

Analysing the dynamics of valued networks

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1. Longitudinal Modeling of Valued Networks

Work on modeling network dynamics

(including statistical modeling in *Siena* program)

has concentrated on binary tie variables \sim (directed) graphs.

However, often using valued ties is more natural: for example,

- *strong and weak ties*
- *positive and negative ties*

This presentation is about the use of **RSiena** for valued ties.



Notation and data

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- 4 Continuous time parameter t ,
observation moments t_1, \dots, t_M .



Model assumptions

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- 1 $X(t)$ is a Markov process in continuous time, although observed at discrete moments.
- 2 Condition on the first observation $X(t_1)$ and do not model it: no assumption of a stationary marginal distribution.
- 3 Micro step:
At any time moment, a tie variable X_{ij} can only change by one step: -1 or $+1$.

cf. Holland & Leinhardt 1977.

Analogous to micro steps in dynamics of binary networks;
more natural for ordered discrete with few values
than for count variables with larger sets of values.



Assumptions: actor-driven models

Each actor “controls” his outgoing ties collected in the row vector $(X_{i1}(t), \dots, X_{in}(t))$.

Actors have full information on all variables.

- ④ The change process is decomposed into sub-models:



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- ④ The change process is decomposed into sub-models:
 1. waiting times until the next opportunity for a change made by actor i :
rate functions $\lambda_i(\alpha, \mathbf{x})$;
 2. probabilities of changing X_{ij} , conditional on such an opportunity for change: depend on *objective functions* $f_i(\beta, \mathbf{x}^0, \mathbf{x})$.

The distinction between rate function and objective function separates the model for *how many* changes are made from the model for *which* changes are made.



A useful approach is not to regard the tie values as numerically meaningful, but as ordered thresholds with potentially qualitative differences.

This enables questions such as:

Do reciprocity, transitivity, covariate values, operate differently for transitions between different thresholds?



Level networks

In this approach the valued networks is considered as a series of *level networks* or *stacked digraphs* $X^{(r)}$.

Each positive tie value defines a dichotomization threshold:

$$i \xrightarrow{r} j \text{ is defined by } X_{ij} \geq r$$

for $r = 1, \dots, R$; define this relation as the digraph $X^{(r)}$.



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Example: friendship in categories 'unknown' = 0, 'acquaintance' = 1, 'friend' = 2, 'close friend' = 3.

The processes leading to network structure and network dynamics are treated as (potentially) qualitatively different for crossing each of the thresholds $r - 1 \Rightarrow r$.



Level networks as a multivariate network

The array of level networks $X = (X^{(1)}, X^{(2)}, \dots, X^{(R)})$

is treated as a multivariate network:

they co-evolve, i.e., their dynamics are interdependent.

Their co-evolution is subject to the **restriction that**

$$X^{(r)} \geq X^{(r+1)} \quad \text{for all } r, 1 \leq r < R.$$

This means that $x_{ij}^{(r)}$ can change from 1 to 0 only if $x_{ij}^{(r+1)} = 0$;

and it can change from 0 to 1 only if $x_{ij}^{(r-1)} = 1$.

This implies that the multinomial choices have smaller option sets.



Dealing with the smaller option set

The option set of tie changes for level r consists of all j for which

$$x_{ij}^{(r+1)} = 0 \text{ and } x_{ij}^{(r-1)} = 1 ,$$

the tie exists at the lower level but not at the higher level.

Most networks are sparse, so that the restriction $x_{ij}^{(r-1)} = 1$, a tie existing at the lower level $r - 1$, will be most severe.

It may be advisable to use the outdegree at level $r - 1$ (perhaps log- or sqrt-transformed) as a 'control' effect for level r by using the `[outActIntn]` effect.

E.g., if the names are X1 (lower level) and X2 (this level),

```
setEffect(..., outActIntn, name="X2", interaction1="X1",
           parameter=0)
```

where `parameter=0` is for a log-transformation.



Definition of the stacked relations model

In this model, each threshold transition $r - 1 \Rightarrow r$,
i.e., dependent network $X^{(r)}$ subject to restriction $X^{(r-1)} \geq X^{(r)} \geq X^{(r+1)}$,
has a specific objective function

$$f_i^{(r)}(\beta, x^0, x) = \sum_{k=1}^L \beta_k^{(r)} s_{ik}^{(r)}(x^0, x).$$

Consider two subsequent states x^0 and x ;
note that these can differ in at most one tie value.

Change from $x_{ij}^0 = r$ to $x_{ij} = r + 1$ is based
on comparing the network states according to objective function $f_i^{(r+1)}$;

change from $x_{ij}^0 = r - 1$ to $x_{ij} = r$ is based
on comparing the network states according to objective function $f_i^{(r)}$.

Also for each transition there is a separate rate function $\lambda_i^{(r)}(\alpha, x)$.



