An Introduction to Stochastic Actor Oriented Models

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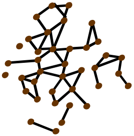
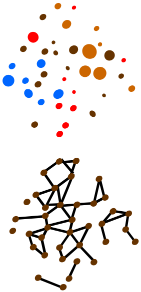
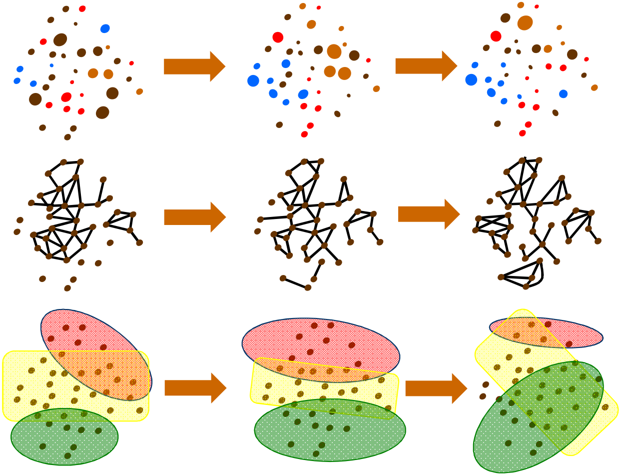
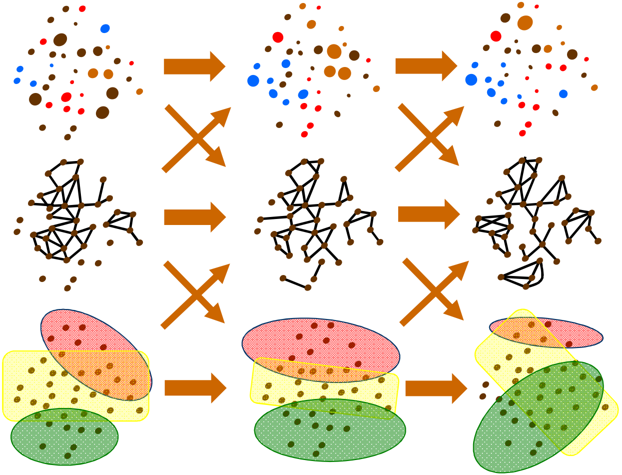
Statistical Models for Social Networks, June 2010









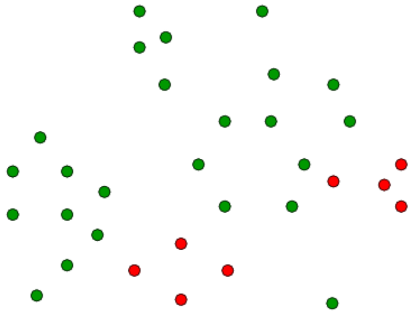
What type of data do we want to explain

Example: Studies Gerhard van de Bunt

Longitudinal study: panel design.

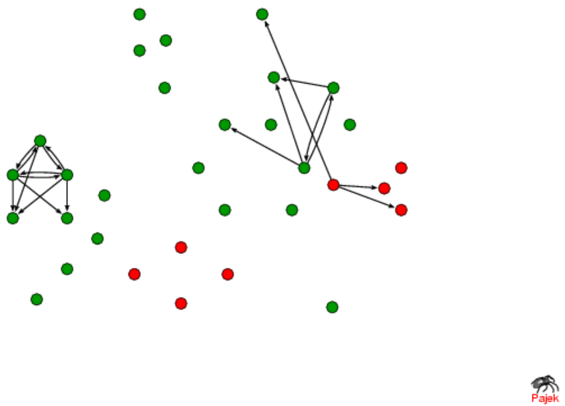
- Study of 32 freshman university students,
  7 waves in 1 year.
  See van de Bunt, van Duijn, & Snijders,
  Computational & Mathematical Organization Theory,
  5 (1999), 167 – 192.

This data set can be pictured by the following graphs
(arrow stands for 'best friends').



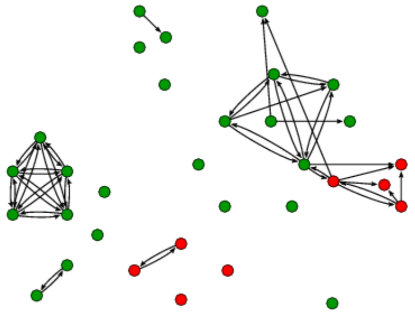
Friendship network time 1.

Average degree 0.0; missing fraction 0.0.



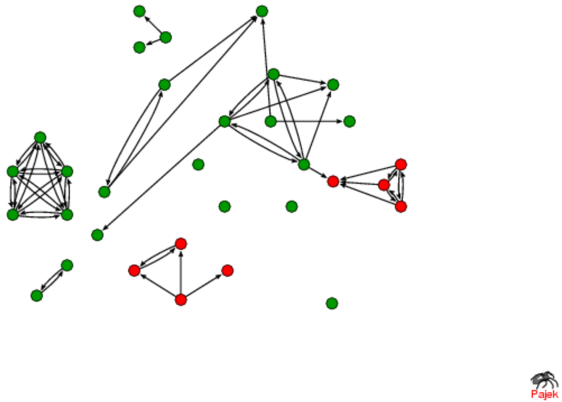
Friendship network time 2.

Average degree 0.7; missing fraction 0.06.



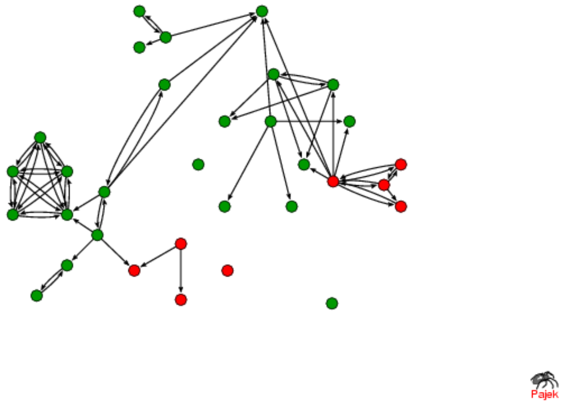
Friendship network time 3.

Average degree 1.7; missing fraction 0.09.



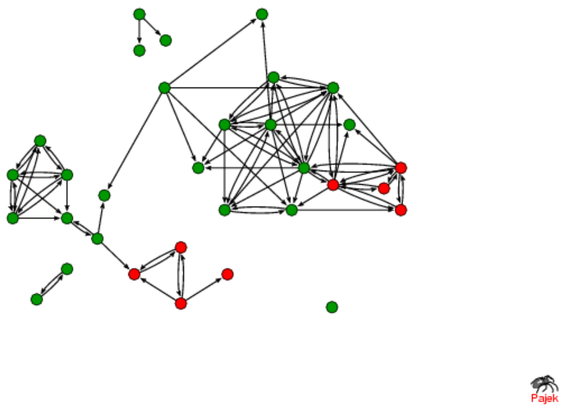
Friendship network time 4.

Average degree 2.1; missing fraction 0.16.



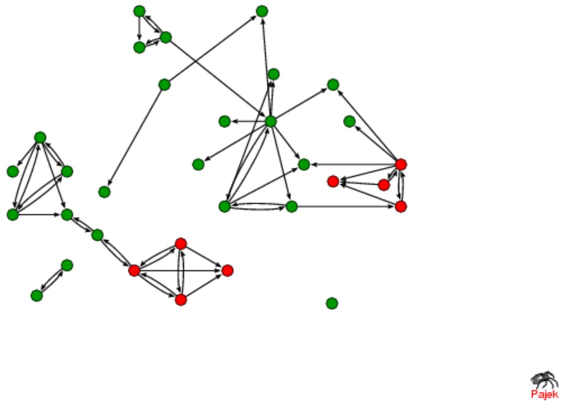
Friendship network time 5.

Average degree 2.5; missing fraction 0.19.



Friendship network time 6.

Average degree 2.9; missing fraction 0.04.



Friendship network time 7.

Average degree 2.3; missing fraction 0.22.

What type of data do we want to explain

Data represented as adjacency matrices
where elements change

$$x(t_0) = \begin{pmatrix} . & 0 & 0 & 0 & 1 \\ 1 & . & 0 & 0 & 0 \\ 1 & 1 & . & 0 & 0 \\ 0 & 0 & 0 & . & 0 \\ 0 & 0 & 1 & 1 & . \end{pmatrix}$$

What type of data do we want to explain

Data represented as adjacency matrices
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$$x(t_1) = \begin{pmatrix} . & 1 & 0 & 0 & 1 \\ 1 & . & 0 & 0 & 0 \\ 1 & 0 & . & 0 & 0 \\ 0 & 0 & 0 & . & 0 \\ 1 & 0 & 1 & 1 & . \end{pmatrix}$$

What type of data do we want to explain

Data represented as adjacency matrices
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$$x(t_2) = \begin{pmatrix} . & 1 & 0 & 1 & 1 \\ 1 & . & 0 & 0 & 1 \\ 1 & 1 & . & 0 & 0 \\ 0 & 0 & 0 & . & 0 \\ 1 & 0 & 0 & 1 & . \end{pmatrix}$$

What type of data do we want to explain

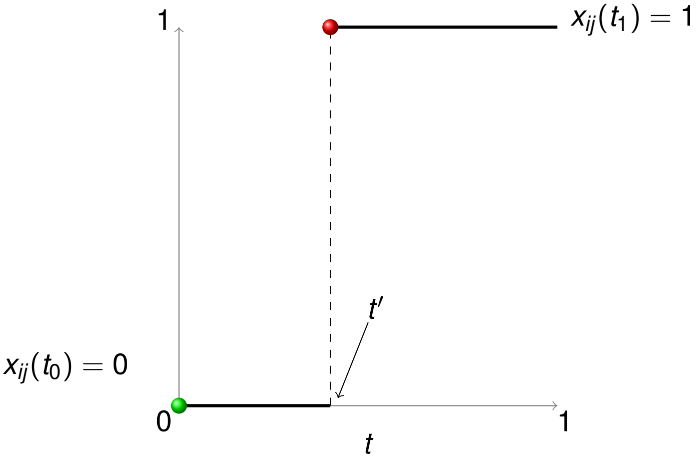
If an element x_{ij} has changed
from

$$x_{ij}(t_0) = 0$$

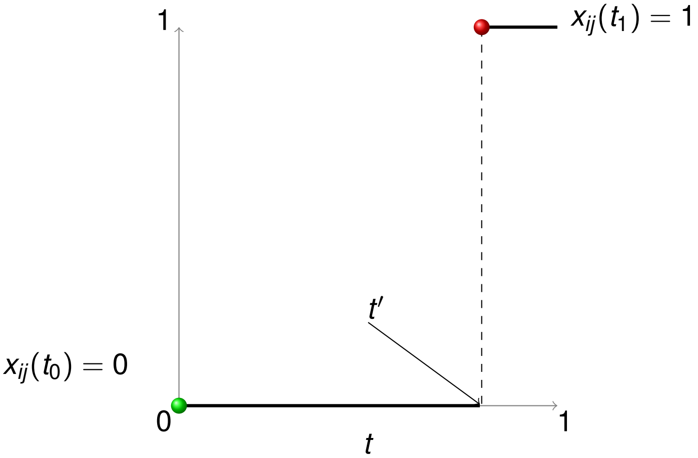
to

$$x_{ij}(t_1) = 1$$

something has changed inbetween t_0 and t_1



Change at t'



