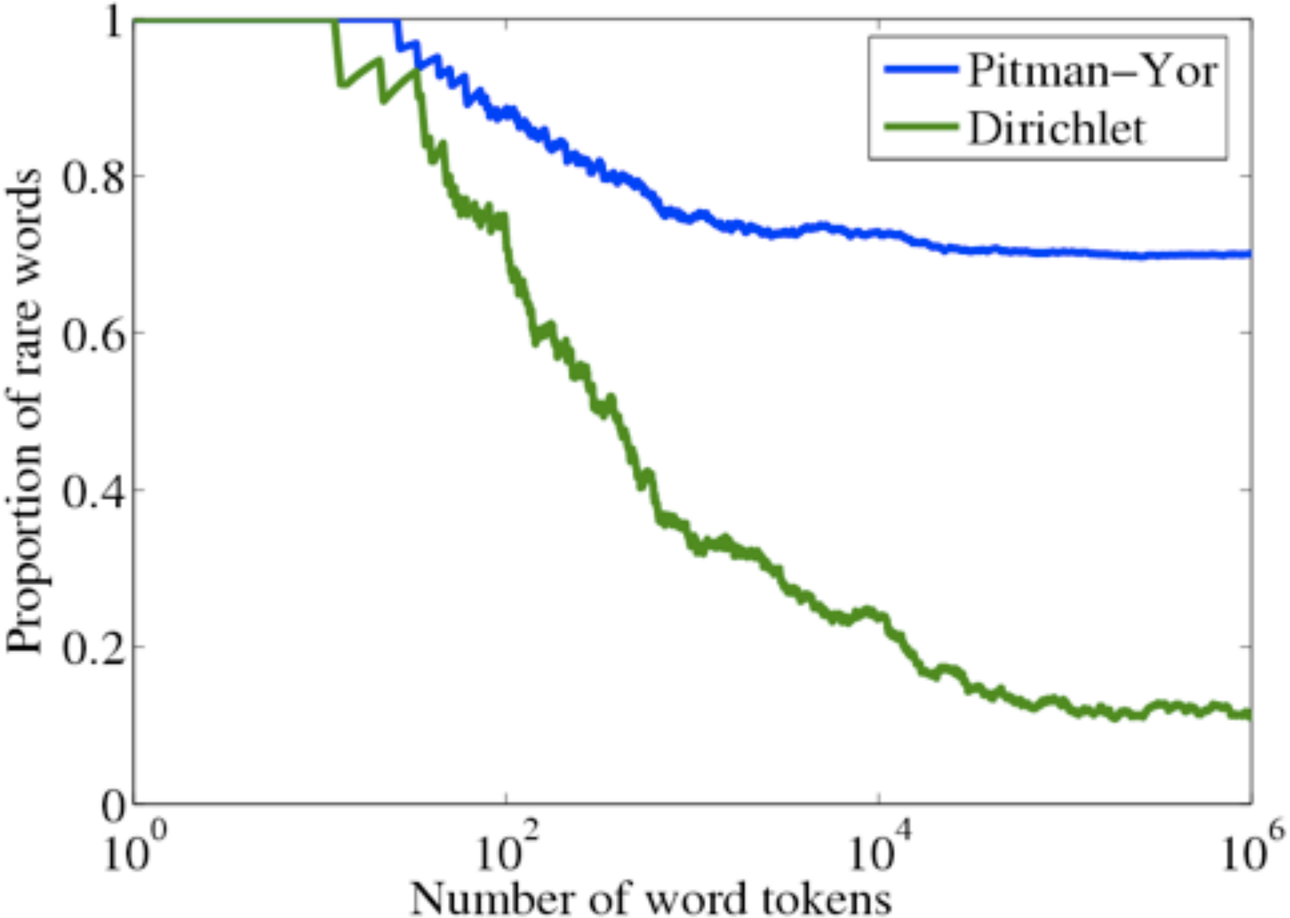

Hierarchical Pitman-Yor Language Models

- Parametrize the conditional probabilities of Markov model:

