Mathematics and Statistics Undergraduate Handbook
Supplement to the Handbook

Honour School of Mathematics and Statistics
Syllabus and Synopses for Part B 2023–2024
for examination in 2024

Contents

Every effort is made to ensure that the list of courses offered is accurate at the time of going online. However, students are advised to check the up-to-date version of this document on the Department of Statistics website.

Notice of misprints or errors of any kind, and suggestions for improvements in this booklet should be addressed to the Academic Administrator in the Department of Statistics (academic.administrator@stats.ox.ac.uk).

Version 1.1 – History of Maths added to other units section.

October 2023

1. Honour School of Mathematics and Statistics

1.1 Units and double-units and methods of examination

See the current edition of the Examination Regulations at
https://examregs.admin.ox.ac.uk
for the full regulations governing these examinations. The examination conventions can be found on the Canvas course examinations and assessment page.

In Part B each candidate shall offer a total of **eight units** from the schedule of units and double units.

(a) Each candidate shall offer the double unit SB1.
(b) Each candidate shall offer a total of at least two units from SB2 and SB3.
(c) Each candidate may offer a total of at most two units from the schedule of 'Other units'.
(d) Each candidate may offer at most one double unit which is an Extended Essay or Structured Project.

Note: Units from the schedule of 'Mathematics Department units' for Part B of the Honour School of Mathematics are also available – see Section 3.

Students are asked to register for the options they intend to take by the end of week 10, Trinity Term 2023 using the Mathematical Institute course management portal. https://courses.maths.ox.ac.uk/ . Students may alter the options they have registered for after this but it is helpful if their registration is as accurate as possible. Students will then be asked to sign up for classes at the start of Michaelmas Term 2023. Students who register for a course or courses for which there is a quota should consider registering for an additional course (by way of a "reserve choice") in case they do not receive a place on the course with the quota.

Every effort will be made when timetabling lectures to ensure that mathematics lectures do not clash. However, because of the large number of options this may sometimes be unavoidable.

1.2 Part B courses in future years

In any year, most courses available in Part B that year will normally also be available in Part B the following year. However, sometimes new options will be added or existing options may cease to run. The list of courses that will be available in Part B in any year will be published by the end of the preceding Trinity Term.

Details of Part C units, examinable in 2025, will be published before Michaelmas Term 2024.

1.3 **Course list by term**

The list of 2023-2024 Part B courses by term is:

Michaelmas Term

SB1.1 Applied Statistics [double unit with SB1.2]
SB2.1 Foundations of Statistical Inference

Hilary Term

SB1.2 Computational Statistics [double unit with SB1.1]
SB2.2 Statistical Machine Learning
SB3.1 Applied Probability

2 Statistics units and double units

2.1 SB1 Applied and Computational Statistics

Level: H-level
Method of Assessment: written examination plus assessed practical assignments. The practical assignments contribute 1/3 of the marks for SB1. Please see below for the hand-in deadlines for practical assignments.
Weight: Double unit.

Prerequisites: A8 Probability and A9 Statistics.

Aims
The course aims to develop the theory of statistical methods, and also to introduce students to the analysis of data using a statistical package. The main topics are: simulation-based inference, practical aspects of linear models, logistic regression and generalized linear models, and computer-intensive methods.

2.1.1 SB1.1 Applied Statistics – 13 MT

Synopsis
The normal linear model: use of matrices, least squares and maximum likelihood estimation, normal equations, distribution theory for the normal model, hypothesis tests and confidence intervals.

Practical aspects of linear models and analysis of variance: multiple regression, categorical variables and interactions, blocks and treatments, orthogonality, model selection (including AIC, but not the derivation of AIC), fit criteria, use of residuals, outliers, leverage, model interpretation.

Normal linear mixed models, hierarchical models.

Generalised Linear Models: logistic regression, linear exponential families and generalized linear models, scale parameter, link functions, canonical link. Maximum likelihood fitting. Iteratively reweighted least squares. Asymptotic theory: statement and applications to inference, analysis of deviance, model checking, residuals.

Reading
A. C. Davison, *Statistical Models*, CUP, 2003
J.J. Faraway, *Linear Models with R*, Chapman and Hall, 2005
A. J. Dobson and A.G Barnett, *An Introduction to Generalized Linear Models*, Chapman and Hall, 2008
J.J. Faraway, *Extending the Linear Model with R: Generalized Linear, Mixed Effects and Nonparametric Regression Models*, Chapman and Hall, 2006

Further Reading
F. L. Ramsey and D. W. Schafer, *The Statistical Sleuth: A Course in Methods of Data Analysis*, 2nd edition, Duxbury, 2002.

2.1.2 SB1.2 Computational Statistics – 13 HT

Synopsis
Smoothing methods (local polynomials). Nonparametric inference (bandwidth and Generalised Cross Validation).
Multivariate smoothers and Generalised Additive Models.

Inference using simulation methods. Monte-Carlo Tests. Permutation tests. Rank statistics.