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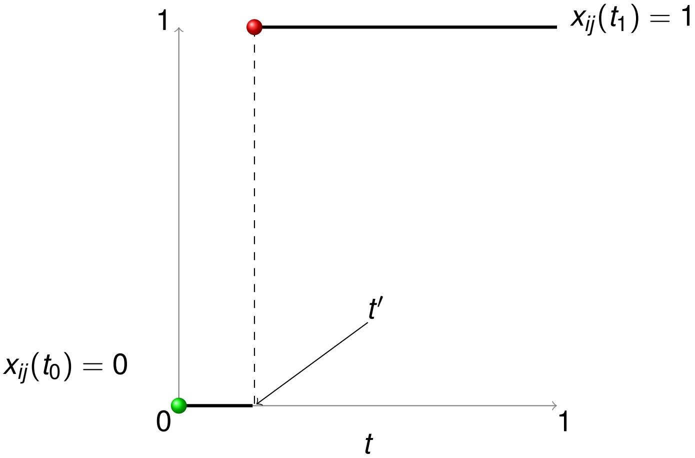
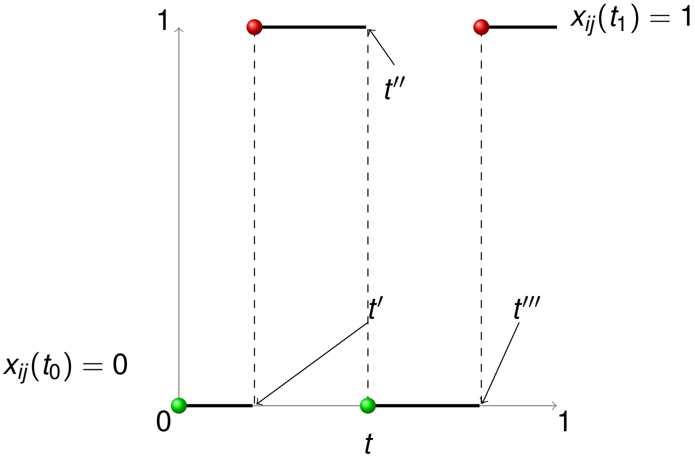
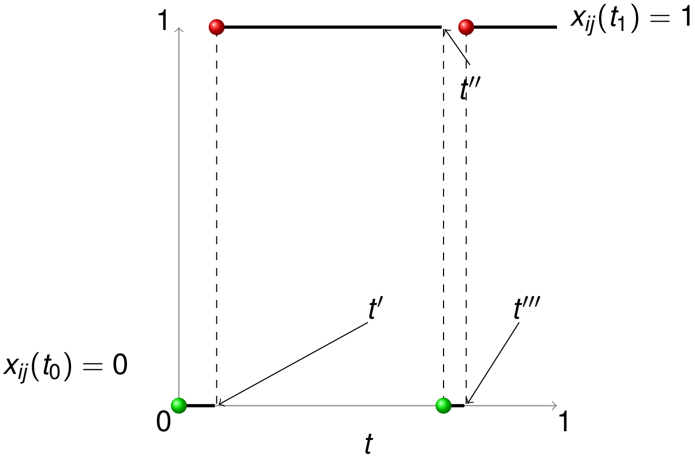


Change at t'



Change at t', t'', *and* t''



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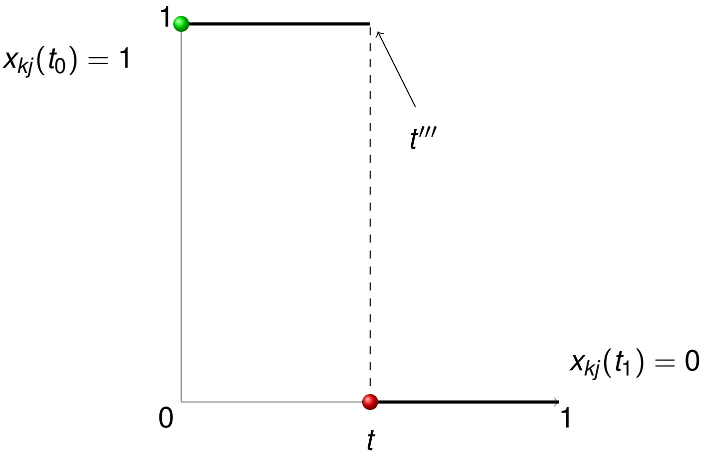
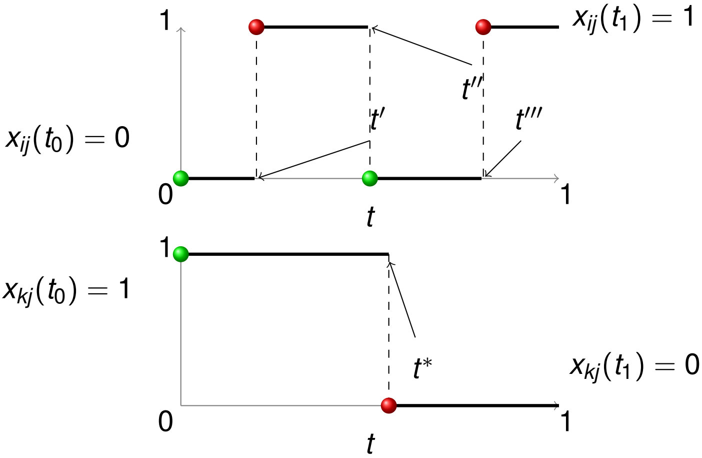
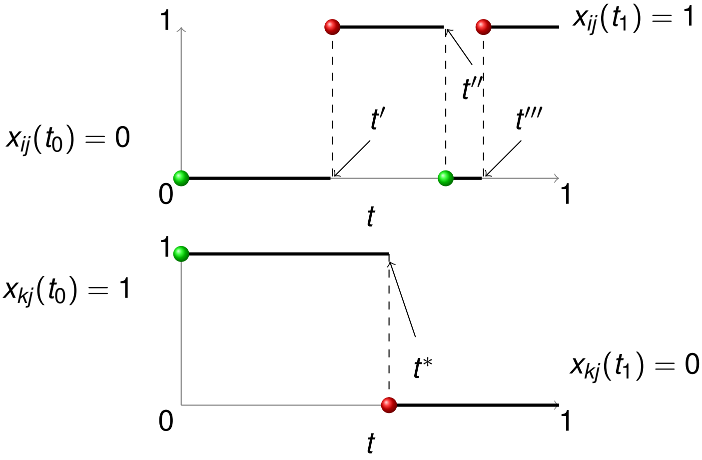


Change at t'''


1
$x_{ij}(t_1) = 1$
t''
t'
t'''
$x_{ij}(t_0) = 0$
0
1
t
1
$x_{kj}(t_0) = 1$
t^*
$x_{kj}(t_1) = 0$
0
1
t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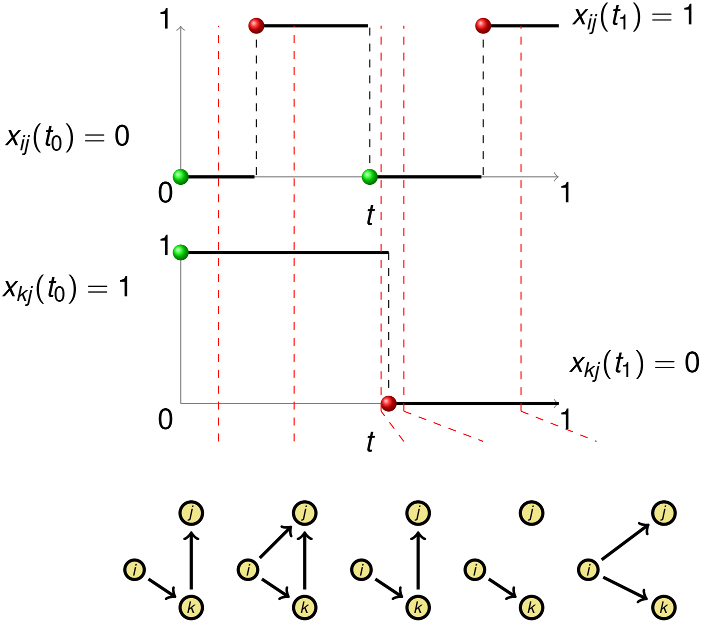
$x_{ij}(t_0) = 0$
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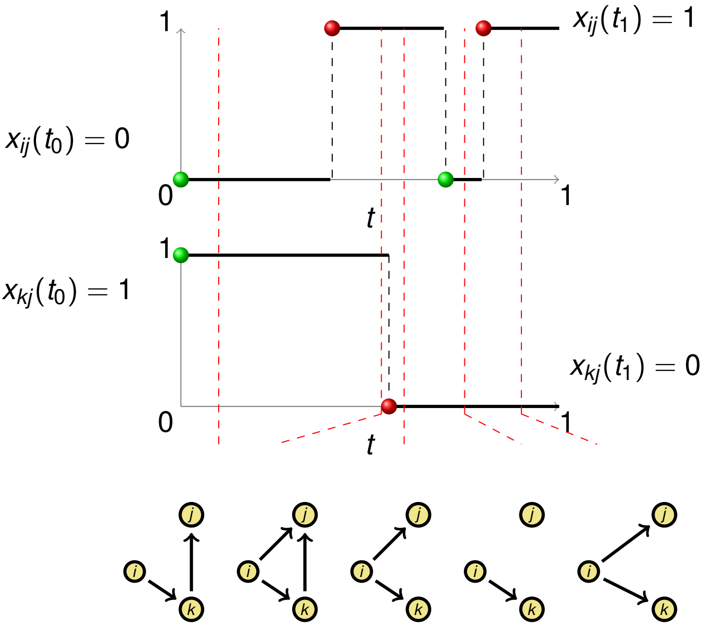
$x_{kj}(t_0) = 1$
$x_{kj}(t_1) = 0$
1
0
1
t
1
0
1
t
i
j
k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0
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i
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j
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- *Ministeps* (one-step change probabilities), and
- Holding times.

Ministeps: moves between states in Walk on 2-cube

Nodes in 2-cube

Ties between neighbouring nodes

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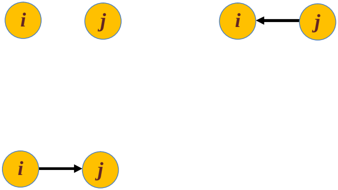

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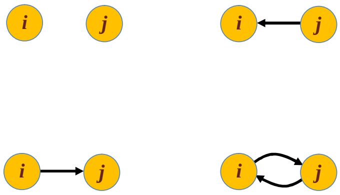
Ministeps: moves between states in Walk on 2-cube



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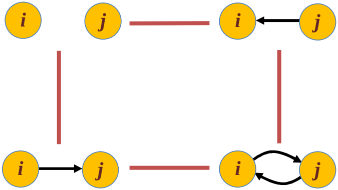

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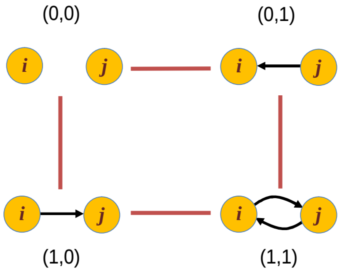
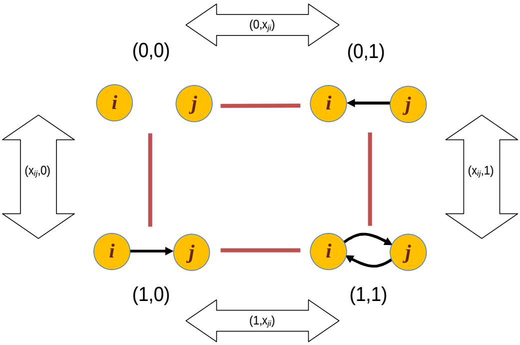


Nodes in 2-cube

Ties between neighbouring nodes correspond to difference

Ministeps: moves between states in Walk on 2-cube



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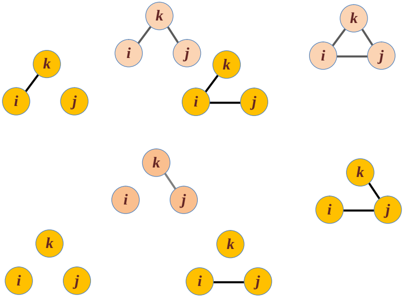
Walk on 3-cube