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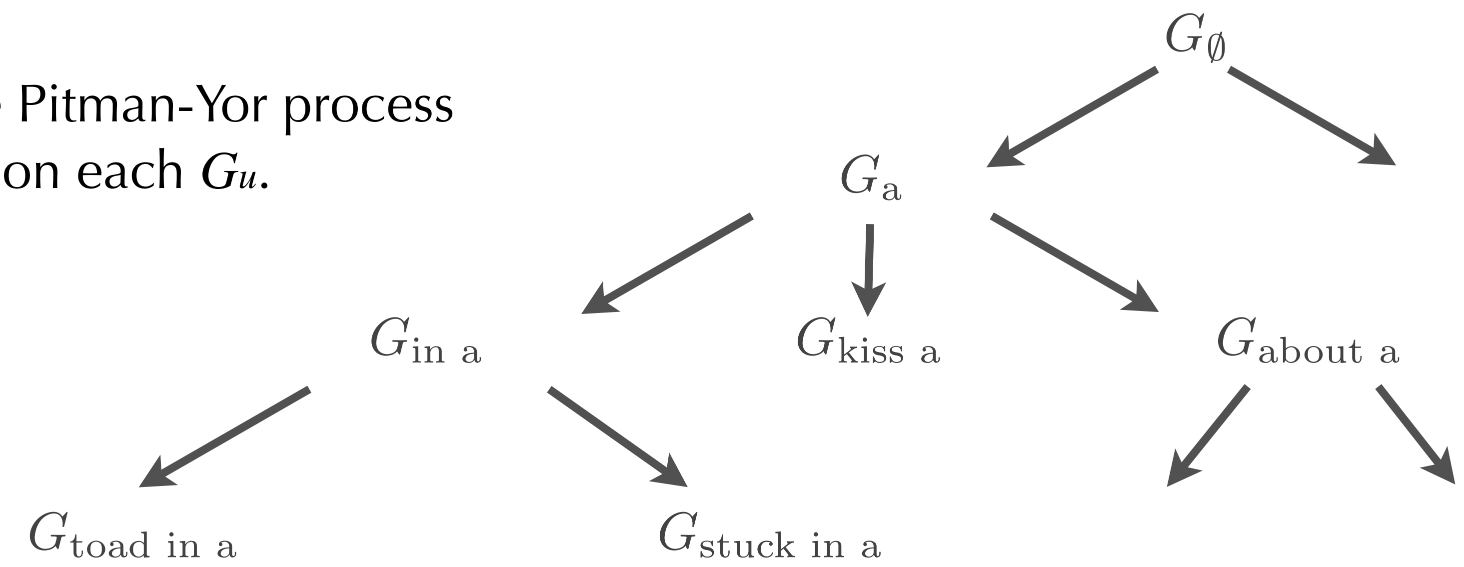
$$G_u = [G_u(w)]_{w \in \text{vocabulary}}$$

- G_u is a probability vector associated with context u.

- Place Pitman-Yor process prior on each G_u.



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Hierarchical Pitman-Yor Language Models

- Significantly improved on the hierarchical Dirichlet language model.
- Results better Kneser-Ney smoothing, state-of-the-art language models.

T	N-1	IKN	MKN	HDLM	HPYLM
2×10^6	2	148.8	**144.1**	191.2	144.3
4×10^6	2	137.1	**132.7**	172.7	**132.7**
6×10^6	2	130.6	126.7	162.3	**126.4**
8×10^6	2	125.9	122.3	154.7	**121.9**
10×10^6	2	122.0	118.6	148.7	**118.2**
12×10^6	2	119.0	115.8	144.0	**115.4**
14×10^6	2	116.7	113.6	140.5	**113.2**
14×10^6	1	169.9	**169.2**	180.6	169.3
14×10^6	3	106.1	102.4	136.6	**101.9**

- Similarity of perplexities not a surprise---Kneser-Ney can be derived as a particular approximate inference method.

