- 1. Let X_1, \ldots, X_n be independent and identically normally distributed as $\mathcal{N}(\mu, \mu^2)$ with $\mu > 0$ being unknown. Thus the observations have constant coefficient of variation $\sqrt{\mathbf{V}(X)/\mathbf{E}(X)}$.
 - (a) Show that $T = (U, V) = (\sum_{i} X_i/n, \sum_{i} X_i^2/n)$ is minimal sufficient;
 - (b) Show that $A = U/\sqrt{V}$ is ancillary;
 - (c) Show that (U, V) is not complete;
 - (d) Discuss inference about μ .
- 2. Let $X = (X_1, \ldots, X_n)$ be a sample of size *n* from the uniform distribution on the interval $(\psi \lambda, \psi + \lambda)$:

$$f(x; \theta) = \frac{1}{2\lambda}$$
 for $\psi - \lambda < x < \psi + \lambda$ and 0 otherwise,

where $\theta = (\psi, \lambda)$ with $-\infty < \psi < \infty$ and $\lambda > 0$ both unknown.

- (a) Show that $(X^{(1)}, X^{(n)})$ is minimal sufficient;
- (b) Show that the maximum likelihood estimator of θ is

$$\hat{\psi} = (X_{(1)} + X_{(n)})/2;, \quad \hat{\lambda} = (X_{(n)} - X_{(1)})/2;$$

- (c) Show that the distribution of $C = (X_{(n)} X_{(1)})/2$ does not depend on ψ ;
- (d) Let $(U, V) = ((X_{(1)} \psi)/\lambda, (X_{(n)} \psi)/\lambda)$ and show that the joint density of (U, V) is

$$f(u,v) = \begin{cases} n(n-1)(v-u)^{n-2}/2^n & \text{if } -1 < u < v < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Hint: Find first $P(U > u, V \leq v)$ *.*

- (e) Find the conditional density of $\hat{\psi} = (X_{(1)} + X_{(n)})/2$, given C = c.
- (f) Discuss conditional inference for ψ in the case where $\lambda = 1$ is known.
- 3. Consider X and Y as independent Poisson random variables with $\mathbf{E}(X) = \gamma$ and $\mathbf{E}(Y) = \delta$, where $\gamma, \delta > 0$ are both unknown. We wish to find a similar test for equality of the two Poisson rates, i.e. the hypothesis $H_0: \gamma = \delta$.
 - (a) Show that under the null hypothesis, C = X + Y is sufficient and complete;
 - (b) Find the conditional distribution of X given C = c;
 - (c) Describe a similar test for the hypothesis H_0 .

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