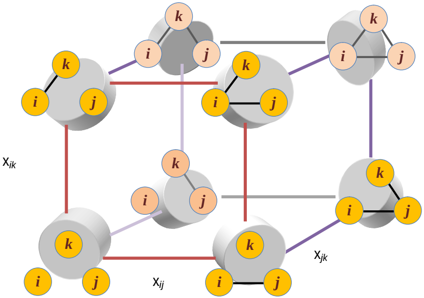


Graphs on 3 vertices

Corresponding 3-cube

Note one-edge difference correspond to coordinate

Walk on 3-cube



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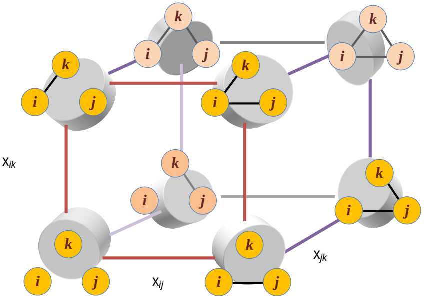
Embedded chain

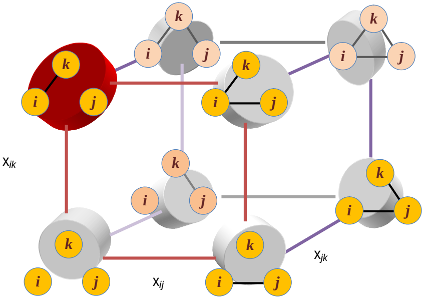


In current state x for time $\sim$exponential $(\lambda(\theta, x))$

Then jumps to neighbouring state x' (ministep)

Chosen w.p. $p(\theta, x, x')$

Embedded chain



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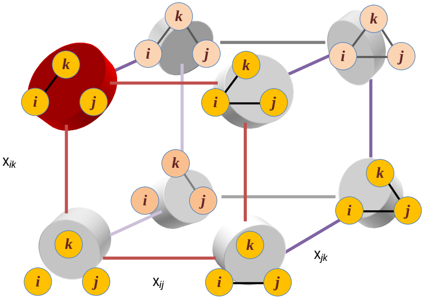

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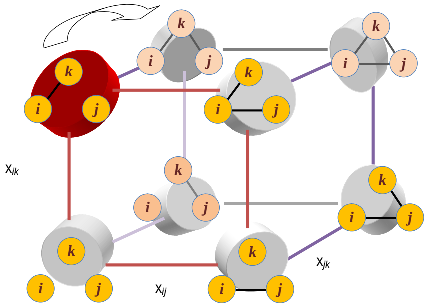

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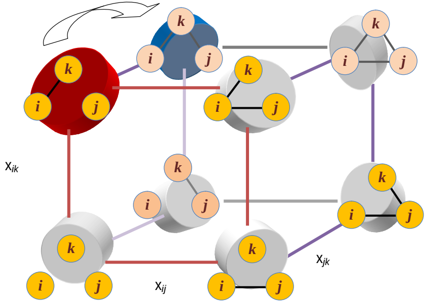

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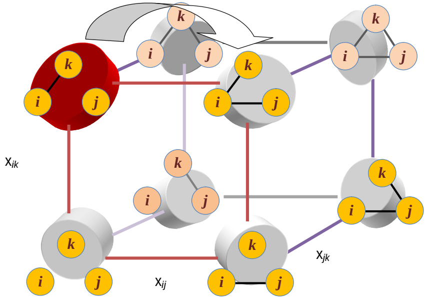

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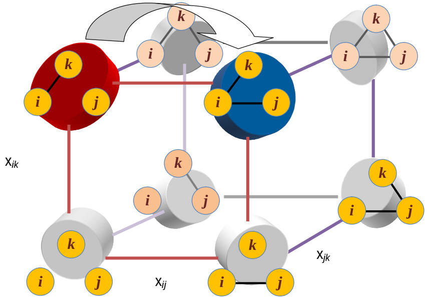

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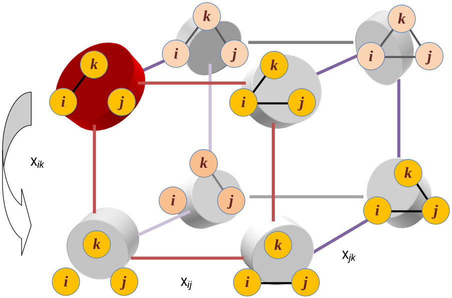

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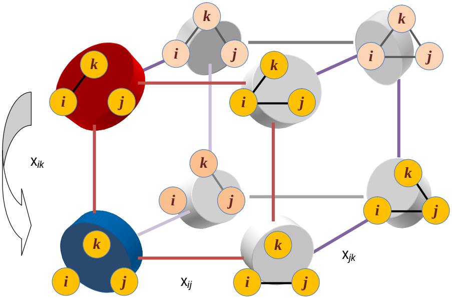

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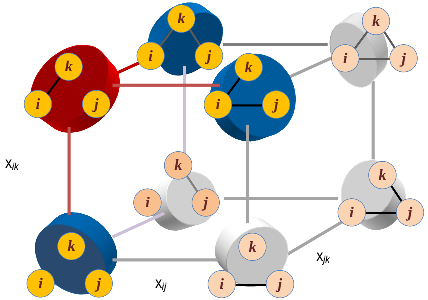

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How model the change in ties?

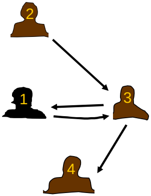
The time inbetween changes mostly a nuisance.

The central part of SAOM is modelling the *Ministeps*.

In SAOM the change of one tie-variable
is seen as the outcome of a decision of one actor
to make a change to one of her out-going ties .

Opportunity for change

$$x = \begin{array}{|c|c|c|c|} \hline - & 0 & 1 & 0 \\ \hline 0 & - & 1 & 0 \\ \hline 1 & 0 & - & 1 \\ \hline 0 & 0 & 0 & - \\ \hline \end{array}$$



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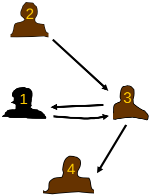
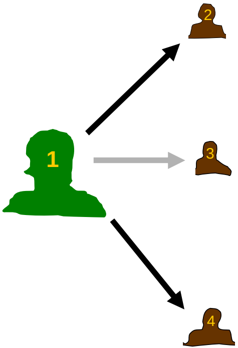
$x =$

-	0	1	0
0	-	1	0
1	0	-	1
0	0	0	-





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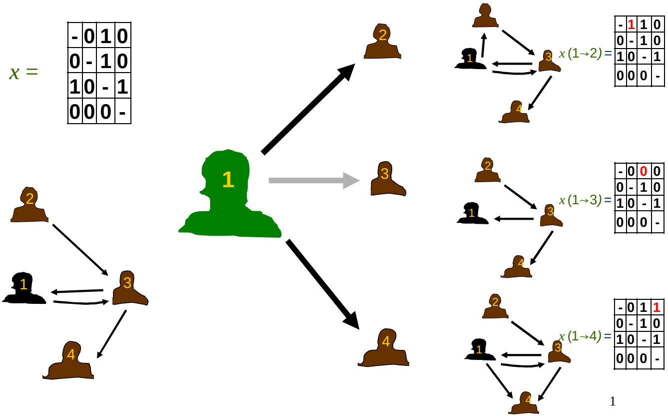

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- modelled like multinomial logistic regression
- reflects the ‘attractiveness’ of the outcome to actor who makes change
- may be regarded as result of myopic optimization.

Purpose of statistical inference:

investigate network evolution *(dependent var.)* as function of

1. structural effects (reciprocity, transitivity, etc.)
2. explanatory actor variables *(independent vars.)*
3. explanatory dyadic variables *(independent vars.)*

simultaneously.

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simultaneously.
By controlling adequately for structural effects, it is possible to test hypothesized effects of variables on network dynamics (without such control these tests would be incomplete).

The structural effects imply that the presence of ties is highly dependent on the presence of other ties.

Principles for this approach to analysis of network dynamics:

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3. use methods of statistical inference for probability models implemented as simulation models
4. for panel data: employ a continuous-time model to represent unobserved endogenous network evolution
5. condition on the first observation and do not model it: no stationarity assumption.

Notation and assumptions

1. *Actors* $i = 1, \ldots, n$ (individuals in the network),
pattern X of *ties* between them : one binary network X;
$X_{ij} = 0$, or 1 if there is no tie, or a tie, from i to j. Matrix X is *adjacency matrix* of digraph.
X_{ij} is a *tie indicator* or *tie variable*.

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3. Continuous time parameter t, observation moments $t_1, \ldots, t_M$.
4. Current state of network $X(t)$ is dynamic constraint for its own change process: Markov process.

Actor-based model:

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6. The ties have inertia: they are *states* rather than *events*.
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Actor-based model:

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6. The ties have inertia: they are *states* rather than *events*.
   At any single moment in time,
   only one variable $X_{ij}(t)$ may change.
7. Changes are modeled as
   choices by actors in their outgoing ties,
   with probabilities depending on '*objective function*'
   of the network state that would obtain after this change.

The change probabilities can (but need not)
be interpreted as arising from goal-directed behavior,
in the weak sense of myopic stochastic optimization.

Assessment of the situation is represented by
objective function, interpreted as
'that which the actors seem to strive after in the short run'.

Next to actor-driven models,
also tie-driven models are possible.

At any given moment, with a given current network structure,
the actors act independently, without coordination.
They also act one-at-a-time.

The subsequent changes ('mini-steps') generate
an endogenous dynamic context
which implies a dependence between the actors over time;
e.g., through reciprocation or transitive closure
one tie may lead to another one.

This implies strong dependence between what the actors do,
but it is completely generated by the time order:
the actors are dependent because they constitute
each other's changing environment.

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The change process is decomposed into two sub-models, formulated on the basis of the idea that the actors *i* control their outgoing ties $(X_{i1}, \ldots, X_{in})$:

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2. probabilities of changing (toggling) X_{ij}, conditional on such an opportunity for change: *objective functions*.

The distinction between rate function and objective function separates the model for *how many* changes are made from the model for *which* changes are made.

This decomposition between the timing model and the model for change can be pictured as follows:

At randomly determined moments t,
actors i have opportunity to change a tie variable X_{ij}:
mini step.
(Actors are also permitted to leave things unchanged.)
Frequency of mini steps is determined by *rate functions*.

When a mini step is taken,
the probability distribution of the result of this step
depends on the *objective function* :
higher probabilities of moving toward new states
that have higher values of the objective function.

Specification: rate function

'how fast is change / opportunity for change ?'

Rate of change of the network by actor *i* is denoted λ_i : expected frequency of changes by actor *i* between observations.

Simple specification: rate functions are constant within periods.

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Rate of change of the network by actor *i* is denoted λ_i :
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More generally, rate functions can depend on observation period $(t_{m-1},\ t_m)$,
actor covariates, and network position (degrees etc.),
through an exponential link function.

Formally, for a certain short time interval $(t, t + \epsilon)$,
the probability that this actor randomly gets an opportunity to change one of his/her outgoing ties, is given by $\epsilon\,\lambda_i$.

Specification: objective function

‘what is the direction of change?’

Specification: objective function

'what is the direction of change?'

The objective function $f_i(\beta, x)$ indicates
preferred 'directions' of change.
β is a statistical parameter, i is the actor (node), x the network.

When actor i gets an opportunity for change,
he has the possibility to change *one* outgoing tie variable X_{ij},
or leave everything unchanged.

By $x(i \rightsquigarrow j)$ is denoted the network obtained
when x_{ij} is changed ('toggled') into $1 - x_{ij}$.
Formally, $x(i \rightsquigarrow i)$ is defined to be equal to x.

Conditional on actor i being allowed to make a change, the probability that X_{ij} changes into $1 - X_{ij}$ is

$$p_{ij}(\beta, x) = \frac{\exp\big(f_i(\beta, x(i \rightsquigarrow j))\big)}{\sum_{h=1}^{n} \exp\big(f_i(\beta, x(i \rightsquigarrow h))\big)} \ ,$$

and p_{ii} is the probability of not changing anything.

Higher values of the objective function indicate the preferred direction of changes.

One way of obtaining this model specification is to suppose that actors make changes such as to optimize the objective function $f_i(\beta, x)$ plus a random disturbance that has a Gumbel distribution, like in random utility models in econometrics:

myopic stochastic optimization, multinomial logit models.

Actor i chooses the "best" j by maximizing

$$f_i\big(\beta, x(i \rightsquigarrow j)\big) \quad + \quad U_i(t, x, j)\,.$$

$$\Uparrow$$

random component

(with the formal definition $x(i \rightsquigarrow i) = x$).

