Part A Statistics HT 2004 **Exercises 3** Dr. Gesine Reinert

1. Le $X_{(n)}$ be the largest value in a sample of size n drawn from the uniform distribution on $[0, \theta]$. Show that $X_{(n)}/\theta$ is a pivot. Using this pivot, find a $100(1-\alpha)\%$ confidence interval for θ.

2. An investigator wants to show that first-born children score higher on IQ tests than second-borns. In one school district, he finds 400 two-child families with both children enrolled in primary school. He gives the children a vocabulary test, consisting of 40 words which the child has to define; 2 points are given for a correct answer, and 1 point for a partially correct answer. The results are that the 400 first-born average 29, and their sample standard deviation is 10; the 400 second-borns average 28, and their sample standard deviation is 10.

   (a) Find a 95% confidence interval for the difference of the means. Which assumptions did you make?

   (b) What can you conclude?

3. Let $X_1, \ldots, X_n$ be a random sample from the distribution with p.d.f.

$$f(x;\theta) = \theta x^{\theta-1}, \quad 0 < x < 1,$$

   where $\theta > 0$ is an unknown parameter.

   (a) Find the m.o.m. estimator for θ using the function $h(x) = x$.

   (b) Find the m.o.m. estimator for θ using the function $h(x) = \ln x$.

   (c) Find the m.l.e. estimator for θ, and explain the relationship to the m.o.m. estimator in (b).

4. A number of customers visit a second-hand bookshop every day, but only few of them actually buy anything. Suppose that the probability that a customer buys a book is p, unknown. The bookshop owner would like to estimate p by counting the number of customers until a customer buys a book. Find a suitable model for this, and find an estimator for the parameter p, using

   (a) the method of moments and

   (b) maximum likelihood.

   If instead the bookshop owner wanted to estimate p based on the number of customers until the second customer buys a book, how would the estimators change?

5. The *Weibull distribution* with parameters λ and β has p.d.f.

$$f(x;\lambda,\beta) = \beta\lambda^{\beta}x^{\beta-1}\exp(-(\lambda x)^{\beta})$$

   for $x > 0$. Let $X_1, \ldots, X_n$ be a random sample from the Weibull distribution with parameter $\lambda = 2$ and unknown β. Explain how to find the m.l.e. for β.