The dynamics of two-mode networks

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Two-mode networks

Next to the well-known one-mode networks, actors in the network can be *affiliated* with various groupings, activities, cognitions, etc.:

this can be represented by *two-mode* ('bipartite') networks, with
a set $\mathcal{N}$ of actors (the 'actor mode') and
a set $\mathcal{M}$ of groupings (the 'group mode');
where the tie $i \rightarrow j$ for $i \in \mathcal{N}, j \in \mathcal{M}$ means that i is a member of grouping j.

By default in RSiena,
the second mode has no agency (i.e., makes no choices).

there are possibilities with agency for the second mode: see script
`TwoModeAsSymmetricOneMode_Siena.R` and
Tom A.B. Snijders and Beata Łopaciuk-Gonczaryk, "Double agency and co-evolution for two-mode networks, with an application to corporate interlocks and firms' environmental performance" (2025), *Social Networks*, 83, 92-104.

Two-mode networks

Borgatti and Everett (*Social Networks*, 1997) have a general paper about *Network analysis of two-mode data*.

A basic notion is the insight by Breiger (1974)
about the *duality between persons and groups*:

A person is defined by the groups s/he is a member of;
a group is defined by its members;
this is simultaneously implied by the two-mode structure.

There is not a conceptual absolute criterion determining
whether a representation as a two-mode network
makes sense for a given set of binary attributes;
this depends on the network-analytic techniques used.

Two-mode networks

A variety of sets have been used for the second mode, e.g.:

1. *durable social groups*
   (clubs, associations, sport teams)
2. *transitory social groupings*
   (meetings, Southern women)
   $\Rightarrow$ *transitory groups cannot be analyzed by the SAOM*
3. *activities* (sports, leisure activities, frequented bars)
4. *behavioral tendencies* (delinquency items, drinking items)
5. *internal structure* (e.g., medical specialties of hospitals –
   Hollway, Pallotti, Lomi, Stadtfeld, *Network Science*, 2017)
6. *cognitions* (opinions, perceptions).

Two-mode networks

Such sets may be regarded as two-mode social networks
because they are relevant also
for the one-mode social networks between the actors.

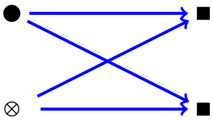
Network delineation for two-mode networks
(different from for one-mode networks):
The second mode should be a sufficiently complete set
of distinct options in non-exclusive choices.

For Stochastic Actor-oriented Models, the nodes of course should be 'states',
which excludes transitory groupings such as meetings.

Transitivity for bipartite networks: 4-cycles

For bipartite networks,
other structures are important than for one-mode networks.
Fewer effects are possible:
e.g., no outdegree popularity or indegree activity.
For assortativity, only out-in assortativity.

We meet each other
in various groups.



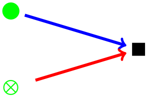
Robins and Alexander (2004):
transitivity in bipartite networks expressed by 4-cycles.

Two-mode homophily

Homophily is a basic mechanism for network dynamics.

(plug: Read the 4-or-5-parameter paper by Snijders & Lomi (*Network Science*, 2019).)

It is also relevant for two-mode networks.



two-mode homophily
'I choose places chosen by my kind of actors'

Groucho Marx:
'I would never become a member of a club that would accept me as a member'

This points to the compatibility between me and the other club members.

Two-mode homophily (2)

Denote the two-mode network by Z.

There are various ways in which an actor covariate V
— i.e., a variable defined on the first mode —
may influence the two-mode network,
in addition to the regular `egoX` effect.

This can be based on the average value of V
for the actors (apart from i) choosing activity j,

$$\check{v}_j^{(-i)} = \begin{cases} \dfrac{\sum_{h \neq i} z_{hj} \, v_h}{z_{+j} - z_{ij}} & \text{if } z_{+j} - z_{ij} > 0 \\ 0 & \text{if } z_{+j} - z_{ij} = 0 . \end{cases}$$

Two-mode homophily (3)

This can be used in the 'alter' form (`altInDist2`)

$$s_{ik}^{(Z)}(z) = \sum_j z_{ij} \check{v}_j^{(-i)}$$

or ininteracted with the popularity of node j (`totInDist2`)

$$s_{ik}^{(Z)}(z) = \sum_j z_{ij} \left(z_{+j} - z_{ij} \right) \check{v}_j^{(-i)} \; = \; \sum_j z_{ij} \sum_{h \neq i} z_{hj} \, v_h \; .$$

both interacted with `egoX`-(V);

Two-mode homophily (4)

or in the 'similarity' form (`simEgoInDist2`)

$$s_{ik}^{(Z)}(z) = \sum_j z_{ij} \left(\mathrm{sim}(\check{v})_{ij} - \widehat{\mathrm{sim}^v} \right) ,$$

where the similarity scores $\mathrm{sim}(\check{v})_{ij}$ are defined as

$$\mathrm{sim}(\check{v})_{ij} = \frac{\Delta - |v_i - \check{v}_j^{(-i)}|}{\Delta}$$

for Δ = range(V).