For a convenient distributional assumption,
(U has type 1 extreme value = Gumbel distribution)
given that i is allowed to make a change,
the probability that i changes the tie variable to j,
or leaves the tie variables unchanged (denoted by $j = i$), is

$$p_{ij}(\beta, x) = \frac{\exp\big(f(i,j)\big)}{\sum_{h=1}^{n} \exp\big(f(i,h)\big)}$$

where

$$f(i,j) = f_i\big(\beta, x(i \rightsquigarrow j)\big)$$

and p_{ii} is the probability of not changing anything.

This is the multinomial logit form of a *random utility* model.

Objective functions will be defined as sum of:

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   that are obtained 'free' but are lost at a value
   (to allow asymmetry between creation and deletion of ties).

Evaluation function and endowment function modeled as linear combinations of theoretically argued components of preferred directions of change. The weights in the linear combination are the statistical parameters.

The focus of modeling is first on the evaluation function; then on the rate and endowment functions.

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The evaluation and endowment functions express how the dynamics of the network process depends on its current state.

Computer simulation algorithm for arbitrary rate function $\lambda_i(\alpha, \rho, x)$

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$$\lambda_+(\alpha, \rho, x) = \sum_i \lambda_i(\alpha, \rho, x) \ .$$
3. Select $i \in \{1, ..., n\}$ using probabilities
$$\frac{\lambda_i(\alpha, \rho, x)}{\lambda_+(\alpha, \rho, x)} \ .$$

4. Select $j \in \{1, ..., n\}$, $j \neq i$ using probabilities $p_{ij}(\beta, x)$.

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5. Set $t = t + S$ and $x = x(i \rightsquigarrow j)$.
6. Go to step 2
   (unless stopping criterion is satisfied).

Model specification :

Simple specification: only evaluation function;
no endowment function, periodwise constant rate function.

Evaluation function f_i reflects network effects
(endogenous) and covariate effects (exogenous).

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Evaluation function f_i reflects network effects
(endogenous) and covariate effects (exogenous).
Covariates can be actor-dependentor dyad-dependent.

Convenient definition of evaluation function is a weighted sum

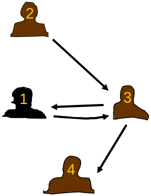
$$f_i(\beta, x) = \sum_{k=1}^{L} \beta_k \, s_{ik}(x) \,,$$

where the weights β_k are statistical parameters
indicating strength of effect $s_{ik}(x)$.

What goes into the evaluation function?

$x =$

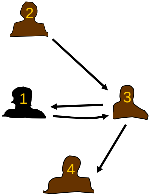
-	0	1	0
0	-	1	0
1	0	-	1
0	0	0	-

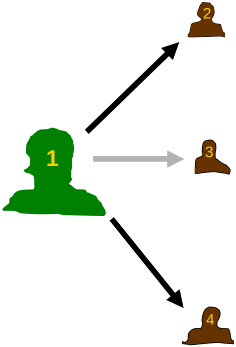


Satisfaction with structure of new state

What goes into the evaluation function?

$$x = \begin{array}{|c|c|c|c|} \hline - & 0 & 1 & 0 \\ \hline 0 & - & 1 & 0 \\ \hline 1 & 0 & - & 1 \\ \hline 0 & 0 & 0 & - \\ \hline \end{array}$$





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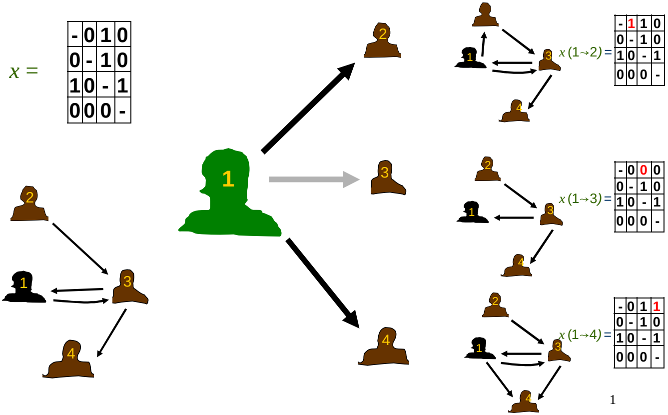


Satisfaction with structure of new state

Choose possible network effects for actor *i*, e.g.:
(others to whom actor *i* is tied are called here *i*'s 'friends')

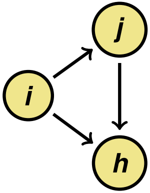
1. *out-degree effect*, controlling the density / average degree,
$s_{i1}(x) = x_{i+} = \sum_j x_{ij}$

Choose possible network effects for actor i, e.g.:
(others to whom actor i is tied are called here i's 'friends')

1. *out-degree effect*, controlling the density / average degree,
   $s_{i1}(x) = x_{i+} = \sum_j x_{ij}$
2. *reciprocity effect*, number of reciprocated ties
   $s_{i2}(x) = \sum_j x_{ij}\, x_{ji}$

Four potential effects representing network closure:

3. *transitive triplets effect*,
   number of transitive patterns in i's ties
   $(i \rightarrow j,\ j \rightarrow h,\ i \rightarrow h)$
   $s_{i3}(x) = \sum_{j,h} x_{ij}\, x_{jh}\, x_{ih}$



transitive triplet

4. *transitive ties effect*,
   number of actors j to whom i is tied indirectly
   (through at least one intermediary: $x_{ih} = x_{hj} = 1$)
   and also directly $x_{ij} = 1$),
   $s_{i4}(x) = \#\{j \mid x_{ij} = 1,\ \max_h(x_{ih}\, x_{hj}) > 0\}$

5. *indirect ties effect*,
   number of actors j to whom i is tied indirectly
   (through at least one intermediary: $x_{ih} = x_{hj} = 1$)
   but not directly $x_{ij} = 0$),
   = number of geodesic distances equal to 2,
   $s_{i5}(x) = \#\{j \mid x_{ij} = 0, \max_h(x_{ih}\, x_{hj}) > 0\}$

6. *balance* or structural equivalence,
similarity between outgoing ties of i
with outgoing ties of his friends,

$$s_{i6}(x) = \sum_{j=1}^{n} x_{ij} \sum_{\substack{h=1 \\ h \neq i,j}}^{g} \big(1 - |x_{ih} - x_{jh}|\big) ,$$

[note that $\big(1 - |x_{ih} - x_{jh}|\big) = 1$ if $x_{ih} = x_{jh}$,
and 0 if $x_{ih} \neq x_{jh}$, so that

$$\sum_{\substack{h=1 \\ h \neq i,j}}^{g} \big(1 - |x_{ih} - x_{jh}|\big)$$

measures agreement between i and j .]

Differences between these three network closure effects:

- transitive triplets effect: i more attracted to j if there are *more* indirect ties $i \rightarrow h \rightarrow j$;

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Differences between these three network closure effects:

- transitive triplets effect: *i* more attracted to *j* if there are *more* indirect ties $i \to h \to j$;
- transitive ties effect: *i* more attracted to *j* if there is *at least one* such indirect connection ;
- balance effect: *i* prefers others *j* who make same choices as *i*.

Non-formalized theories usually do not distinguish between these different closure effects.
It is possible to 'let the data speak for themselves' and see what is the best formal representation of closure effects.

7. *in-degree related popularity effect*, sum friends' in-degrees
$s_{i7}(x) = \sum_j x_{ij} \sqrt{x_{+j}} = \sum_j x_{ij} \sqrt{\sum_h x_{hj}}$
related to dispersion of in-degrees
(can also be defined without the $\sqrt{}$ sign);

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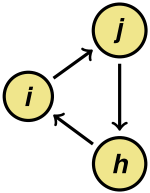
   related to dispersion of in-degrees
   (can also be defined without the $\sqrt{}$ sign);
8. *out-degree related popularity effect*,
   sum friends' out-degrees
   $s_{i8}(x) = \sum_j x_{ij} \sqrt{x_{j+}} = \sum_j x_{ij} \sqrt{\sum_h x_{jh}}$
   related to association in-degrees — out-degrees;
9. *Outdegree-related activity effect* ,
   $s_{i9}(x) = \sum_j x_{ij} \sqrt{x_{i+}} = x_{i+}^{1.5}$
   related to dispersion of out-degrees;
10. *Indegree-related activity effect* ,
    $s_{i10}(x) = \sum_j x_{ij} \sqrt{x_{+i}} = x_{i+} \sqrt{x_{+i}}$
    related to association in-degrees — out-degrees;

11. *three-cycle effect*,
   number of three-cycles in i's ties
   $(i \rightarrow j,\ j \rightarrow h,\ h \rightarrow i)$
   $s_{i11}(x) = \sum_{j,h} x_{ij}\, x_{jh}\, x_{hi}$



three-cycle

   This represents a kind of generalized reciprocity, and absence of hierarchy.

12. ... and potentially many others ...

Assortativity effects:

Preferences of actors dependent on their degrees.
Depending on their own out- and in-degrees,
actors can have differential preferences for ties
to others with also high or low out- and in-degrees.

Together this yields 4 possibilities:

- out ego - out alter degrees
- out ego - in alter degrees
- in ego - out alter degrees
- in ego - in alter degrees

All these are product interactions between the two degrees.
Here also the degrees could be replaced by their square roots.

Four kinds of evaluation function effect associated with actor covariate v_i.

This applies also to behavior variables Z_h.

13. *covariate-related popularity*, ‘alter’
sum of covariate over all of i ’s friends
$s_{i13}(x) = \sum_j x_{ij} \, v_j$;

Four kinds of evaluation function effect associated with actor covariate v_i.

This applies also to behavior variables Z_h.

13. *covariate-related popularity*, 'alter'
    sum of covariate over all of i 's friends
    $s_{i13}(x) = \sum_j x_{ij} v_j$;
14. *covariate-related activity*, 'ego'
    i's out-degree weighted by covariate
    $s_{i14}(x) = v_i x_{i+}$;

15. *covariate-related similarity*,
sum of measure of covariate similarity
between i and his friends,
$s_{i15}(x) = \sum_j x_{ij} \, \mathrm{sim}(v_i, v_j)$
where $\mathrm{sim}(v_i, v_j)$ is the similarity between v_i and v_j,

$$\mathrm{sim}(v_i, v_j) \; = \; 1 - \frac{|v_i - v_j|}{R_V} \; ,$$

R_V being the range of V;

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R_V being the range of V;

16. *covariate-related interaction*, 'ego $\times$ alter'
$s_{i16}(x) = v_i \sum_j x_{ij}\, v_j$;

Evaluation function effect for dyadic covariate w_{ij} :

17. *covariate-related preference*,
sum of covariate over all of i's friends,
i.e., values of w_{ij} summed over all others to whom i is tied,
$s_{i17}(x) = \sum_j x_{ij} w_{ij}$.
If this has a positive effect, then the value of a tie $i \rightarrow j$ becomes higher when w_{ij} becomes higher.

Example

Data collected by Gerhard van de Bunt:
group of 32 university freshmen,
24 female and 8 male students.

Three observations used here (t_1, t_2, t_3) :
at 6, 9, and 12 weeks after the start of the university year.
The relation is defined as a 'friendly relation'.

Missing entries $x_{ij}(t_m)$ set to 0
and not used in calculations of statistics.

Densities increase from 0.15 at t_1 via 0.18 to 0.22 at t_3 .

Very simple model: only out-degree and reciprocity effects

	Model 1	
Effect	par.	(s.e.)
Rate $t_1 - t_2$	3.51	(0.54)
Rate $t_2 - t_3$	3.09	(0.49)
Out-degree	-1.10	(0.15)
Reciprocity	1.79	(0.27)

rate parameters:
per actor about 3 opportunities for change between observations;

out-degree parameter negative:
on average, cost of friendship ties higher than their benefits;

reciprocity effect strong and highly significant ($t = 1.79/0.27 = 6.6$).

Evaluation function is

$$f_i(x) = \sum_j \Big(-1.10\, x_{ij} + 1.79\, x_{ij}\, x_{ji} \Big).$$

This expresses 'how much actor *i* likes the network'.

