Mathematics and Statistics Undergraduate Handbook
Supplement to the Handbook

Honour School of Mathematics and Statistics
Syllabus and Synopses for Part A 2023–2024
for examination in 2024

Contents

Notice of misprints or errors of any kind, and suggestions for improvements in this booklet should be addressed to the Academic Administrator in the Department of Statistics academic.administrator@stats.ox.ac.uk.

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1. **Honour School of Mathematics and Statistics**

Please see the current edition of the
https://examregs.admin.ox.ac.uk for the full regulations governing these examinations.

In Part A Mathematics and Statistics each candidate shall be required to offer the following **six** compulsory written papers:

- A0 Linear Algebra (1.5 hours)
- A1 Differential Equations 1 (1.5 hours)
- A2 Metric Spaces and Complex Analysis (3 hours)
- A8 Probability (1.5 hours)
- A9 Statistics (1.5 hours)
- ASO Short Options (1.5 hours)

and also three or four papers from the Long Options (each 1.5 hours long)

- A3 Rings and Modules
- A4 Integration
- A5 Topology
- A6 Differential Equations 2
- A7 Numerical Analysis
- A10 Fluids and Waves
- A11 Quantum Theory
- A12 Simulation and Statistical Programming

Paper ASO will examine the nine Short Options (Number Theory, Group Theory, Projective Geometry, Multidimensional Analysis and Geometry, Integral Transforms, Calculus of Variations, Graph Theory, Special Relativity, Mathematical Modelling in Biology). Students are recommended to take three of these Short Options.

Taking 3 or 4 long options.
A student taking 3 long options will still have each of them counting as a unit's weight towards their overall second year mark.
For a student taking 4 long options, their best 2 papers (following the exams) will count one unit each and their worst 2 papers will count half a unit each. Thus these 4 papers will overall still have a weight of 3 units.
The results from all 4 papers will appear on the student's exam transcript.

The aim of the above scoring system is to ensure anyone taking on an extra option will not do so lightly (all marks will be reported and all count to some extent) but also that a student will not get a lower overall mark for having taken on the extra workload (the given scoring system was a fair compromise looking at several years' data sets). We are anticipating that most students will not wish to take on the extra workload, and that in most years it would be some subset of the first-class students wishing to take 4 long options.

Part A shall be taken on one occasion only (there will be no resits). At the end of the Part A examination, a candidate will be awarded nine or ten 'University Standardised Marks' (USMs) for their performance in Part A – one USM for each paper taken in Part A. The

USM from papers A2 will have twice the weight of the USMs awarded for the other papers. A weighted average of the USMs will be carried forward for the classification awarded at the end of the third year, with this average from the second year papers counting for 40%.

Syllabus and Synopses

The syllabus details in this booklet are those referred to in the Examination Regulations and have been approved by the Statistics Teaching Committee for examination in Trinity Term 2024. The synopses in this booklet give some additional detail, and show how the material is split between the different lecture courses. They also include details of recommended reading.

2. CORE MATERIAL

2.1 Syllabi

The examination syllabi of the core papers A0, A1 and A2 shall be the mathematical content of the synopses for the courses

Linear Algebra
Differential Equations
Metric Spaces and Complex Analysis

For the synopses for these courses, see those for Part A of the Honour School of Mathematics, which are available on the web at
https://courses.maths.ox.ac.uk/course/index.php?categoryid=741

The examination syllabi of the core papers A8 and A9 shall be the mathematical content of the synopses for the courses

Probability
Statistics

2.2. Synopses of lectures

This section contains the lecture synopses associated with the two core papers A8 and A9.

2.2.1 A8 Probability – 16 lectures MT

Aims and objectives
The first half of the course takes further the probability theory that was developed in the first year. The aim is to build up a range of techniques that will be useful in dealing with mathematical models involving uncertainty. The second half of the course is concerned with Markov chains in discrete time and Poisson processes in one dimension, both with developing the relevant theory and giving examples of applications.

Synopsis
Continuous random variables. Jointly continuous random variables, independence, conditioning, functions of one or more random variables, change of variables. Examples including some with later applications in statistics.

Moment generating functions and applications. Statements of the continuity and uniqueness theorems for moment generating functions. Characteristic functions (definition only). Convergence in distribution and convergence in probability. Weak law of large numbers and central limit theorem for independent identically distributed random variables. Strong law of large numbers (proof not examinable).

Discrete-time Markov chains: definition, transition matrix, n-step transition probabilities, communicating classes, absorption, irreducibility, periodicity, calculation of hitting probabilities and mean hitting times. Recurrence and transience. Invariant distributions, mean return time, positive recurrence, convergence to equilibrium (proof not examinable), ergodic theorem (proof not examinable). Reversibility, detailed balance equations. Random walks (including symmetric and asymmetric random walks on Z, and symmetric random walks on Z^d).

Reading

G. R. Grimmett and D. R. Stirzaker, *Probability and Random Processes* (3rd edition, OUP, (2001). Chapters 4, 6.1-6.5, 6.8.
G.R. Grimmett and D. R. Stirzaker, *One Thousand Exercises in Probability* (OUP, 2001).
G. R. Grimmett and D J A Welsh, *Probability: An Introduction* (OUP, 1986). Chapters 6, 7.4, 8, 11.1-11.3.
J. R. Norris, *Markov Chains* (CUP, 1997). Chapter 1.
D. R. Stirzaker, *Elementary Probability* (Second edition, CUP, 2003). Chapters 7-9 excluding 9.9.

