1. Consider the causal Bayesian network with variables A, B, C, D, E, F determined by A, B, C being mutually independent and binary with values $\{-1, 1\}$ and $P(A = 1) = P(B = 1) = P(C = 1) = 1/2$ and $D = AB$, $E = BC$, $F = DE$.
   (a) Draw the graph of the associated Bayesian network
   (b) Find $P(C = 1 \mid A = -1)$.
   (c) Find $P(E = 1 \mid F = 1, B = -1)$.
   (d) Find the intervention probability $P(E = 1 \mid F \leftarrow 1, B \leftarrow -1)$.

2. Let X_1, X_2, X_3, X_4, X_5 be independent with $X_i \sim \mathcal{N}(0, 1)$. Define recursively the structural equation system

   $$Y_1 \leftarrow X_1, \quad Y_2 \leftarrow X_2 + Y_1, \quad Y_3 \leftarrow X_3 + Y_2, \quad Y_4 \leftarrow X_4 + Y_2 + Y_3, \quad Y_5 \leftarrow X_5 + Y_1 + Y_4$$

   and assume intervention in the system is made by replacement, so the associated Bayesian network is causal
   (a) Draw the causal DAG associated with this system
   (b) Find the concentration matrix $K = \Sigma^{-1}$ of Y.
   (c) Construct the dependence graph of Y;
   (d) Find the conditional distribution of Y_5 given $Y_3 = 0, Y_1 = 0$.
   (e) Find the intervention distribution of Y_5 given $Y_3 \leftarrow 0, Y_1 \leftarrow 0$.

3. Let $\mathcal{A} = \mathcal{C}$ be the cliques of a chordal graph $\mathcal{G} = (V, E)$. For $A \subseteq V$ let $H(A)$ denote the entropy of X_A.

   Show that
   $$H(V) = \sum_{C \in \mathcal{C}} H(C) - \sum_{S \in \mathcal{S}} \nu(S) H(S)$$
   where $\mathcal{S}$ are the minimal complete separators of $\mathcal{G}$ and $\nu(S)$ the number of times that the set S appears as an intersection between neighbouring cliques in a junction tree for $\mathcal{A}$.

4. Consider the following directed acyclic graphs, and in each case, list all DAGs in their Markov equivalence class and verify in every single case whether they are Markov equivalent to an undirected graph.
   (a) $1 \to 2, 3 \to 2, 2 \to 4, 4 \to 5, 2 \to 5$;
   (b) $1 \to 2, 2 \to 3, 2 \to 4, 4 \to 5, 6 \to 5$;
   (c) $1 \to 2, 2 \to 3, 2 \to 4, 4 \to 5, 5 \to 6$;