Adding a reciprocated tie (i.e., for which $x_{ji} = 1$) gives

$$-1.10 + 1.79 = 0.69.$$

Adding a non-reciprocated tie (i.e., for which $x_{ji} = 0$) gives

$$-1.10,$$

i.e., this has negative benefits.

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Adding a non-reciprocated tie (i.e., for which $x_{ji} = 0$) gives

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i.e., this has negative benefits.
Gumbel distributed disturbances are added:
these have variance $\pi^2/6 = 1.645$ and s.d. 1.28.

Conclusion: reciprocated ties are valued positively,
unreciprocated ties negatively;
actors will be reluctant to form unreciprocated ties;
by ‘chance’ (the random term),
such ties will be formed nevertheless
and these are the stuff on the basis of which
reciprocation by others can start.

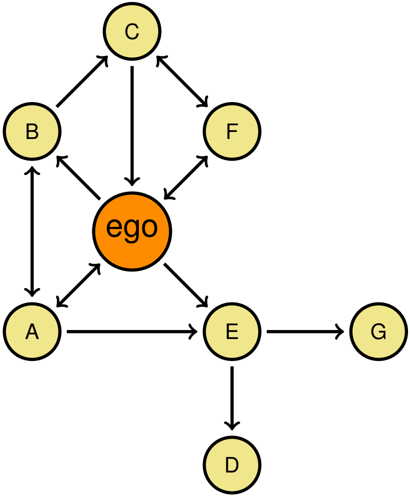
(Incoming unreciprocated ties, $x_{ji} = 1, x_{ij} = 0$ do not play a role
because for the objective function
only those parts of the network are relevant
that are under control of the actor,
so terms not depending on the outgoing relations of the actor
are irrelevant.)

For an interpretation, consider the simple model with only the transitive ties network closure effect. The estimates are:

Structural model with one network closure effect

	Model 3	
Effect	par.	(s.e.)
Rate $t_1 - t_2$	3.89	(0.60)
Rate $t_2 - t_3$	3.06	(0.47)
Out-degree	-2.14	(0.38)
Reciprocity	1.55	(0.28)
Transitive ties	1.30	(0.41)

Example: Personal network of ego.



for ego:

out-degree x_{i+} = 4
#{recipr. ties} = 2,
#{trans. ties } = 3.

The evaluation function is

$$f_i(x) = \sum_j \Big(-2.14\, x_{ij} + 1.55\, x_{ij}\, x_{ji} + 1.30\, x_{ij} \max_h \big(x_{ih}\, x_{hj}\big)\Big)$$

$\Big($ note: $\sum_j x_{ij} \max_h \big(x_{ih}\, x_{hj}\big)$ is #{trans. ties } $\Big)$

so its current value for this actor is

$$f_i(x) = -2.14 \times 4 + 1.55 \times 2 + 1.30 \times 3 = -1.56.$$

Options when ‘ego’ has opportunity for change:

	out-degr.	recipr.	trans. ties	gain	prob.
current	4	2	3	0.00	0.061
new tie to C	5	3	5	+2.01	0.455
new tie to D	5	2	4	+0.46	0.096
new tie to G	5	2	4	+0.46	0.096
drop tie to A	3	1	0	–3.31	0.002
drop tie to B	3	2	1	–0.46	0.038
drop tie to E	3	2	2	+0.84	0.141
drop tie to F	3	1	3	+0.59	0.110

The actor adds random influences to the gain (with s.d. 1.28), and chooses the change with the highest total ‘value’.

Model with more structural effects

	Model 3	
Effect	par.	(s.e.)
Rate $t_1 - t_2$	4.64	(0.80)
Rate $t_2 - t_3$	3.53	(0.57)
Out-degree	–0.90	(0.58)
Reciprocity	2.27	(0.41)
Transitive triplets	0.35	(0.06)
Transitive ties	0.75	(0.45)
Three-cycles	–0.72	(0.21)
In-degree popularity ($\sqrt{}$)	–0.71	(0.27)

Conclusions:

Reciprocity, transitivity; negative 3-cycle effect; negative popularity effect.

Add effects of gender & program, smoking similarity

	Model 4	
Effect	par.	(s.e.)
Rate $t_1 - t_2$	4.71	(0.80)
Rate $t_2 - t_3$	3.54	(0.59)
Out-degree	–0.81	(0.61)
Reciprocity	2.14	(0.45)
Transitive triplets	0.33	(0.06)
Transitive ties	0.67	(0.46)
Three-cycles	–0.64	(0.22)
In-degree popularity ($\sqrt{}$)	–0.72	(0.28)
Sex (M) alter	0.52	(0.27)
Sex (M) ego	–0.15	(0.27)
Sex similarity	0.21	(0.22)
Program similarity	0.65	(0.26)
Smoking similarity	0.25	(0.18)

Conclusions:

Trans. ties now not needed any more to represent transitivity; men more popular; program similarity.

To interpret the three effects of actor covariate *gender*, it is more instructive to consider them simultaneously. Gender was coded originally by with 1 for *F* and 2 for *M*. This dummy variable was centered (mean was subtracted) but this only adds a constant to the values presented next, and does not affect the differences between them.

Therefore we may do the calculations with $F = 0, \ M = 1$.

The joint effect of the gender-related effects
for the tie variable x_{ij} from i to j is

$$-0.15\, z_i \,+\, 0.52\, z_j \,+\, 0.21\, I\{z_i = z_j\}\ .$$

$i \setminus j$	F	M
F	0.21	0.52
M	−0.15	0.58

Conclusion:
men seem not to like female friends...?

Extended model specification

1. Endowment effect $g_i(\gamma, x, j)$

This represents the value of a tie
that is lost when the tie $i \rightarrow j$ is dissolved,
but that did not play a role when the tie was created.

This model component is used when certain effects
work differently for *creation* of ties $(0 \Rightarrow 1)$
than for *termination* of ties $(1 \Rightarrow 0)$.

With this extension, the relative log-probabilities are

$$f_i\big(\beta, x(i \leadsto j)\big) \; - \; x_{ij}\, g_i(\gamma, x, j) \; .$$

(Note that x_{ij} is the indicator of the current tie, before the change.)

The endowment function again can be a weighted sum

$$g_i(\gamma, x, j) = \sum_{h=1}^{H} \gamma_h \, r_{ijh}(x) \; .$$

Examples of components of endowment function:

1. $\gamma_1 \, x_{ji}$

   γ_1 extra benefits of a reciprocated tie.

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1. $\gamma_1 \, x_{ji}$

   γ_1 extra benefits of a reciprocated tie.

2. $\gamma_2 \, w_{ij}$

   effect of dyadic covariate w_{ij}

   different for creating than for breaking a tie.

3. ... all other effects used also in the evaluation function.

Add endowment effect of reciprocated tie

	Model 5	
Effect	par.	(s.e.)
Rate $t_1 - t_2$	5.45	(1.00)
Rate $t_2 - t_3$	4.05	(0.67)
Out-degree	–0.62	(0.59)
Reciprocity	1.39	(0.48)
Transitive triplets	0.38	(0.06)
Three-cycles	–0.60	(0.26)
In-degree popularity ($\surd$)	–0.70	(0.26)
Sex (M) alter	0.63	(0.26)
Sex (M) ego	–0.29	(0.30)
Sex similarity	0.29	(0.24)
Program similarity	0.78	(0.28)
Smoking similarity	0.34	(0.17)
Endowment reciprocated tie	2.18	(0.95)

Transitive ties effect omitted.

Evaluation effect reciprocity: 1.39
Endowment reciprocated tie: 2.18

The overall (combined) reciprocity effect was 2.14.
With the split between the evaluation and endowment effects,
it appears now that the value of reciprocity
for creating a tie is 1.39,
and for withdrawing a tie 1.39 + 2.18 = 3.57.

Thus, there is a very strong barrier
against the dissolution of reciprocated ties.

Extended model specification

2. Non-constant rate function $\lambda_i(\alpha, x)$.

This means that some actors change their ties
more quickly than others,
depending on covariates or network position.

Dependence on covariates:

$$\lambda_i(\alpha, x) = \rho_m \exp(\sum_h \alpha_h v_{hi}) .$$

ρ_m is a period-dependent base rate.

(Rate function must be positive; $\Rightarrow$ exponential function.)

Dependence on network position:
e.g., dependence on out-degrees:

$$\lambda_i(\alpha, x) = \exp(\alpha_1 \, x_{i+}) \ .$$

Also, in-degrees and $\sharp$ reciprocated ties of actor *i* may be used.

Now the parameter is $\theta = (\rho, \alpha, \beta, \gamma)$.

Continuation example

Rate function depends on out-degree:
those with higher out-degrees
also change their tie patterns more quickly.

endowment function depends on tie reciprocation

$$g_i(\gamma, x, j) = \gamma_1 \, x_{ji}$$

Reciprocity operates differently
for tie initiation than for tie withdrawal.

Parameter estimates model with rate and endowment effects

	Model 6	
Effect	par.	(s.e.)
Rate (period 1)	3.99	(0.70)
Rate (period 2)	2.93	(0.48)
Out-degree effect on rate	0.041	(0.034)
Out-degree	-0.79	(0.57)
Reciprocity	1.51	(0.54)
Transitive triplets	0.35	(0.05)
Three-cycles	–0.57	(0.19)
In-degree popularity ($\sqrt{}$)	–0.59	(0.27)
Gender ego	-0.33	(0.31)
Gender alter	0.57	(0.27)
Gender similarity	0.30	(0.24)
Program similarity	0.80	(0.26)
Smoking similarity	0.36	(0.19)
Endowment recipr. tie	1.82	(0.97)

Conclusion:

non-significant tendency that actors with higher out-degrees change their ties more often ($t = 0.041/0.034 = 1.2$),

value of reciprocation is larger for termination of ties than for creation ($t = 1.82/0.97 = 1.88$).

Non-directed networks

Working paper available from:

http://www.stats.ox.ac.uk/~snijders/PoliticalAnalysis_NetDyn.pdf

The actor-driven modeling is less straightforward
for non-directed relations,
because two actors are involved in deciding about a tie.

Various modeling options are possible:

1. Forcing model:
   one actor takes the initiative and unilaterally imposes that a tie is created or dissolved.

2. Unilateral initiative with reciprocal confirmation:
   one actor takes the initiative and proposes a new tie
   or dissolves an existing tie;
   if the actor proposes a new tie, the other has to confirm,
   otherwise the tie is not created.

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   otherwise the tie is not created.
3. Pairwise conjunctive model:
   a pair of actors is chosen and reconsider whether a tie
   will exist between them; a new tie is formed if both agree.
4. Pairwise disjunctive (forcing) model:
   a pair of actors is chosen and reconsider whether a tie
   will exist between them;
   a new tie is formed if at least one wishes this.

5. Pairwise compensatory (additive) model:
   a pair of actors is chosen and reconsider whether a tie will exist between them; this is based on the sum of their utilities for the existence of this tie.

Option 1 is close to the actor-driven model for directed relations.

In options 3–5, the pair of actors (i, j) is chosen depending on the product of the rate functions $\lambda_i \, \lambda_j$ (under the constraint that $i \neq j$).

The numerical interpretation of the ratio function differs between options 1–2 compared to 3–5.

The decision about the tie is taken on the basis of the objective functions $f_i \, f_j$ of both actors.

2. Estimation

Suppose that at least 2 observations on $X(t)$ are available, for observation moments $t_1, \; t_2$.
(Extension to more than 2 observations is straightforward.)

How to estimate θ?

Condition on $X(t_1)$:
the first observation is accepted as given,
contains in itself no observation about θ.

No assumption of a stationary network distribution.

Thus, simulations start with $X(t_1)$.

2A. Method of moments

Choose a suitable statistic $Z = (Z_1, \ldots, Z_K)$,
i.e., K variables which can be calculated from the network;
the statistic Z must be *sensitive* to the parameter θ
in the sense that higher values of θ_k
lead to higher values of the expected value $\mathrm{E}_\theta(Z_k)$;

determine value $\hat{\theta}$ of $\theta = (\rho, \beta)$ for which
observed and expected values of suitable Z statistic are equal:

$$E_{\hat{\theta}}\{Z\} = z \, .$$

Questions:

- What is a suitable (K-dimensional) statistic?
  Corresponds to objective function.
- How to find this value of θ?
  By stochastic approximation (Robbins-Monro process)
  based on repeated simulations of the dynamic process,
  with parameter values
  getting closer and closer to the moment estimates.