2.2.2 **A9 Statistics – 16 lectures HT**

Overview

Building on the first year course, this course develops statistics for mathematicians, emphasising both its underlying mathematical structure and its application to the logical interpretation of scientific data. Advances in theoretical statistics are generally driven by the need to analyse new and interesting data which come from all walks of life.

Learning Outcomes

At the end of the course students should have an understanding of: the use of probability plots to investigate plausible probability models for a set of data; maximum likelihood estimation and large sample properties of maximum likelihood estimators; hypothesis tests and confidence intervals (and the relationship between them). They should have a corresponding understanding of similar concepts in Bayesian inference.

Synopsis

Order statistics, probability plots.

Estimation: observed and expected information, statement of large sample properties of maximum likelihood estimators in the regular case, methods for calculating maximum likelihood estimates, large sample distribution of sample estimators using the delta method.

Hypothesis testing: simple and composite hypotheses, size, power and p-values, Neyman-Pearson lemma, distribution theory for testing means and variances in the normal model, generalized likelihood ratio, statement of its large sample distribution under the null hypothesis, analysis of count data.

Confidence intervals: exact intervals, approximate intervals using large sample theory, relationship to hypothesis testing.

Probability and Bayesian Inference. Posterior and prior probability densities. Constructing priors including conjugate priors, subjective priors, Jeffreys priors. Bayes estimators and credible intervals. Statement of asymptotic normality of the posterior. Model choice via posterior probabilities and Bayes factors.

Examples: statistical techniques will be illustrated with relevant datasets in the lectures.

Reading
F Daly, D J Hand, M C Jones, A D Lunn and K J McConway, *Elements of Statistics*, Addison Wesley (1995) Chapters 7-10 (and Chapters 1-6 for background)
J A Rice*, Mathematical Statistics and Data Analysis*, 2nd edition, Wadsworth (1995) Sections 8.5, 8.6, 9.1-9.7, 9.9, 10.3-10.6, 11.2, 11.3, 12.2.1, 13.3, 13.4.
T Leonard and J S J Hsu *Bayesian Methods*, Cambridge, Chapters 2 and 3.

Further Reading
G Casella and R L Berger, *Statistical Inference,* 2nd edition, Wadsworth (2001)
A C Davison, *Statistical Models*, Chapter 11.

3 OPTIONS

3.1 Syllabi

The examination syllabi of the options papers A3-A7 and A10-A12 shall be the mathematical content of the synopses for the courses

- A3 Rings and Modules
- A4 Integration
- A5 Topology
- A6 Differential Equations 2
- A7 Numerical Analysis
- A10 Fluids and Waves
- A11 Quantum Theory
- A12 Simulation and Statistical Programming

For the synopses of A3-A7 and A10-A11, see those for Part A of the Honour School of Mathematics, which are available on the web at
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3.2 Synopsis of Lectures

3.2.1 A12 Simulation and Statistical Programming – 14 lectures and 6 practicals HT

The workload of this course is equivalent to a 16-lecture course.

Aims and Objectives
Building on Part A probability and first year statistics, this course introduces Monte Carlo methods, collectively one of the most important toolkits for modern statistical inference. In parallel, students are taught programming in R, a programming language widely used in statistics. Lectures alternate between Monte Carlo methods and Statistical Programming so that students learn to programme by writing simulation algorithms.

Synopsis
Simulation: Transformation methods. Rejection sampling including proof for a scalar random variable, Importance Sampling. Unbiased and consistent IS estimators. MCMC including the Metropolis-Hastings algorithm.

Statistical Programming: Numbers, strings, vectors, matrices, data frames and lists, and Boolean variables in R. Calling functions. Input and Output. Writing functions and flow control. Scope. Recursion. Runtime as a function of input size. Solving systems of linear equations, Cholesky decomposition. Numerical stability. Regression and least squares, QR factorisation. Implementation of Monte Carlo methods for elementary Bayesian inference.

Course Structure
The course will consist of fourteen lectures. Six of these will be held in a computer laboratory and are followed by an associated practical session. There will be four classes on problem sheets.

Reading
W J Braun and D J Murdoch, *A First Course in Statistical Programming with R*, CUP 2007
S M Ross, *Simulation,* Elsevier, 4th edition, 2006
J R Norris, *Markov Chains*, CUP, 1997

Reference
C P Robert and G Casella, *Monte Carlo Statistical Methods*, Springer, 2004
B D Ripley, *Stochastic Simulation*, Wiley, 1987

4. SHORT OPTIONS

4.1 Syllabi and Synopses

The examination syllabi of the short options paper ASO shall be the mathematical content of the synopses for the courses

Number Theory
Group Theory
Projective Geometry
Multidimensional Analysis and Geometry
Integral Transforms
Calculus of Variations
Graph Theory
Special Relativity
Mathematical Modelling in Biology

For the synopses for these courses, see those for Part A of the Honour School of Mathematics, which are available on the web at
https://courses.maths.ox.ac.uk/course/index.php?categoryid=741