1. Consider the DAG $\mathcal{D}$ with arrows $A \to C, B \to C, B \to D, C \to E, D \to F, E \to G, E \to H, F \to G, G \to J, I \to J$.

   (a) Find the moral graph $\mathcal{D}^m$ of $\mathcal{D}$;

   (b) Find a *minimal chordal cover* $\mathcal{G}$ of $\mathcal{D}^m$, i.e. a chordal graph $\mathcal{G} \supset \mathcal{D}^m$ with the property that removal of any edge in $\mathcal{G}$ which is not an edge in $\mathcal{D}^m$ will not be chordal;

   (c) Arrange the cliques of $\mathcal{G}$ in a junction tree;

   (d) For a specification of all conditional distributions $p_{v \mid \mathrm{pa}(v)}, v \in V$, allocate appropriate potentials to the junction tree to prepare for probability propagation.

2. Consider random variables $X_1, \ldots, X_6$ taking values in $\{-1, 1\}$ and having distribution P with joint probability mass function determined as

$$p(x) \propto \exp\{\theta(x_1x_2 + x_2x_3 + x_2x_5 + x_3x_4 + x_3x_5 + x_3x_6)\},$$

   where $\theta \neq 0$.

   (a) Find the dependence graph of P and identify its cliques;

   (b) Set up an appropriate junction tree for probability propagation;

   (c) Allocate potentials to cliques;

   (d) Calculate $P(X_6 = 1 \,|\, X_1 = 1, X_4 = 1)$ by probability propagation.

3. Consider a Gaussian distribution $\mathcal{N}_4(0, \Sigma)$ with

$$K = \Sigma^{-1} = \begin{pmatrix} 5 & 1 & 4 & 4 \\ 1 & 12 & 2 & 5 \\ 4 & 2 & 14 & 2 \\ 4 & 5 & 2 & 8 \end{pmatrix}.$$

   (a) What is the marginal distribution of X_1?

   (b) Find the conditional distribution of (X_2, X_3) given $(X_1 = 0, X_4 = 1)$;

   (c) Find the conditional distribution of X_4 given $X_2 = 0$.