Suitable statistics for method of moments

Assume first that $\lambda_i(x) = \rho = \theta_1$,
and 2 observation moments.

This parameter determines the expected “amount of change”.

A sensitive statistic for $\theta_1 = \rho$ is

$$C = \sum_{\substack{i,j=1 \\ i \neq j}}^{g} \mid X_{ij}(t_2) - X_{ij}(t_1) \mid ,$$

the “observed total amount of change”.

For the weights β_k in the evaluation function

$$f_i(\beta, x) = \sum_{k=1}^{L} \beta_k \, s_{ik}(x) \, ,$$

a higher value of β_k means that all actors
strive more strongly after a high value of $s_{ik}(x)$,
so $s_{ik}(x)$ will tend to be higher for all i, k.

This leads to the statistic

$$S_k = \sum_{i=1}^{n} s_{ik}(X(t_2)) \ .$$

This statistic will be sensitive to β_k :
a high β_k will to lead to high values of S_k.

Moment estimation will be based on the vector of statistics

$$Z = (C, S_1, ..., S_{K-1}) \; .$$

Denote by z the observed value for Z.
The moment estimate $\hat{\theta}$ is defined as the parameter value for which the expected value of the statistic is equal to the observed value:

$$\mathrm{E}_{\hat{\theta}} \{Z\} \; = \; z \; .$$

Robbins-Monro algorithm

The moment equation $E_{\hat{\theta}}\{Z\} = z$ cannot be solved by analytical or the usual numerical procedures, because

$$\mathrm{E}_{\theta}\{Z\}$$

cannot be calculated explicitly.

However, the solution can be approximated by the Robbins-Monro (1951) method for stochastic approximation.

Iteration step:

$$\hat{\theta}_{N+1} = \hat{\theta}_N - a_N\, D^{-1}(z_N - z)\ , \qquad (1)$$

where z_N is a simulation of Z with parameter $\hat{\theta}_N$,
D is a suitable matrix, and $a_N \to 0$.

Covariance matrix

The method of moments yields the covariance matrix

$$\mathrm{cov}(\hat{\theta}) \approx D_\theta^{-1} \, \Sigma_\theta \, D_\theta'^{\,-1}$$

where

$$\begin{aligned} \Sigma_\theta &= \mathrm{cov}\{Z \,|\, X(t_1) = x(t_1)\} \\ D_\theta &= \frac{\partial}{\partial\theta} \mathrm{E}\{Z \,|\, X(t_1) = x(t_1)\} \,. \end{aligned}$$

Matrices Σ_θ and D_θ can be estimated from MC simulations with fixed θ.

After the presumed convergence of the algorithm for approximately solving the moment equation, extra simulations are carried out

(*a*) to check that indeed $\mathrm{E}_{\hat{\theta}}\{Z\} \approx z$,

(*b*) to estimate Σ_θ,

(*c*) and to estimate D_θ
using a score function algorithm
(earlier algorithm used
difference quotients and common random numbers).

Modified estimation method:

conditional estimation .

Condition on the observed numbers of differences between successive observations,

$$c_m = \sum_{i,j} \mid x_{ij}(t_{m+1}) - x_{ij}(t_m) \mid \quad .$$

For continuing the simulations do not mind the values of the time variable t,
but continue between t_m and t_{m+1} until
the observed number of differences

$$\sum_{i,j} \mid X_{ij}(t) - x_{ij}(t_m) \mid$$

is equal to the observed c_m .
This is defined as time moment t_{m+1} .

This procedure is a bit more stable;
requires modified estimator of ρ_m .

Computer algorithm has 3 phases:

1. brief phase for preliminary estimation of $\partial \mathrm{E}_\theta \{Z\} / \partial \theta$ for defining D;

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Computer algorithm has 3 phases:

1. brief phase for preliminary estimation of $\partial \mathrm{E}_\theta \{Z\} / \partial\theta$ for defining D;
2. estimation phase with Robbins-Monro updates, where a_N remains constant in *subphases* and decreases between subphases;
3. final phase where θ remains constant at estimated value; this phase is for checking that

$$\mathrm{E}_{\hat{\theta}} \{Z\} \approx z \ ,$$

and for estimating D_θ and Σ_θ to calculate standard errors.

Extension: more periods

The estimation method can be extended
to more than 2 repeated observations:
observations $x(t)$ for $t = t_1, ..., t_M$.

Parameters remain the same in periods between observations
except for the basic rate of change ρ
which now is given by ρ_m for $t_m \leq t < t_{m+1}$.

For the simulations,
the simulated network $X(t)$ is reset to the observation $x(t_m)$
whenever the time parameter t passes the observation time t_m .

The statistics for the method of moments are defined as
sums of appropriate statistics calculated per period (t_m, t_{m+1}).

2B. ML Estimation

skipped

The Siena program (version 4) is available as an R package, programmed by Ruth Ripley and Krists Boitmanis.

This is the package RSiena, which now includes most of the functionality in the earlier standalone version Siena 3.

3. Networks as dependent and independent variables

Simultaneous endogenous dynamics of networks and behavior:
e.g.,

- individual humans & friendship relations:
  attitudes, behavior (lifestyle, health, etc.)
- individual humans & cooperation relations:
  work performance
- companies / organisations & alliances, cooperation:
  performance, organisational success.

Two-way influence between networks and behavior

Relational embeddedness is important
for well-being, opportunities, etc.

Actors are influenced in their behavior, attitudes, performance
by other actors to whom they are tied
e.g., network resources (social capital), social control.

(N. Friedkin, *A Structural Theory of Social Influence*, C.U.P., 1998).

In return, many types of tie
(friendship, cooperation, liking, etc.)
are influenced positively by
similarity on relevant attributes: *homophily*
(e.g., McPherson, Smith-Lovin, & Cook, *Ann. Rev. Soc.*, 2001.)

More generally, actors choose relation partners
on the basis of their behavior and other characteristics
(similarity, opportunities for future rewards, etc.).

Influence, network & behavior effects on *behavior*;
Selection, network & behavior effects on *relations*.

Terminology

relation = network = pattern of ties in group of actors;
behavior = any individual-bound changeable attribute
(including attitudes, performance, etc.).

Relations and behaviors are endogenous variables
that develop in a simultaneous dynamics.

Thus, there is a feedback relation in the dynamics
of relational networks and actor behavior / performance:
macro $\Rightarrow$ mini $\Rightarrow$ macro $\cdot\ \cdot\ \cdot\cdot$

(although network perhaps is meso rather than macro)

The investigation of such social feedback processes is difficult:

- Both the *network* $\Rightarrow$ *behavior*
  and the *behavior* $\Rightarrow$ *network* effects
  lead ‘network autocorrelation’:
  “friends of smokers are smokers”
  “high-reputation firms don’t collaborate
  with low-reputation firms”.
  It is hard to ascertain the strengths
  of the causal relations in the two directions.
- For many phenomena
  quasi-continuous longitudinal observation is infeasible.
  Instead, it may be possible to observe
  networks and behaviors at a few discrete time points.

Data

One bounded set of actors
(e.g. school class, group of professionals, set of firms);

several discrete observation moments;

for each observation moment:

- network: who is tied to whom
- behavior of all actors

Aim: disentangle effects *networks* $\Rightarrow$ *behavior*
from effects *behavior* $\Rightarrow$ *networks*.

Notation:

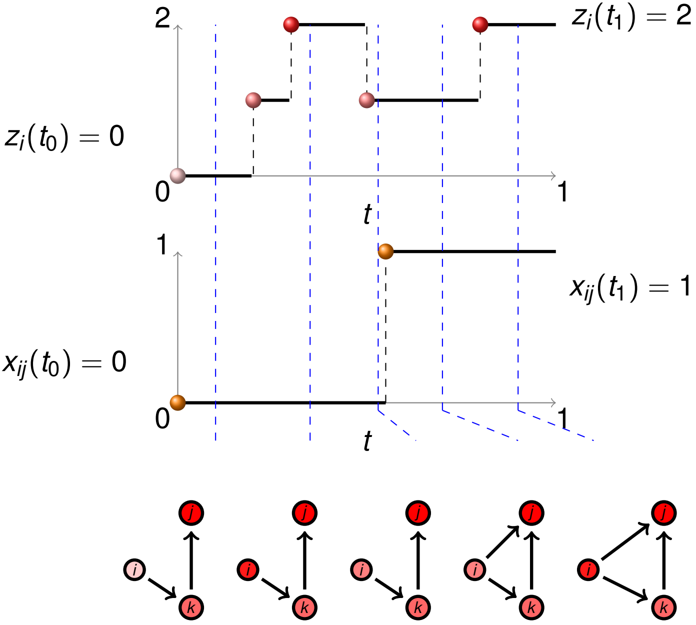
Integrate the *influence* (dep. var. = behavior)
and *selection* (dep. var. = network) processes.

In addition to the network X, associated to each actor i
there is a vector $Z_i(t)$ of actor characteristics
indexed by $h = 1, \ldots, H$.
Assumption: ordered discrete
(simplest case: one dichotomous variable).

Actor-driven models

Each actor "controls" not only his outgoing ties,
collected in the row vector $\big(X_{i1}(t), ..., X_{in}(t)\big)$,
but also his behavior $Z_i(t) = \big(\ Z_{i1}(t), ..., Z_{iH}(t)\ \big)$
(H is the number of dependent behavior variables).

Network change process and behavior change process
run simultaneously, and influence each other
being each other's changing constraints.



2
0
1
t
1
0
1
t
$z_i(t_0) = 0$
$z_i(t_1) = 2$
$x_{ij}(t_0) = 0$
$x_{ij}(t_1) = 1$
i
j
k

At stochastic times
(*rate functions* λ^X for changes in network,
λ^Z_h for changes in behavior h),
the actors may change a tie or a behavior.

Probabilities of change are increasing functions of
objective functions of the new state,
defined specifically for network, f^X,
and for behavior, f^Z .

Again, only the smallest possible steps are allowed:
change one tie variable,
or move one step up or down on a behavior variable.

For network change, change probabilities are as before.

For the behaviors, the formula of the change probabilities is

$$p_{ihv}(\beta, z) = \frac{\exp(f(i,h,v))}{\sum_{k,u} \exp(f(i,k,u))}$$

where $f(i,h,v)$ is the objective function calculated for the potential new situation after a behavior change,

$$f(i,h,v) = f_i^Z(\beta, z(i, h \rightsquigarrow v)) \; .$$

Again, multinomial logit form.

Again, a 'maximizing' interpretation is possible.

mini-step for change in network:

At random moments occurring at a rate λ^X,
a random actor is designated
to make a change in one tie variable:
the *mini-step* (on $\Rightarrow$ off, or off $\Rightarrow$ on.)

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mini-step for change in behavior:

At random moments occurring at a rate λ^Z_h,
a random actor is designated to make a change in behavior h
(one component of Z_i, assumed to be ordinal):
the *mini-step* is a change to an adjacent category.

Again, many mini-steps can *accumulate* to big differences.

Optimizing interpretation:

When actor i 'may' change an outgoing tie variable to some other actor j, he/she chooses the 'best' j by maximizing the evaluation function $f_i^X(\beta, X, z)$ of the situation obtained after the coming network change
plus a random component representing unexplained influences;

and when this actor 'may' change behavior h,
he/she chooses the "best" change (up, down, nothing)
by maximizing the evaluation function $f_i^Z(\beta, x, Z)$ of the situation obtained after the coming behavior change
plus a random component representing unexplained influences.

Optimal network change:

The new network is denoted by $x(i \rightsquigarrow j)$.
The attractiveness of the new situation
(evaluation function plus random term)
is expressed by the formula

$$f_i^X(\beta, x(i \rightsquigarrow j), z) \quad + \quad U_i^X(t, x, j)\,.$$

$$\Uparrow$$

random component

(Note that the network is also permitted to stay the same.)