Bootstrapping.

Hidden Markov Models: specification. Forward-backward algorithm. Kalman filter.

Reading
J. D. Gibbons, *Nonparametric Statistical Inference*, Marcel Dekker, 1985, pp 1-193, 273-290.
G.H. Givens and J.A. Hoeting, *Computational Statistics*, 2nd edition, Wiley, 2012.
G. James, D. Witten, T. Hastie, R. Tibshirani, *An Introduction to Statistical Learning*, Springer , 2013. This book is freely available online: http://www-bcf.usc.edu/~gareth/ISL/
R. H. Randles and D. A. Wolfe, *Introduction to the Theory of Nonparametric Statistics*, Wiley 1979, pp 1-322.
L. Wasserman, *All of Nonparametric Statistics*, Springer, 2005.
L. Wasserman, *All of Statistics*, Springer, 2004.

Further Reading
A.C. Davison and D.V. Hinkley, *Bootstrap Methods and their Application,* CUP, 1997.
C.R. Shalizi, *Advanced Data Analysis from an Elementary Point of View*, http://www.stat.cmu.edu/~cshalizi/ADAfaEPoV/.

Practicals
In addition to the lectures there will be five supervised practicals. Four of these contain problems whose written solutions will be assessed as part of the unit examination.

The hand-in deadlines for the four assessed practicals are:
1st practical: 12 noon Monday week 8, Michaelmas Term 2023
2nd practical: 12 noon Monday week 2, Hilary Term 2024
3rd practical: 12 noon Monday week 8, Hilary Term 2024
4th practical: 12 noon Monday week 2, Trinity Term 2024.

It is expected that submissions will be made online via the Inspera system. Details of how to submit will be provided during Michaelmas Term.

Candidates who miss the above deadlines may ask their college to apply to the Proctors' Office for permission to submit late.

2.2.1 SB2.1 Foundations of Statistical Inference – 16 MT

Level: H-level
Method of Assessment: written examination
Weight: Unit

Prerequisites: A9 Statistics, A8 Probability.

Learning outcomes
Understanding how data can be interpreted in the context of a statistical model. Working knowledge and understanding of key-elements of model-based statistical inference, including awareness of similarities, relationships and differences between Bayesian and frequentist approaches.

Synopsis
Exponential families: Curved and linear exponential families; canonical parametrization; likelihood equations. Sufficiency: Factorization theorem; sufficiency in exponential families.

Frequentist estimation: unbiasedness; method of moments; the Cramer-Rao information inequality; Rao-Blackwell theorem: Lehmann-Scheffé Theorem and Rao-Blackwellization; Statement of complete sufficiency for Exponential families.

The Bayesian paradigm: likelihood principal; subjective probability; prior to posterior analysis; asymptotic normality; conjugacy; examples from exponential families. Choice of prior distribution: proper and improper priors; Jeffreys' and maximum entropy priors. Hierarchical Bayes models.

Decision theory: risk function; Minimax rules, Bayes rules. Point estimators and admissibility of Bayes rules. The James-Stein estimator, shrinkage estimators and Empirical Bayes. Hypothesis testing as decision problem.

Reading
P. H. Garthwaite, I. T. Jolliffe and Byron Jones, *Statistical Inference,* 2nd edition, Oxford University Press, 2002.
G.A. Young and R.L. Smith, *Essentials of Statistical Inference*, Cambridge University Press, 2005.
T. Leonard and J.S.J. Hsu, *Bayesian Methods*, Cambridge University Press, 2005.

Further reading
D. Barber, *Bayes Reasoning and Machine Learning*, Cambridge University Press, 2012.
D. R. Cox, *Principles of Statistical Inference*, Cambridge University Press, 2006.
H. Liero and S Zwanzig, *Introduction to the Theory of Statistical Inference*, CRC Press, 2012.

2.2.2 SB2.2 Statistical Machine Learning – 16 HT

Level: H-level
Method of Assessment: Written examination
Weight: Unit

Recommended prerequisites: Part A A9 Statistics and A8 Probability. SB2.1 Foundations of Statistical Inference useful but not essential.

Aims and Objectives
Machine learning studies methods that can automatically detect patterns in data, and then use these patterns to predict future data or other outcomes of interest. It is widely used across many scientific and engineering disciplines.

This course covers statistical fundamentals of machine learning, with a focus on supervised learning and empirical risk minimisation. Both generative and discriminative learning frameworks are discussed and a variety of widely used classification and regression algorithms are overviewed.