Representation in RSiena by the 'higher' attribute

For multivariate networks, the ordering of the level networks is represented by the attribute **higher**, as explained in Section 5.6 of the manual.

This ordering is automatically noted by **RSiena**, and maintained during the simulations so that all simulated networks can be regarded as level networks of a valued network.



For the model specification

For each level r there are separate coefficients $\beta_k^{(r)}$, reflecting the potential qualitative differences between the dichotomized relations.

This gives the possibility, e.g., to investigate how homophily for a given covariate depends on the strength of the tie to be created.

It may be attractive, but it is not necessary, to use the same effects s_{ik} for modeling each of the $R - 1$ transitions.

If this leads to a model which is too extensive, one possibility is to specify some of the effects with `fix=TRUE`, `test=TRUE`, so they will not complicate the estimation but you do have a check on whether they are ignorable indeed.



Hierarchy in the model specification

For the Stochastic Actor-oriented Model in general, there is the **hierarchy requirement** that for interpretation it is helpful that for each effect in the model also the effects expressing sub-configurations are included.

Like for other multivariate Stochastic Actor-oriented Models, this can come at the cost of a large number of parameters, which may be hard to estimate from the data.

Here again, if this leads to a model which is too extensive, one possibility is to specify some of the effects for the sub-configurations with `fix=TRUE`, `test=TRUE`, so they will not complicate the estimation but you do have a check on whether they are ignorable indeed.



Estimation

Estimation can be carried out by **siena07** using the method of moments, similarly to estimation for other network data.

Maximum likelihood estimation is also possible using MCMC methods.



2. Example: Studies Gerhard van de Bunt

Longitudinal study: panel design.

- Study of 32 freshman university students,
7 waves (numbered 0–6) in one year.

van de Bunt, van Duijn, & Snijders, *Comp. & Math. Org. Theory*, **5** (1999), 167 – 192.

We use waves 2–5 (omitting startup processes).

Categories recoded here as follows:

0	unknown or troubled relation
1	known, neutral relation
2	friendly relation
3	friend or best friend.



Average degrees for separate tie values:

observation 1	2	3	4	5
av. degree, value ≥ 1	17.9	17.3	18.7	20.4
av. degree, value ≥ 2	4.5	5.4	6.7	7.5
av. degree, value 3	1.7	2.0	2.4	2.8

Aggregated changes between subsequent observations concentrated about diagonal (as expected):

	to			
from	0	1	2	3
0	1920	548	122	21
1	15	1265	164	3
2	0	114	271	73
3	0	1	22	189

Note that transitions between values r and q for $|r - q| \geq 2$ will be modeled as the result of at least 2 micro-steps.



RSiena data set

The three level networks are given to **RSiena** as a multivariate network.

```
> vdb.ordered2345 <- sienaDataCreate(friendly2345, friends2345, cfriends2345,
+   sex, program, smoke)
Network friendly2345 is higher than network friends2345.
Network friendly2345 is higher than network cfriends2345.
Network friends2345 is higher than network cfriends2345.
```

This will be respected in the simulations. If this is not desired, change attribute 'higher' by function `sienaDataConstraint`.

The attribute 'higher' is

```
> attr(vdb.ordered2345, "higher")
friendly2345,friendly2345  friends2345,friendly2345  cfriends2345,friendly2345
                             FALSE                             FALSE                             FALSE
friendly2345,friends2345  friends2345,friends2345  cfriends2345,friends2345
                             TRUE                             FALSE                             FALSE
friendly2345,cfriends2345 friends2345,cfriends2345 cfriends2345,cfriends2345
                             TRUE                             TRUE                             FALSE
```



```
> vdb.ordered2345
```

```
Dependent variables: friendly2345, friends2345, cfriends2345
```

```
Number of observations: 4
```

```
Nodeset          Actors
Number of nodes      32
```

```
Dependent variable friendly2345
Type                oneMode
Observations        4
Nodeset             Actors
Densities           0.58 0.56 0.6 0.66
```

```
Dependent variable friends2345
Type                oneMode
Observations        4
Nodeset             Actors
Densities           0.14 0.18 0.22 0.24
```

```
Dependent variable cfriends2345
Type                oneMode
Observations        4
Nodeset             Actors
Densities           0.053 0.066 0.077 0.091
```

```
Constant covariates: sex, program, smoke
```

```
Network friendly2345 is higher than network friends2345.
Network friendly2345 is higher than network cfriends2345.
Network friends2345 is higher than network cfriends2345.
```

```
This will be respected in the simulations.
```

```
If this is not desired, change attribute 'higher'.
```



Results for a basic model

Effect	null \Rightarrow known		known \Rightarrow friendly		friendly \Rightarrow friend	
	par.	(s.e.)	par.	(s.e.)	par.	(s.e.)
outdegree (density)	-0.801 [†]	(0.434)	-1.458***	(0.156)	-1.710***	(0.379)
reciprocity	-0.573 [†]	(0.330)	0.995***	(0.213)	1.064*	(0.484)
transitive triplets	0.108***	(0.021)	0.169***	(0.031)	0.283*	(0.119)
same sex	0.872*	(0.341)	0.178	(0.148)	1.016**	(0.382)
program similarity	2.480***	(0.476)	0.643***	(0.195)	0.108	(0.398)
lower outd. activity	—		0.016	(0.013)	0.029	(0.040)

[†] $p < 0.1$; * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$;

convergence t ratios all < 0.05 . Overall maximum convergence ratio 0.08.