Optimal behavior change:

Whenever actor i may make a change in variable h of Z, he changes only one behavior, say z_{ih}, to the new value v. The new vector is denoted by $z(i, h \rightsquigarrow v)$.
Actor i chooses the "best" h, v by maximizing the objective function of the situation obtained after the coming behavior change plus a random component:

$$f_i^Z(\beta, x, z(i, h \rightsquigarrow v)) \quad + \quad U_i^Z(t, z, h, v)\,.$$

$$\Uparrow$$

random component

(behavior is permitted to stay the same.)

Specification of the behavior model

Many different reasons why networks
are important for behavior:

1. *imitation* :
   individuals imitate others
   (basic drive; uncertainty reduction).
2. *social capital* :
   individuals may use resources of others;
3. *coordination* :
   individuals can achieve some goals
   only by concerted behavior;

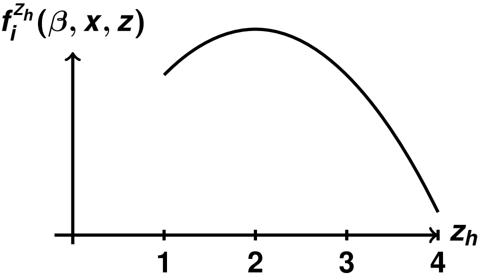
In this presentation, only imitation is considered,
but the other two reasons are also of eminent importance.

Basic effects for dynamics of behavior f_i^Z:

$$f_i^Z(\beta, x, z) = \sum_{k=1}^{L} \beta_k \, s_{ik}(x, z) \,,$$

1. *tendency* ,
   $s_{i1}^Z(x, z) = z_{ih}$
2. *quadratic tendency*, ‘effect behavior on itself’,
   $s_{i2}^Z(x, z) = z_{ih}^2$
   Quadratic tendency effect important for model fit.

For a negative quadratic tendency parameter,
the model for behavior is a unimodal preference model.



For positive quadratic tendency parameters ,
the behavior objective function can be bimodal
('positive feedback').

3. *behavior-related average similarity*,
   average of behavior similarities between i and friends
   $s_{i3}(x) = \frac{1}{x_{i+}} \sum_j x_{ij} \, \text{sim}(z_{ih}, z_{jh})$
   where $\text{sim}(z_{ih}, z_{jh})$ is the similarity between v_i and v_j,

$$\text{sim}(z_{ih}, z_{jh}) \;=\; 1 - \frac{|z_{ih} - z_{jh}|}{R_{Z^h}} \;,$$

   R_{Z^h} being the range of Z^h;

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   where $\mathrm{sim}(z_{ih}, z_{jh})$ is the similarity between v_i and v_j,
   $$\mathrm{sim}(z_{ih}, z_{jh}) \, = \, 1 - \frac{|z_{ih} - z_{jh}|}{R_{Z^h}} \ ,$$
   R_{Z^h} being the range of Z^h;
4. *average behavior alter* — an alternative to similarity:
   $s_{i4}(x, z) = z_{ih} \frac{1}{x_{i+}} \sum_j x_{ij} z_{jh}$

Effects 3 and 4 are alternatives for each other:
they express the same theoretical idea of influence
in mathematically different ways.
The data will have to differentiate between them.

Network position can also have influence on behavior dynamics
e.g. through degrees rather than through behavior
of those to whom one is tied:

5. *popularity-related tendency*, (in-degree)
   $s_{i5}(x, z) = z_{ih}\, x_{+i}$

Network position can also have influence on behavior dynamics
e.g. through degrees rather than through behavior
of those to whom one is tied:

7. *popularity-related tendency*, (in-degree)
   $s_{i7}(x, z) = z_{ih}\, x_{+i}$
8. *activity-related tendency*, (out-degree)
   $s_{i8}(x, z) = z_{ih}\, x_{i+}$

7. *dependence on other behaviors* $(h \neq \ell)$,
   $s_{i7}(x, z) = z_{ih} \, z_{i\ell}$

For both the network and the behavior dynamics,
extensions are possible depending on the network position.

Now focus on the *similarity effect* in evaluation function :

sum of absolute behavior differences between i and his friends
$s_{i2}(x, z) = \sum_j x_{ij} \, \mathrm{sim}(z_{ih}, z_{jh})$.

This is fundamental both
to network selection based on behavior,
and to behavior change based on network position.

A positive coefficient for this effect means that the actors prefer friends with similar Z_h values (*network autocorrelation*).

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Actors can attempt to attain this by changing their own Z_h value to the average value of their friends (*network influence*, *contagion*),

A positive coefficient for this effect means that the actors prefer friends with similar Z_h values (*network autocorrelation*).

Actors can attempt to attain this by changing their own Z_h value to the average value of their friends (*network influence*, *contagion*),

or by becoming friends with those with similar Z_h values (*selection on similarity*).

Statistical estimation: networks & behavior

Procedures for estimating parameters in this model are similar to estimation procedures for network-only dynamics: Methods of Moments & Stochastic Approximation, conditioning on the first observation $X(t_1), Z(t_1)$.

The two different effects,
networks $\Rightarrow$ behavior and behavior $\Rightarrow$ networks,
both lead to network autocorrelation of behavior;

but they can be (in principle)
distinguished empirically by the time order: respectively
association between ties at t_m and behavior at t_{m+1};
and association between behavior at t_m and ties at t_{m+1}.

Statistics for use in method of moments:

for estimating parameters in network dynamics:

$$\sum_{m=1}^{M-1} \sum_{i=1}^{n} s_{ik}(X(t_{m+1}), Z(t_m)) \; ,$$

and for the behavior dynamics:

$$\sum_{m=1}^{M-1} \sum_{i=1}^{n} s_{ik}(X(t_m), Z(t_{m+1})) \; .$$

The data requirements for these models are strong:
few missing data; enough change on the behavioral variable.

Currently, work still is going on about good ways
for estimating parameters in these models.

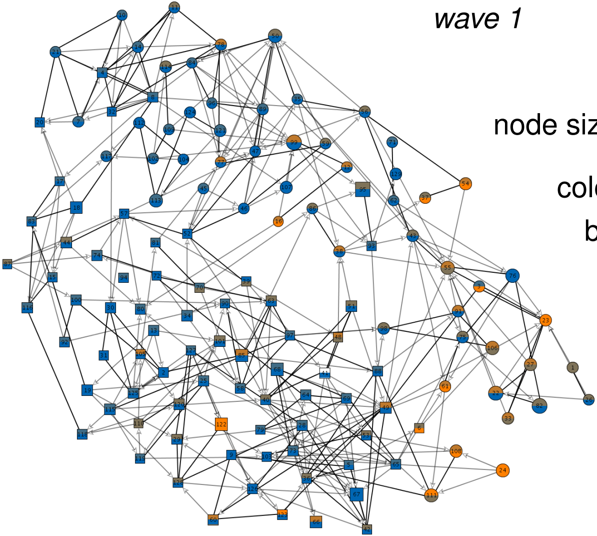
Maximum likelihood estimation procedures
(currently even more time-consuming; under construction...)
are preferable for small data sets.

Example :

Study of smoking initiation and friendship
(following up on earlier work by P. West, M. Pearson & others).
One school year group from a Scottish secondary school starting at age 12-13 years, was monitored over 3 years; total of 160 pupils, of which 129 pupils present at all 3 observations; with sociometric & behavior questionnaires at three moments, at appr. 1 year intervals.

Smoking: values 1–3;
drinking: values 1–5;

covariates:
gender, smoking of parents and siblings (binary),
money available (range 0–40 pounds/week).



wave 1

girls: circles

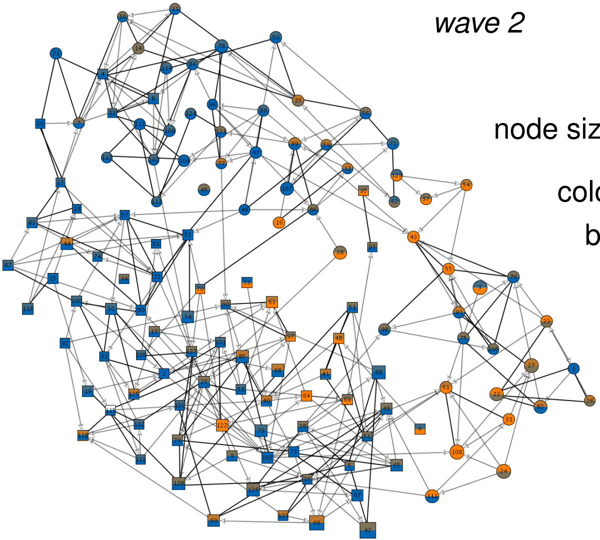
boys: squares

node size: pocket money

color: top = drinking

bottom = smoking

(orange = high)



wave 2

girls: circles

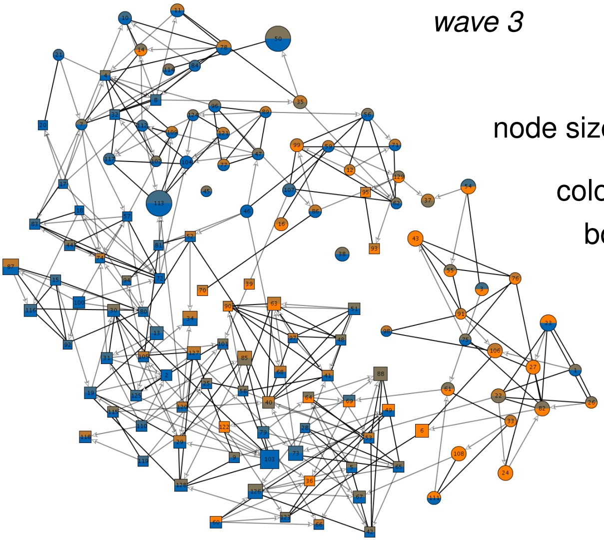
boys: squares

node size: pocket money

color: top = drinking

bottom = smoking

(orange = high)



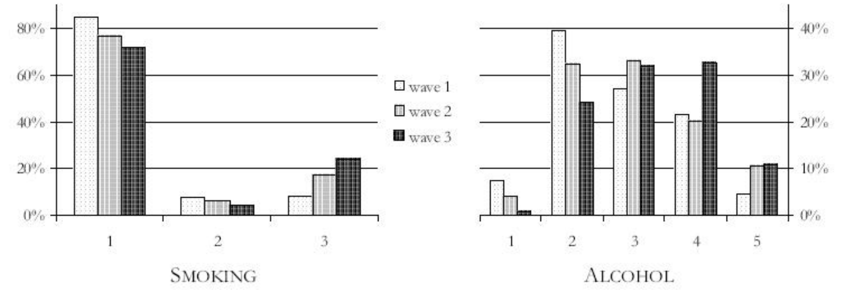
wave 3

girls: circles
boys: squares

node size: pocket money

color: top = drinking
bottom = smoking
(orange = high)

FIGURE 2. — OBSERVED DISTRIBUTION OF SUBSTANCE USE IN THE THREE WAVES.



Simple model: friendship dynamics

Friendship dynamics	Rate 1	14.24	(1.52)
	Rate 2	10.51	(1.04)
	Outdegree	–2.95	(0.06)
	Reciprocity	1.96	(0.10)
	Popularity	0.35	(0.07)
	Transitive triplets	0.27	(0.02)
	Sex similarity	0.97	(0.10)
	Drinking alter	0.01	(0.07)
	Drinking ego	0.01	(0.08)
	Drinking ego $\times$ drinking alter	0.17	(0.06)
	Smoking alter	–0.04	(0.08)
	Smoking ego	–0.03	(0.08)
	Smoking ego $\times$ smoking alter	0.05	(0.09)

Simple model: smoking and drinking dynamics

Smoking dynamics	Rate 1	5.16	(1.88)
	Rate 2	3.59	(1.24)
	Linear tendency	–3.43	(0.48)
	Quadratic tendency	2.69	(0.40)
	Ave. alter	1.89	(0.75)

Alcohol consumption dynamics	Rate 1	1.56	(0.34)
	Rate 2	2.45	(0.44)
	Linear tendency	0.47	(0.17)
	Quadratic tendency	–0.70	(0.30)
	Ave. alter	1.59	(0.83)

Preliminary conclusion:

Significant evidence for friendship selection
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However, this model controls insufficiently
for other influences and for the
endogenous network dynamics.

