1. Let $X = (X_1, \dots, X_n)$ be a sample from the *Weibull* distribution with individual densities

$$f(x;\theta) = \theta x^{\theta-1} e^{-x^\theta} \text{ for } x > 0$$

where $\theta > 0$ is unknown.

   (a) Find the score statistic and the likelihood equation;

   (b) Show that if the likelihood equation has a solution, it must be the MLE;

   (c) Find an iterative method for solving the likelihood equation;

   (d) Find the LRT for the null hypothesis $H_0 : \theta = 1$ vs. the alternative $H_A : \theta = 2$.

   (e) Find the maximized LRT for the null hypothesis $H_0 : \theta = 1$ vs. the alternative $H_A : \theta \neq 1$;

   (f) Find the score test for the null hypothesis $H_0 : \theta = 1$ vs. the alternative $H_A : \theta \neq 1$;

   (g) Discuss alternative large sample tests for the null hypothesis $H_0 : \theta = 1$ vs. the alternative $H_A : \theta \neq 1$.

2. Let $X = (X_1, \dots, X_k)$ be independent and Poisson distributed with parameter λ_i as

$$f(x_i;\lambda_i) = \frac{\lambda_i^{x_i}}{x_i!} e^{-\lambda_i}, x_i = 0, 1, \dots$$

where $\lambda_i > 0$ are unknown.

   (a) Show that the MLE of $\lambda = (\lambda_1, \dots, \lambda_k)$ is

$$\hat{\lambda} = (\hat{\lambda}_1, \dots, \hat{\lambda}_k) = (X_1, \dots, X_k).$$

   (b) Consider the composite hypothesis $H_0 : \lambda_i = \alpha d_i, i = 1, \dots, k$, where d_i are known numbers and $\alpha > 0$ is unknown. Show that the MLE $\hat{\hat{\lambda}}$ under this hypothesis is

$$\hat{\hat{\lambda}} = \left\{ \frac{\sum X_i}{\sum d_i} d_1, \dots, \frac{\sum X_i}{\sum d_i} d_k \right\}.$$

   (c) Find the maximized LRT statistic under this hypothesis and indicate its asymptotic distribution;

   (d) Find the Wald test statistic for H_0;

   (e) Find the χ^2 test statistic for H_0.

   (f) Consider next the hypothesis $H_1 : \lambda_i = d_i, i = 1, \dots, k$. Find the maximized likelihood ratio test for this hypothesis under the assumption that H_0 is true.