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Markov Models for Language and Text

- Usually makes a Markov assumption to simplify model:

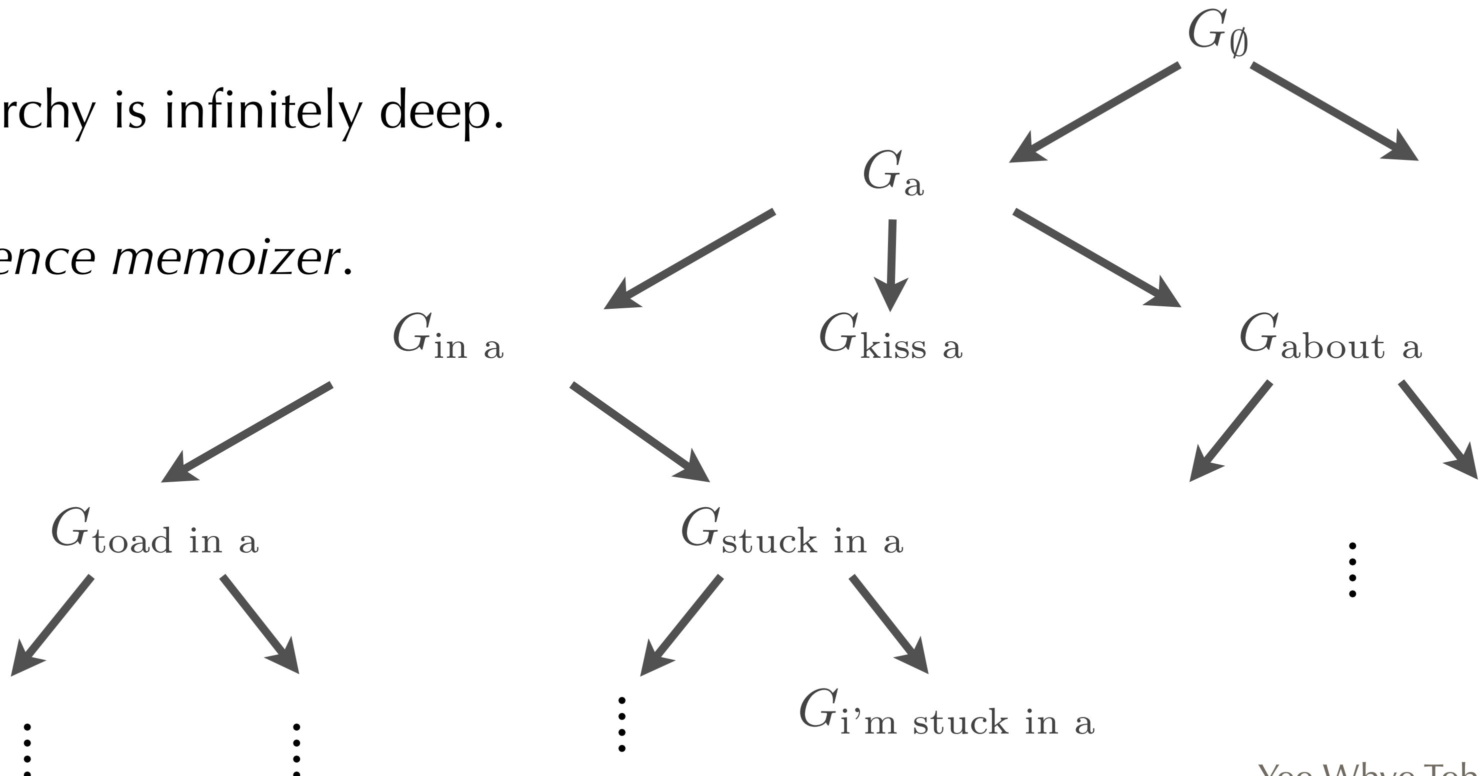
$$
\begin{gathered}
P(\text{toad in a hole}) \sim \\
P(\text{toad})* \\
P(\text{in} \mid \text{toad})* \\
P(\text{a} \mid \text{in})* \\
P(\text{hole} \mid \text{a})
\end{gathered}
$$

- Language models: usually Markov models of order 2-4 (3-5-grams).

- How do we determine the order of our Markov models?
- Is the Markov assumption a reasonable assumption?
    - Be nonparametric about Markov order...