More realistic model

Friendship dynamics	Rate 1	18.67	(2.17)
	Rate 2	12.42	(1.30)
	Outdegree	–1.57	(0.27)
	Reciprocity	2.04	(0.13)
	Transitive triplets	0.35	(0.04)
	Transitive ties	0.84	(0.09)
	Three-cycles	–0.41	(0.10)
	In-degree based popularity ($\sqrt{}$)	0.05	(0.07)
	Out-degree based popularity ($\sqrt{}$)	–0.45	(0.16)
	Out-degree based activity ($\sqrt{}$)	–0.39	(0.07)
	Sex alter	–0.14	(0.08)
	Sex ego	0.08	(0.10)
	Sex similarity	0.66	(0.08)
	Romantic exp. similarity	0.10	(0.06)
	Money alter (unit: 10 pounds/w)	0.11	(0.05)
	Money ego	–0.06	(0.06)
	Money similarity	0.98	(0.27)

More realistic model (continued)

Friendship dynamics	Drinking alter	–0.01	(0.07)
	Drinking ego	0.09	(0.09)
	Drinking ego × drinking alter	0.14	(0.06)
	Smoking alter	–0.08	(0.08)
	Smoking ego	–0.14	(0.09)
	Smoking ego × smoking alter	0.03	(0.08)

Smoking dynamics	Rate 1	4.74	(1.88)
	Rate 2	3.41	(1.29)
	Linear tendency	–3.39	(0.45)
	Quadratic tendency	2.71	(0.40)
	Ave. alter	2.00	(0.95)
	Drinking	–0.11	(0.24)
	Sex (F)	–0.12	(0.35)
	Money	0.10	(0.20)
	Smoking at home	–0.05	(0.29)
	Romantic experience	0.09	(0.33)

Alcohol consumption dynamics	Rate 1	1.60	(0.32)
	Rate 2	2.50	(0.42)
	Linear tendency	0.44	(0.17)
	Quadratic tendency	–0.64	(0.22)
	Ave. alter	1.34	(0.61)
	Smoking	0.01	(0.21)
	Sex (F)	0.04	(0.22)
	Money	0.17	(0.16)
	Romantic experience	–0.19	(0.27)

Conclusion:

In this case, the conclusions from a more elaborate model – i.e., with better control for alternative explanations – are similar to the conclusions from the simple model.

There is evidence for friendship selection based on drinking, and for social influence with respect to smoking and drinking.

Parameter interpretation for behavior change

Omitting the non-significant parameters yields
the following objective functions.
For smoking

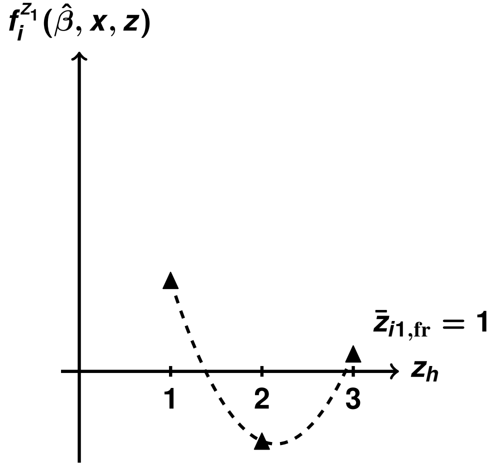
$$f_i^{Z_1}(\hat{\beta}, x, z) =$$

$$-3.39\,(z_{i1} - \bar{z}_1) + 2.71\,(z_{i1} - \bar{z}_1)^2 + 2.00\,(z_{i1} - \bar{z}_1)\,(\bar{z}_{i1,\mathrm{fr}} - \bar{z}_1)\ ,$$

where z_{i1} is smoking of actor i: values 1–3, mean 1.4.
$\bar{z}_{i1,\mathrm{fr}}$ is the average smoking behavior of i's friends.

Convex function – consonant with addictive behavior.

$$-3.39\,(z_{i1} - \bar{z}_1) + 2.71\,(z_{i1} - \bar{z}_1)^2 + 2.00\,(z_{i1} - \bar{z}_1)\,(\bar{z}_{i1,\mathrm{fr}} - \bar{z}_1)$$



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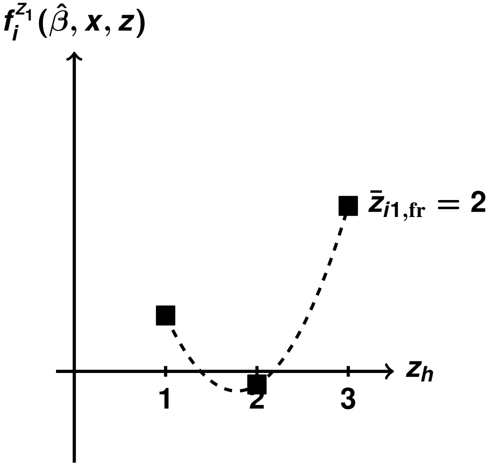

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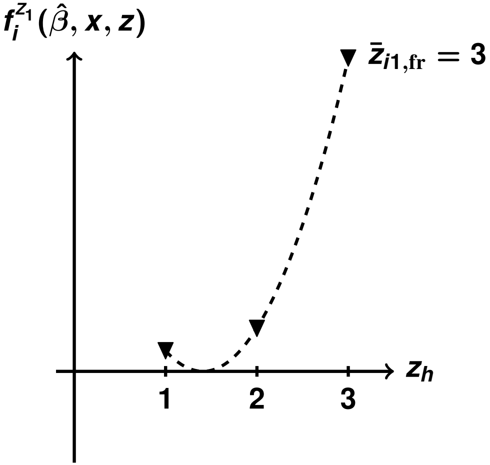

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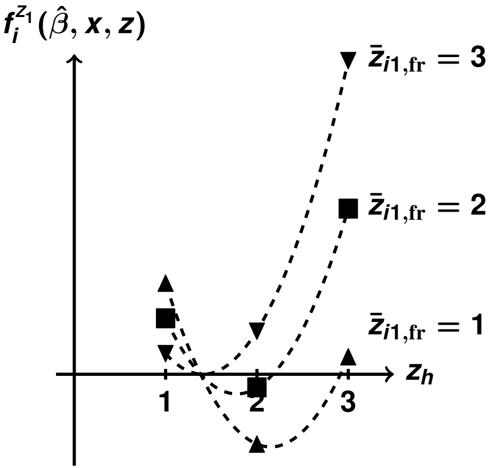


For drinking the objective function (significant terms only) is

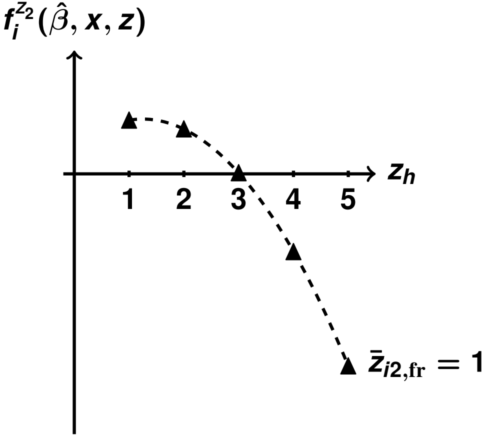
$$f_i^{z_2}(\hat{\beta}, x, z) =$$

$$0.44\,(z_{i2} - \bar{z}_2) \;-\; 0.64\,(z_{i2} - \bar{z}_2)^2 \;+\; 1.34\,(z_{i2} - \bar{z}_2)\,(\bar{z}_{i2,\mathrm{fr}} - \bar{z}_2)\ ,$$

where z_{i2} is drinking of actor i: values 1–5, mean 3.0.

Unimodal function – consonant with non-addictive behavior.

$$0.44\,(z_{i2} - \bar{z}_2) \,-\, 0.64\,(z_{i2} - \bar{z}_2)^2 \,+\, 1.34\,(z_{i2} - \bar{z}_2)\,(\bar{z}_{i2,\mathrm{fr}} - \bar{z}_2)$$



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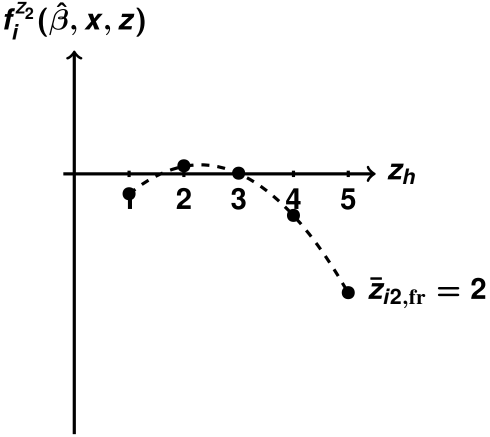

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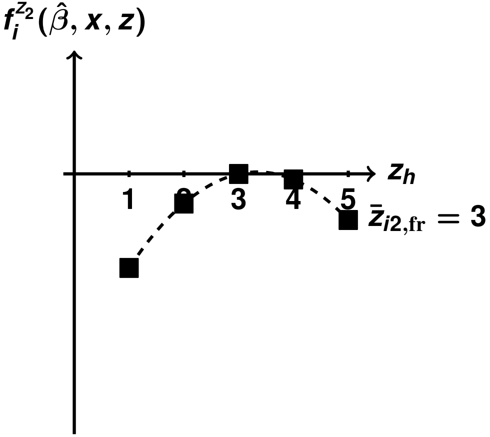

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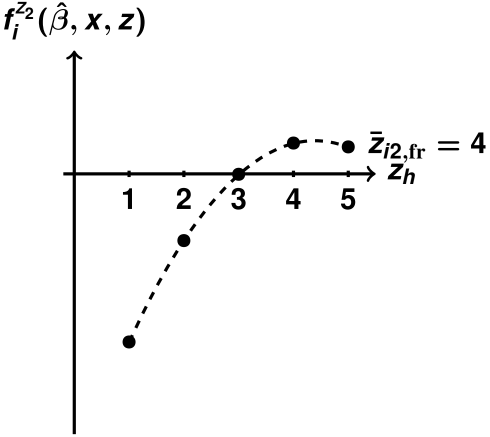

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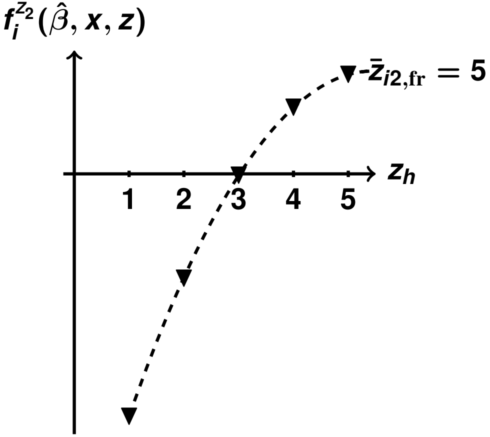

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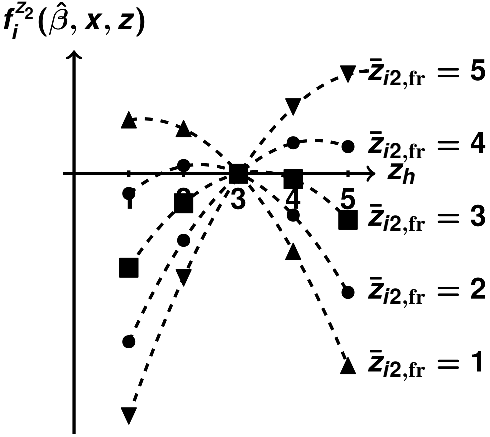

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- More research is needed for mathematical properties
  such as consistency, asymptotic normality, etc.

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9. Valued relations; Multivariate relations.

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  In press, *Sociological Methodology*.

A lot of material
– programs, manuals, papers, workshop announcements –
can be found at the Siena website:

http://www.stats.ox.ac.uk/siena/

There is also a user's group:

http://groups.yahoo.com/groups/stocnet/