1. Consider the following generating class over $V = \{a, b, c, d, e, f, g, h\}$:

$$\mathcal{A} = \{\{a, c, e\}, \{b, c\}, \{b, h\}, \{d, e\}, \{d, f, h\}, \{d, g, h\}, \{f, g\}\}.$$

   (a) Find the dependence graph $G(\mathcal{A})$;

   (b) Find the factor graph $F(\mathcal{A})$;

   (c) Find the cliques of $G(\mathcal{A})$;

   (d) Show $\mathcal{A}$ is not conformal;

   (e) which of the following three statements are implied by $\mathcal{A}$?

$$\{a, b, c\} \perp\!\!\!\perp \{d, f, g\} \mid \{e, h\}, \quad \{a, b, c\} \perp\!\!\!\perp \{d, f, g, h\} \mid \{e\}, \quad d \perp\!\!\!\perp h \mid \{f, g\}.$$

2. Prove the *information inequality:* For non-negative numbers $a(x)_{x \in \mathcal{X}}$ and $b(x)_{x \in \mathcal{X}}$ with $\sum_x a(x) = \sum_x b(x)$ it holds that

$$\sum_{x \in \mathcal{X}} a(x) \log b(x) \leq \sum_{x \in \mathcal{X}} a(x) \log a(x)$$

   where the inequality is strict unless $a(x) = b(x)$ for all $x \in \mathcal{X}$. For the expressions to make sense we use the convention that $0 \log 0 = 0$.

   Hint: Show first the inequality $\log y \leq y - 1$.

3. The *entropy* $H(X)$ of a discrete random variable X is

$$H(X) = \mathbf{E}\{-\log f(X)\} = \sum_{x \in \mathcal{X}} -f(x) \log f(x).$$

   (a) Show that $H(X) \geq 0$;

   (b) Show that the entropy of the uniform distribution with $f(x) = 1/|\mathcal{X}|$ is $H(X) = \log |\mathcal{X}|$.

   (c) Show that the uniform distribution has maximal entropy, i.e. that

$$H(X) \leq \log |\mathcal{X}|.$$

4. Consider a three-way contingency table with variables A, B, C and cell counts

		C		
A	B	0	1	total
1	1	3	5	8
1	0	12	10	22
0	1	10	10	20
0	0	1	4	5

   Perform one cycle of the IPS algorithm for the hierarchical log-linear model with generator $\{\{A, B\}, \{B, C\}, \{A, C\}\}$.

5. Consider a generating class $\mathcal{A} = \{a, b\}$ with only two elements. Show that the IPS algorithm converges after a single cycle.