Non-Markov Models for Language and Text

- Model the conditional probabilities of each possible word occurring after each possible context.
- Use hierarchical Pitman-Yor process prior to share information across all contexts.
- Hierarchy is infinitely deep.
- *Sequence memoizer.*



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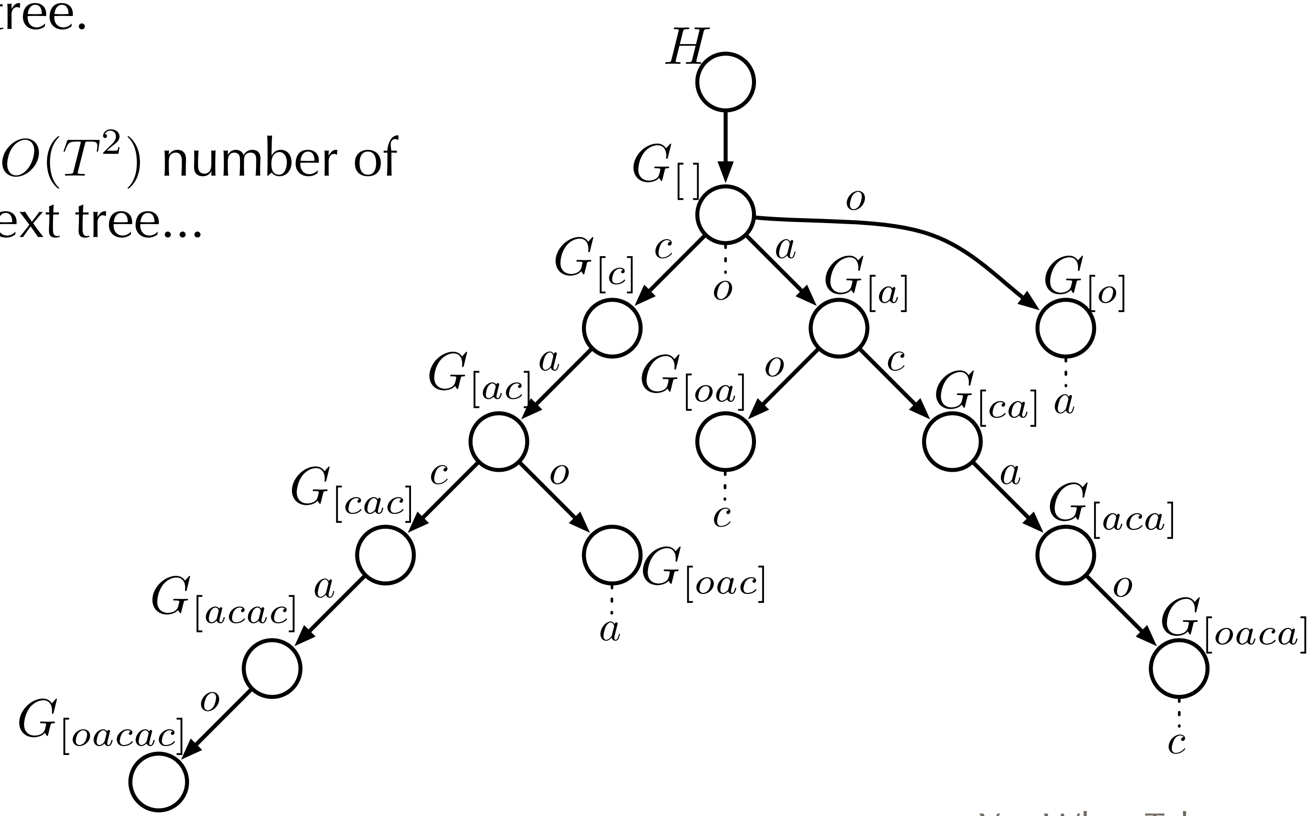
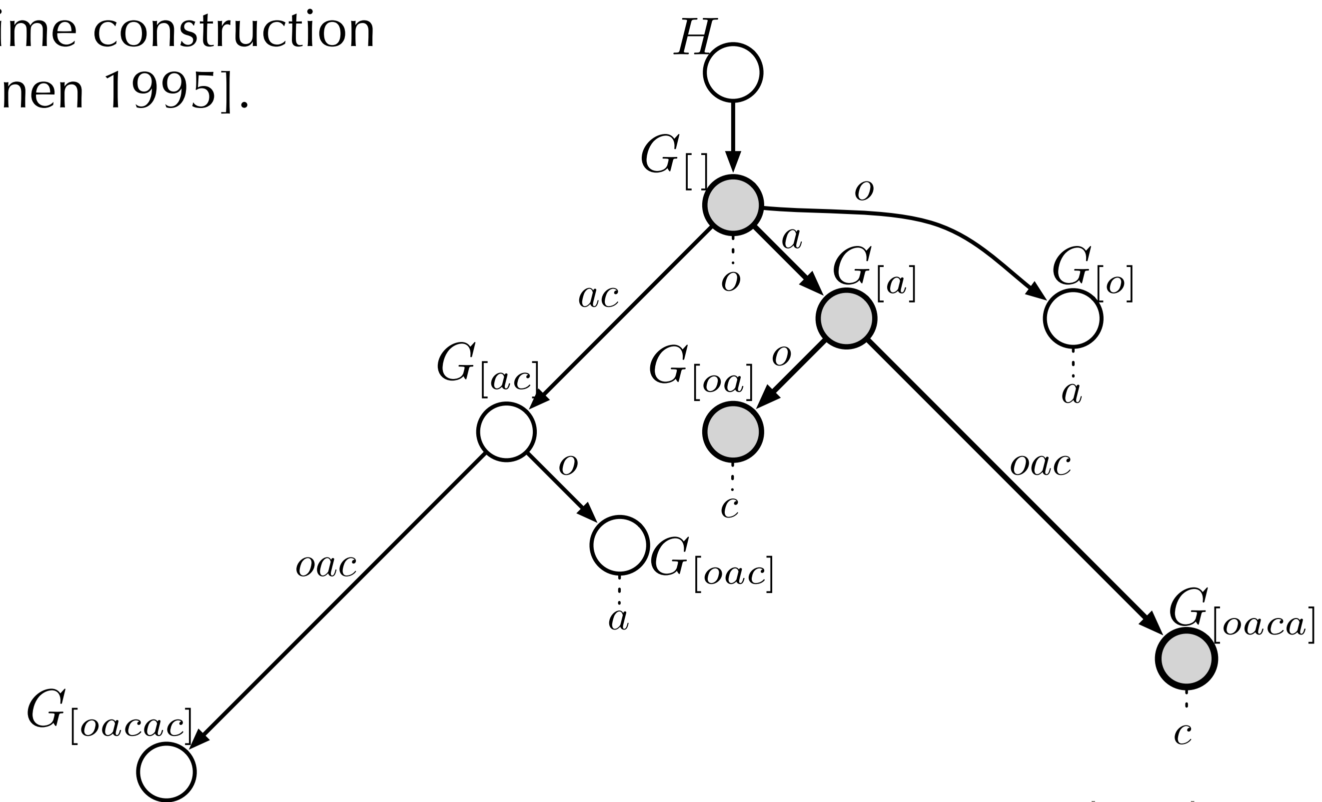
Model Size: Infinite -> O(T^2)