Synopsis

Introduction to unsupervised learning. Principal component analysis, dimensionality reduction, singular value decomposition. K-means clustering.
Fundamentals of statistical learning for supervised learning: loss function, risk, Bayes classifier. Plug-in estimators and empirical risk minimisation. Generative and discriminative methods.
Generative classifiers: Linear and quadratic discriminant analysis, naive Bayes.
Further concepts in statistical machine learning: feature expansion, overfitting and bias-variance trade-off, double descent phenomenon and overparameterisation; cross-validation, performance measures, ROC curves
Optimisation for Machine learning: gradient descent and stochastic gradient descent, early stopping
Linear classifiers: least-squares, perceptron, logistic regression
K-nearest neighbours
Decision trees, bagging, random forests, boosting
Neural networks and deep learning

Reading
C. Bishop, *Pattern Recognition and Machine Learning*, Springer, 2007.
T. Hastie, R. Tibshirani, J Friedman, *Elements of Statistical Learning*, Springer, 2009.
K. Murphy, *Machine Learning: a Probabilistic Perspective*, MIT Press, 2012.

Further Reading
B. D. Ripley, *Pattern Recognition and Neural Networks*, Cambridge University Press, 1996.
G. James, D. Witten, T. Hastie, R. Tibshirani, *An Introduction to Statistical Learning*, Springer, 2013.

2.3.1 SB3.1 Applied Probability – 16 HT

Level: H-level
Method of Assessment: written examination
Weight: Unit.

Prerequisite: A8 Probability.

Aims and Objectives
This course is intended to show the power and range of probability by considering real examples in which probabilistic modelling is inescapable and useful. Theory will be developed as required to deal with the examples.

Synopsis
Poisson processes and birth processes. Continuous-time Markov chains. Transition rates, jump chains and holding times. Forward and backward equations. Class structure, hitting times and absorption probabilities. Recurrence and transience. Invariant distributions and limiting behaviour. Time reversal.

Renewal theory. Limit theorems: strong law of large numbers, strong law and central limit theorem of renewal theory, elementary renewal theorem, renewal theorem, key renewal theorem. Excess life, inspection paradox.

Applications in areas such as: queues and queueing networks - M/M/s queue, Erlang's formula, queues in tandem and networks of queues, M/G/1 and G/M/1 queues; insurance ruin models; stochastic epidemic models.

Reading
J.R. Norris: *Markov Chains*. Cambridge University Press, 1997.
G.R. Grimmett and D.R. Stirzaker: *Probability and Random Processes*, 3rd edition, Oxford University Press, 2001.
S.M. Ross: I*ntroduction to Probability Models*, 4th edition, Academic Press, 1989.

3 Mathematical and Other units

The other units that students in Part B Mathematics and Statistics may take are drawn from Part B of the Honour School of Mathematics.
For full details of these units, see
https://courses.maths.ox.ac.uk/course/index.php?categoryid=735

3.1 Mathematics units

The Mathematics units that are available are as follows:

B1.1: Logic	16 MT
B1.2: Set Theory	16 HT
B2.1: Introduction to Representation Theory	16 MT
B2.2: Commutative Algebra	16 HT
B2.3: Lie Algebras	16 MT
B3.1: Galois Theory	16 MT
B3.2: Geometry of Surfaces	16 MT
B3.3: Algebraic Curves	16 HT
B3.4: Algebraic Number Theory	16 HT
B3.5: Topology and Groups	16 MT
B4.1: Functional Analysis I	16 MT
B4.2: Functional Analysis II	16 HT
B4.3 Distribution Theory	16 MT
B4.4 Fourier Analysis	16 HT
B5.1 Stochastic Modelling of Biological Processes	16 HT
B5.2: Applied Partial Differential Equations	16 MT
B5.3: Viscous Flow	16 MT
B5.4: Waves and Compressible Flow	16 HT
B5.5: Further Mathematical Biology	16 MT
B5.6: Nonlinear Dynamics, Bifurcations and Chaos	16 HT
B6.1 Numerical Solution of Partial Differential Equations	16 MT
B6.2 Optimisation for Data Science	16 HT
B6.3 Integer Programming	16 MT
B7.1 Classical Mechanics	16 MT
B7.2 Electromagnetism	16 HT
B7.3 Further Quantum Theory	16 HT
B8.1: Probability, Measure and Martingales	16 MT
B8.2: Continuous Martingales and Stochastic Calculus	16 HT
B8.3: Mathematical Models of Financial Derivatives	16 HT
B8.4: Information Theory	16 HT
B8.5: Graph Theory	16 MT
OCS2: Computational Complexity	16 HT
BEE Mathematical Extended Essay	MT & HT [double unit]
or	
BSP: Structured Projects	MT & HT [double unit]
[Note: Students **cannot take both** BEE and BSP]	

The units above are the units referred to in Section 1 as ‘Mathematics Department units’ for Part B of the Honour School of Mathematics.

See the “Projects Guidance Notes” on the web at

https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/projects for more information on the Extended Essay option.

Please note that the following **are not permitted options** in Part B of the Honour School of Mathematics and Statistics:
BOE ''Other Mathematical'' Extended Essay

3.2 Other units

The units in this subsection, Section 3.2, are those referred to in (c) of Section 1 as the schedule of 'Other units'.

The other units available are as follows:

BO1.1 History of Mathematics

(BN1.1 Mathematics Education and BN 1.2 Undergraduate Ambassadors' Scheme are not running this year.)