

# Some effects for categorical covariates

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## Overview

Effects for categorical covariates

# 1. Effects for categorical covariates

This is part of my presentation at the 2025 Sunbelt in Paris, modified to be independently understood.

Tom A.B. Snijders and Beata Łopaciuk-Gonczaryk (2025).

Double agency and co-evolution for two-mode networks, with an application to corporate interlocks and firms' environmental performance. *Social Networks*, 83, 92-104.

## Expressions reflecting heterogeneity between nodes

Especially in two-mode networks — but also more generally, there can be important differences between 'kinds' of nodes.

*'Senders differ in the kind of receivers they choose.'*

*'Receivers differ in the kind of senders choosing them.'*

*'Different kinds of receivers are chosen  
by different kinds of senders.'*

Such expectations can be implemented in models when 'kinds' are operationalized by categorical variables.

Consider categorical nodal covariates  $U, V$ ,  
with values  $0, 1, 2, \dots$ ,  
which divide the node set into several groups.

*especially helpful for more than two groups*

Suppose that  $U$  is for the first node set (senders),  
and  $V$  is for the second node set (receivers).

*Node sets may be the same: one-mode.*

For two-mode networks especially, such category systems can be important.  
The following pages give formulas for effects for actor  $i$  in network  $X$ .

*‘senders differ in the kinds of receivers they choose.’*

*[outAct]* restricted to ties to nodes of the same  $V$  :

*outdegree activity to homogeneous covariate*

(internal effect parameter  $p = 3$ )

*[homXOutAct2]* defined by the sum of  $i$ ’s outgoing ties  
weighted by the proportion of  $i$ ’s outgoing ties  
to nodes with the same covariate value (as receiver  $j$ ),

$$s_i(x) = \sum_j x_{ij} \frac{\sum_h x_{ih} I\{v_h = v_j > 0\}}{\sum_h I\{v_h = v_j > 0\}} .$$

(The restriction  $v_j > 0$  is to give a possibility for handling missing data,  
or irrelevant categories.)

$\beta_k > 0$ : actors specialize in specific  $V$ -categories;

$\beta_k < 0$ : actors do not want many ties to same  $V$ -category.

*‘receivers differ in the kinds of senders choosing them.’*

*[inPop]* restricted to ties from actors of the same  $U$  :

*indegree popularity from same covariate*

(internal effect parameter  $p = 3$ )

*[sameXInPop]* defined by the sum over  $j$  of outgoing ties  $i \rightarrow j$  weighted by the proportion of the incoming ties of  $j$  from actors with the same covariate value (as sender  $i$ ),

$$s_i(x) = \sum_j x_{ij} \frac{\sum_h x_{hj} I\{u_h = u_i\}}{\sum_h I\{u_h = u_i\}}.$$

$\beta_k > 0$ : actors follow others' choices in same  $U$ -category;

$\beta_k < 0$ : actors repelled by others' choices in same  $U$ -category.

*‘Different kinds of receivers chosen by different kinds of senders.’*

*indegree popularity from same cov1 to same cov2, [(sameXVInPop)]*

(name ‘popularity’ is not very appropriate....)

(internal effect parameter  $p = 3$ )

defined by the sum over  $j$  of ties  $i \rightarrow j$

weighted by the density of the  $(u_i, v_j)$  block,

$$s_i(x) = \sum_j x_{ij} \frac{\sum_{h,k} x_{hk} I\{u_h = u_i > 0, v_k = v_j > 0\}}{\sum_{h,k} I\{u_h = u_i > 0, v_k = v_j > 0\}}.$$

This is similar to a **pre-specified stochastic block model**.

It is an elementary effect.

(The restriction  $u_i > 0, v_j > 0$  is to give a possibility for handling missing data, or irrelevant categories.)

$\beta_k > 0$ : actors adapt to the current block structure;

$\beta_k < 0$ : actors equalize the current block structure.

Since  $[(\text{sameXVInPop})]$  is an elementary effect,  
it really should be indicated as

$$e_{kij}^{\text{el}}(x) = \frac{\sum_{h,k} x_{hk} I\{u_h = u_i > 0, v_k = v_j > 0\}}{\sum_{h,k} I\{u_h = u_i > 0, v_k = v_j > 0\}} \dots$$

It might be regarded as a contextual effect,  
the context being given by the current  $(U, V)$  block structure.

The motivation to have these effects  
is the ambiguity that sometimes occurs in the delineation  
of the second mode of a two-mode network,  
which may lead to quite heterogeneous node sets.

But they may be useful quite generally.