- The sequence memoizer model is very large (actually, infinite).
- Given a training sequence (e.g.: o,a,c,a,c), most of the model can be integrated out, leaving a finite number of nodes in context tree.
- But there are still $O(T^2)$ number of nodes in the context tree...



Model Size: Infinite -> O(T^2) -> 2T

- Idea: integrate out non-branching, non-leaf nodes of the context tree.
- Resulting tree has at most *2T* nodes.
- There are linear time construction algorithms [Ukkonen 1995].



Closure under Marginalization

- In marginalizing out non-branching interior nodes, need to ensure that resulting conditional distributions are still tractable.

$$
\begin{tikzcd}
G_{[a]} \arrow[d, "\mathrm{PY}(\theta_2, d_2, G_{[a]})"] & G_{[a]} \arrow[dd, "?"] \\
G_{[ca]} \arrow[d, "\mathrm{PY}(\theta_3, d_3, G_{[ca]})"] & \\
G_{[aca]} & G_{[aca]}
\end{tikzcd}
$$

- E.g.: If each conditional is Dirichlet, resulting conditional is not of known analytic form.

Closure under Marginalization

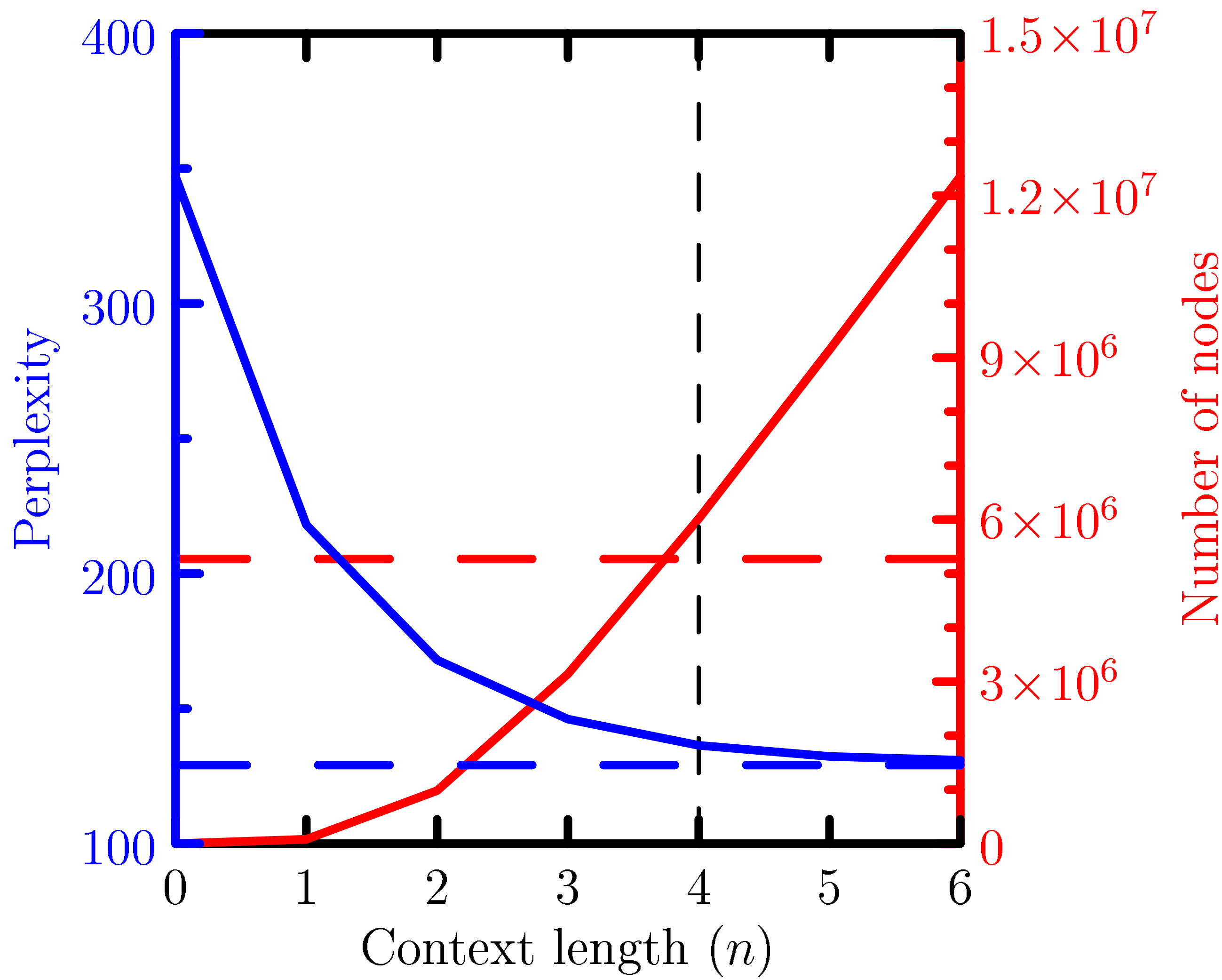
- In marginalizing out non-branching interior nodes, need to ensure that resulting conditional distributions are still tractable.

$$\begin{tikzcd} G_{[a]} \arrow[d, "\mathrm{PY}(\theta_2, d_2, G_{[a]})"] \\ G_{[ca]} \arrow[d, "\mathrm{PY}(\theta_2 d_3, d_3, G_{[ca]})"] \\ G_{[aca]} \end{tikzcd} \qquad \begin{tikzcd} G_{[a]} \arrow[dd, "\mathrm{PY}(\theta_2 d_3, d_2 d_3, G_{[a]})"] \\ \\ G_{[aca]} \end{tikzcd}$$

- For certain parameter settings, Pitman-Yor processes are closed under marginalization!

