1. Let $X_1, \ldots, X_n$ be independent and identically normally distributed as $\mathcal{N}(\mu, \mu^2)$ with $\mu > 0$ being unknown. Thus the observations have *constant coefficient of variation* $\sqrt{\mathbf{V}(X)/\mathbf{E}(X)}$.

   (a) Show that $T = (U, V) = (\sum_i X_i/n, \sum_i X_i^2/n)$ is minimal sufficient;

   (b) Show that $A = U/\sqrt{V}$ is ancillary;

   (c) Show that (U, V) is not complete;

   (d) Discuss inference about μ.

2. Let $X = (X_1, \ldots, X_n)$ be a sample of size n from the uniform distribution on the interval $(\psi - \lambda, \psi + \lambda)$:

$$f(x; \theta) = \frac{1}{2\lambda} \text{ for } \psi - \lambda < x < \psi + \lambda \text{ and 0 otherwise,}$$

   where $\theta = (\psi, \lambda)$ with $-\infty < \psi < \infty$ and $\lambda > 0$ both unknown.

   (a) Show that $(X^{(1)}, X^{(n)})$ is minimal sufficient;

   (b) Show that the maximum likelihood estimator of θ is

$$\hat{\psi} = (X_{(1)} + X_{(n)})/2; , \quad \hat{\lambda} = (X_{(n)} - X_{(1)})/2;$$

   (c) Show that the distribution of $C = (X_{(n)} - X_{(1)})/2$ does not depend on ψ;

   (d) Let $(U, V) = ((X_{(1)} - \psi)/\lambda, (X_{(n)} - \psi)/\lambda)$ and show that the joint density of (U, V) is

$$f(u, v) = \begin{cases} n(n-1)(v-u)^{n-2}/2^n & \text{if } -1 < u < v < 1 \\ 0 & \text{otherwise.} \end{cases}$$

   Hint: Find first $P(U > u, V \leq v)$.

   (e) Find the conditional density of $\hat{\psi} = (X_{(1)} + X_{(n)})/2$, given $C = c$.

   (f) Discuss conditional inference for ψ in the case where $\lambda = 1$ is known.

3. Consider X and Y as independent Poisson random variables with $\mathbf{E}(X) = \gamma$ and $\mathbf{E}(Y) = \delta$, where $\gamma, \delta > 0$ are both unknown. We wish to find a similar test for equality of the two Poisson rates, i.e. the hypothesis $H_0 : \gamma = \delta$.

   (a) Show that under the null hypothesis, $C = X + Y$ is sufficient and complete;

   (b) Find the conditional distribution of X given $C = c$;

   (c) Describe a similar test for the hypothesis H_0.