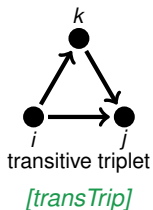
Here, 'lower outd. activity' refers to the *[outActIntn]* effect.

Here `parameter=1` was used (untransformed outdegrees).

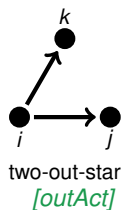
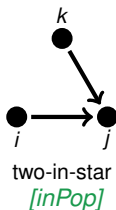
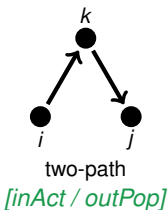
Note the differences for reciprocity, transitivity, same sex, and same program.



The previous ('basic') model is not hierarchical:



but it does not
include



To make conclusions about transitivity as a mechanism,
the other three effects should be added, at all levels.

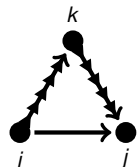


Some cross-network effects

We also study mixed closure effects of the stronger networks:

Do two-paths of strong ties lead to direct weak ties? Granovetter's thesis

Strong ties indicated by 



mixed closure

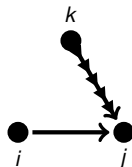
[closure]

has
component
effects

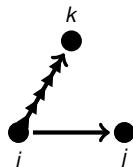


(for strong ties)

[inAct / outPop]



[inPopIntn]



[outActIntn]

Also included: **Does reciprocation by a strong tie lead to a direct tie?**

where 'leading to' is understood as creating and/or maintaining.



Results for extended model

Effect	null \Rightarrow known		known \Rightarrow friendly		friendly \Rightarrow friend	
	par.	(s.e.)	par.	(s.e.)	par.	(s.e.)
outdegree (density)	0.330	(0.979)	-1.697***	(0.191)	-1.707***	(0.403)
reciprocity	-0.684	(0.428)	0.595	(0.581)	0.886 [†]	(0.526)
transitive triplets	0.051	(0.038)	0.204***	(0.047)	0.274*	(0.119)
same sex	1.214*	(0.476)	0.056	(0.161)	1.064**	(0.390)
program similarity	2.898***	(0.742)	0.778***	(0.233)	0.133	(0.404)
reciproc. with stronger	0.624	(0.864)	3.387	(9.202)	—	
indeg. stronger pop.	0.238 [†]	(0.133)	-0.352**	(0.111)	—	
outdeg. stronger activ.	0.147	(0.144)	-0.060	(0.062)	—	
closure of stronger	0.613 [†]	(0.337)	1.279 [†]	(0.773)	—	
outdeg. weaker activ.	—		0.023	(0.015)	0.032	(0.042)

[†] $p < 0.1$; * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$; convergence t ratios all < 0.06 . Overall maximum convergence ratio 0.10.

‘stronger’ indicates the stronger relation, and ‘weaker’ the weaker relation, as an explanatory variable.

‘outdeg. weaker activ.’ refers to the *[outActIntn]* effect.



The comparison between the basic and the extended model shows that effects of covariates (same sex, same program) are quite robust, while effects of reciprocity and transitivity are a bit different, in part because of larger standard errors (extended model may be a bit too extended), in part because the effects of transitivity for the transition null \Rightarrow known is taken up by the mixed closure of the friendly relation, and the effect of reciprocity for the friendly relation is taken up by the mixed reciprocity with 'real' friendship.

For tests of reciprocity and transitivity, note that p -values given here are two-sided, whereas the test should be a one-sided test so the p -values for positive estimates can be divided by 2.



Discussion

- ⇒ Multivariate models can become quite ‘full’ in the sense of having many parameters because of the hierarchy principle.
- ⇒ *How much complexity should we entertain in practice?*
- ⇒ Score-type tests can help reducing some of the complexity of the model.
- ⇒ Signed (i.e., positive & negative) networks can be handled in a similar way, using the ‘disjoint’ attribute:
use the network of positive ties and the network of negative ties as two interdependent networks in the multivariate approach.
- ⇒ Constraints of the option set do not work for multi-group data (including *sienaBayes*);
instead, they can be implemented by fixing parameters of the relevant creation and endowment effects to large negative values (e.g., -5 or -20).