[Pitman 1999, Ho, James and Lau 2006]

Comparison to Finite Order HPYLM



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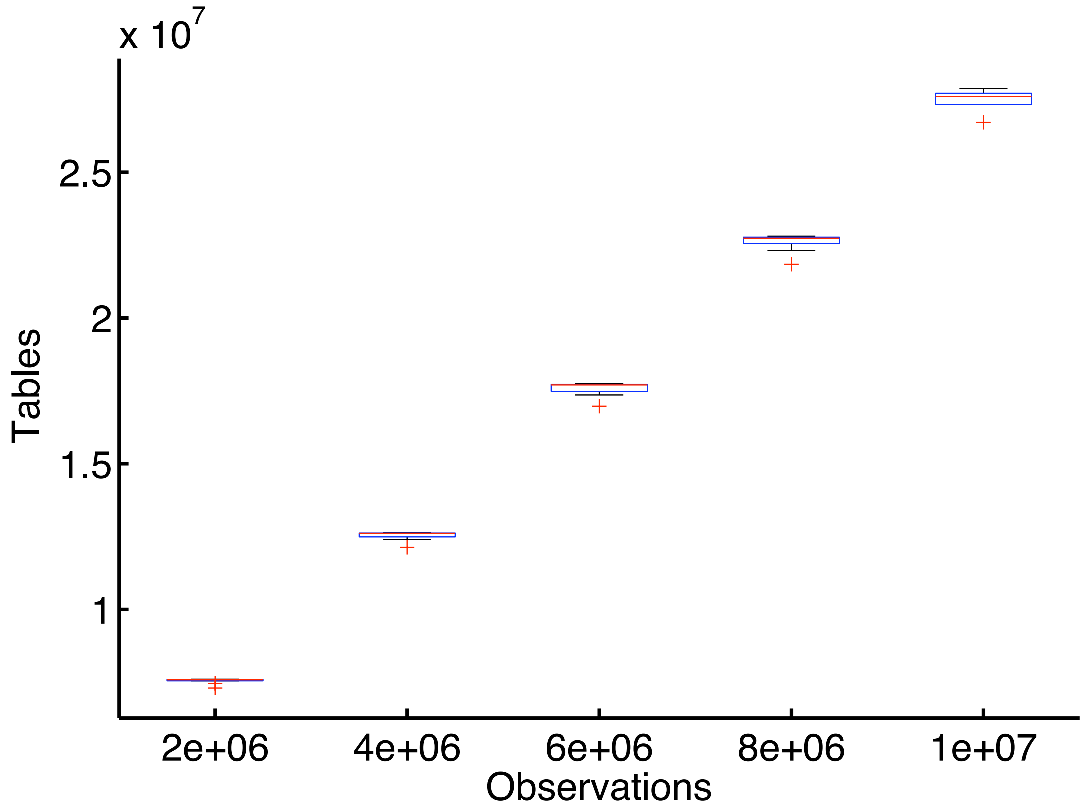
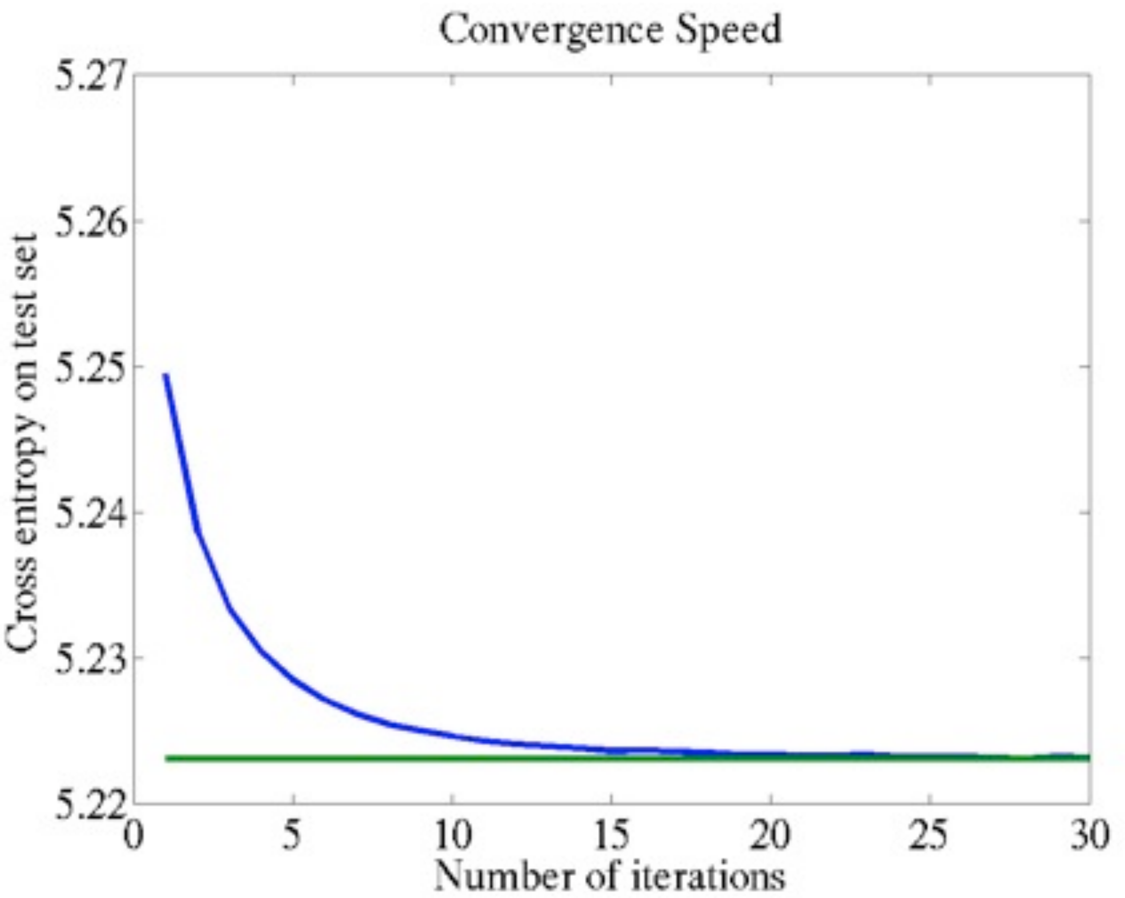
Inference using Gibbs Sampling

