A comparison of various correlation measures

Load data from the attached csv file with the prices of each of the following financial instruments {'SPY','^VIX','^TNX', 'OIL','GLD','GOLD','^N225','^FTSE','SPXS','SPXL'}

Let P denote the prices matrix, of size $T \times n$, where T is the number of days in history, and n is the number of instruments. For each time series of a given instrument, compute the so called log-returns

$$R_i(t) = log(R_i(t)/R_i(t-1))$$

You can do so, for all instruments at once, in R via the command

$$R = \log(P[2:T,]/P[1:(T-1),])$$

1) Plot a histogram of the returns, for each of the ten instruments.

2) Compute the sample mean and variance of the return, for each instrument.

Next, we will be computing various measures of correlation between the above instruments, namely the following six different correlation measures: Pearson, Spearman, Hoeffding, Maximal Correlation, dCor, and MIC. Here are some useful tips to keep in mind when implementing this in R:

- Spearman and Pearson correlation (standard; *cor* in R)
- maximal correlation: *library(acepack)* d(the corresponding R package). Note that you first need to compute *q = ace(x,y)*, and then the maximal correlation via *cor(atx, aty)*. Note that you cannot do this for all pairs of instruments at once, but only sequentially for each pair of instruments, i.e., run:

  transfVars = ace(R[,i], R[,j]);

  for every pair of instruments i, j, and compute the final maximal correlation between (i, j) as

  cor(transfVars$tx, transfVars$ty)

  (the usual Pearson correlation between the transformed values)
- Hoeffding's D: *library(Hmisc); hoeffd(R)$D* returns the entire correlation matrix
- distance correlation: *library(energy)*; using *dcor(x,y)*. Note that you have to do this for all pairs (columns of R) individually.
- MIC: *library(minerva)*; the command *mine(R, n.cores = 4)$MIC* returns the entire correlation matrix at once.

Note: to install a given R package, use the command *install.packages()*. For example, if you wish to install the acepack package, just run *install.packages('acepack')*;

3) Convince yourself that there is a trivial (positive/negative) relationship between any pair from SPY, $SPXS$, and $SPXL$ (in other words, $SPXS$ and $SPXL$ are leveraged and inverse leveraged ETFs for SPY). Compute the Pearson, Spearman, Hoeffding, Maximal Correlation, dCor, and MIC correlation coefficients between these three instruments. Can you guess from the data what do $SPXS$ and $SPXL$

represent, in relationship to SPY? What can you conclude from the results, regarding the performance of these different six different correlation measure?

4) Consider the subset {'SPY','^VIX','^TNX', 'OIL','GLD','GOLD','^N225','^FTSE'} (note I left out SPXS and SPXL). For each pair of instruments, plot their relationship as well as the values of the six different measures of correlation. I attached here an example of how the end result should look like, for two different pairs. Feel free to use the R script I put together for this particular format (with the legend containing the correlation values), in particular the function `plot_results()`.

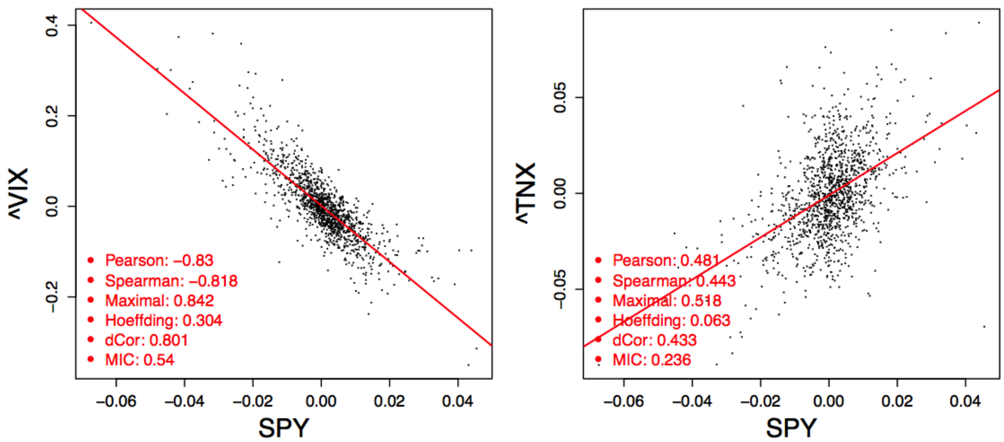

Figure 1: Analysis of pairs of instruments.

Note: In R, when printing to a pdf, you can combine multiple plots on the same graph in a table format, using the command par(mfrow = c(3,5))) which prints to table with 3 rows and 5 columns, for example. See for example `http://www.statmethods.net/advgraphs/layout.html`

5) Name a few of the strongest relationships that you observe in (4).

6) If your goal was to predict future stock returns, and you only had access to the above different types of correlation measures, what would you do to discover variables that affect future returns? Can you give a few simple examples? (Hint: lag, causation).

(7) Pick your favorite such new variable, and illustrate it on your favorite pair of instruments. Show your results in a plot similar to the one in Figure 1, where the y-axis is the future return of some instrument A and the x-axis is your newly defined variable corresponding to some instrument B.