1. Consider samples x and y and let M_1 denote the model considering X and Y independent with binomial distributions

$$P(X = x) = \binom{n}{x} \theta^x (1-\theta)^{n-x}, \quad P(Y = y) = \binom{n}{y} \eta^y (1-\eta)^{n-y},$$

where $0 \leq \theta, \eta \leq 1$ are both unknown.

Let similarly M_2 denote the model where $\theta = \eta$ and everything else is as for M_1.

   (a) Calculate the maximized log-likelihood ratio statistic $-2 \log \Lambda$ for comparing the two models;

   (b) For uniform prior distributions on θ, η, calculate the Bayes factor for comparing M_1 to M_2;

   (c) Find the BIC approximation to the Bayes factor, with or without including all terms, and comment on its accuracy;

   (d) Find the AIC for the two models

   (e) Compare the model determination procedures using the three criteria for large values of n.

2. Consider regression data $(x, y) = ((x_1, y_1), \ldots, (x_n, y_n))$ with x considered fixed and the responses Y_i being independent with

$$Y_i \sim \mathcal{N}\{\mu_k(x_i), \phi\},$$

where μ_k is determined by model M_k as

$$M_1 : \mu_1(x_i) = \alpha; \quad M_2 : \mu_2(x_i) = \beta x_i; \quad M_3 : \mu_3(x_i) = \gamma x_i^2.$$

   (a) For (improper) prior distributions $\pi_i(\eta, \phi) \propto \phi^{-1}$, where either $\eta = \alpha$, $\eta = \beta$, or $\eta = \gamma$, calculate expressions for the Bayes factor for comparing any pair of these models;

   (b) Find expressions for the BIC approximation to these models;

   (c) Find expressions for the AIC for these models;

   (d) Find expressions for Mallows' C_p.

   (e) Compare the model determination procedures.

   (f) What happens if also models $M_{uv} : \mu_{uv}(x_i) = \mu_u(x_i) + \mu_v(x_i)$ etc. are considered?