- Gibbs sampling in Chinese restaurant process representation of Pitman-Yor processes.



Inference

- Very efficient: posterior is unimodal and variables are weakly coupled.



Online Algorithms for Compression

- Possible to construct context tree in online fashion as sequence of training symbols observed one at a time.
- Use particle filtering or interpolated Kneser-Ney approximation for inference.
- Linear sized storage and average computation time, but quadratic computation time in worse case.
- Online construction, inference and prediction of next symbol necessary for use in text compression using entropic coding.

Compression

Model	Average bits / byte
gzip	2.61
bzip2	2.11
CTW	1.99
PPM	1.93
Sequence Memoizer	1.89

Calgary corpus
SM inference: particle filter
PPM: Prediction by Partial Matching
CTW: Context Tree Weigting
Online inference, entropic coding.

Conclusions

- Probabilistic models of sequence models without making Markov assumptions with efficient construction and inference algorithms.
- State-of-the-art text compression and language modelling results.
- Hierarchical Bayesian modelling leads to improved performance.
- Pitman-Yor processes allow us to encode prior knowledge about power-law properties, leading to improved performance.
- Extensions to the probabilisitic models straightforward.

Related Works

- Text compression: Prediction by Partial Matching [Cleary & Witten 1984], Context Tree Weighting [Willems et al 1995]...
- Language model smoothing algorithms [Chen & Goodman 1998, Kneser & Ney 1995].
- Variable length/order/memory Markov models [Ron et al 1996, Buhlmann & Wyner 1999, Begleiter et al 2004...].
- Hierarchical Bayesian nonparametric models [Teh & Jordan 2010].

Publications

- **A Hierarchical Bayesian Language Model based on Pitman-Yor Processes**.
  Y.W. Teh. Coling/ACL 2006.
- **A Bayesian Interpretation of Interpolated Kneser-Ney**.
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- **Hierarchical Bayesian Nonparametric Models with Applications**.
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Thank You!

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