Two-mode homophily (5)

For categorical V, there are further possibilities:

indegree-popularity for same V (`sameXInPop` (internal effect parameter = 3))

$$s^{(Z)}_{ik}(x) = \sum_j z_{ij} \, \frac{\sum_h I\{v_h = v_i\} \, z_{hj}}{\sum_h I\{v_h = v_i\}}$$

(also applicable to one-mode networks),

and four-cycles to the same V (`sameXCycle4`)

$$s^{(Z)}_{ik}(x) = \sum_{j,k} z_{ij} \, z_{ik} \sum_h I\{v_h = v_i\} \, z_{hj} \, z_{hk} \ .$$

"We meet others of the same V category in various groups".

These may be interacted with `egoX` for <u>non-centered</u> dummy variables for a particular category of V,
to indicate that these tendencies may differ across categories of V,

Example: Glasgow friends and pastimes

Example:
West of Scotland 11-16 Study; West et al. (1996 and later).

One school year group from a Scottish secondary school
starting at age 12-13 years, monitored over more than 2 years;
total of 160 pupils, sociometric & behavior questionnaires
at three moments, at appr. 1 year intervals.

Two-mode network: activities.

covariate: gender.

Descriptives for leisure activities

Three waves $\sim$ two periods.

Average degrees 4.7; 4.0; 3.9.

Amount of stability in activities also measured by Jaccard coefficient

$$J = \frac{N_{11}}{N_{01} + N_{10} + N_{11}}$$

where $N_{hk} =$ number of tie variables
with value h at one wave and value k at the next.

J = 0.51 for both periods.

Second mode: Leisure time activities

	daily	weekly	monthly	less
I listen to tapes or CDs	**388**	23	5	16
I look around in the shops	**65**	290	48	30
I read comics, mags or books	**186**	121	65	60
I go to sport matches	**30**	**113**	90	200
I take part in sports	**218**	117	30	68
I hang round in the streets	**216**	64	26	125
I play computer games	**157**	109	45	122
I spend time on hobby (e.g. art, instrument)	**114**	113	36	170
I go to something like B.B., Guides or Scouts	**36**	**81**	1	314
I go to cinema	**11**	**81**	269	71
I go to pop concerts, gigs	**7**	**6**	**92**	326
I go to church, mosque or temple	**2**	**52**	**10**	368
I look after a pet animal	**197**	25	6	203
I go to dance clubs or raves	**15**	**44**	**104**	266
I do nothing much (am bored)	37	39	24	331

Number of students participating in each of a list of activities, summed over three waves, for Glasgow data.
Bold-faced are categories counted as a tie.

The RSiena specification of homophily used:

```
includeInteraction(...  , egoX, totInDist2,
           interaction1=c('girls','girls'))
includeInteraction(..., egoX, totInDist2,
           interaction1=c('boys','boys'))
```

This allows different tendencies toward homophily for boys and girls.

Instead of `totInDist2`, also `sameXInPop` could be used.
This gives the same results, but also has the possibility
of using only its main effect, not distinguishing between boys and girls.

Results

Effect	Model 1 par.	Model 1 (s.e.)	Model 2 par.	Model 2 (s.e.)
rate (period 1)	4.232	(0.271)	4.303	(0.283)
rate (period 2)	4.023	(0.276)	4.143	(0.290)
outdegree (density)	–2.662***	(0.265)	–2.265***	(0.306)
four-cycles	0.0366***	(0.0059)	0.0330***	(0.0062)
indegree-popularity	0.0471***	(0.0057)	0.0310***	(0.0066)
outdegree-activity	0.5102***	(0.0736)	0.4437***	(0.0765)
out-in degree assortativity	–0.0161***	(0.0024)	–0.0147***	(0.0025)
girl ego			–0.821**	(0.293)
girl × outdegree-activity			0.0615*	(0.0251)
girl × number girls participating			0.0395***	(0.0061)
boy × number boys participating			0.0229***	(0.0057)

* $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$;

Convergence t-ratios all < 0.03; overall maximum convergence ratios 0.05, 0.07.

Estimation results for activity participation of Glasgow students.

Interpretation

- Four-cycles $> 0 \Rightarrow$ clustering of activities
- Outdegree-actvity $> 0 \Rightarrow$ large differences between students in number of activities mentioned.
- Out-in degree assortativity $< 0 \Rightarrow$ those who mention more activities, tend to mention especially the infrequent ones.
- Homophily both for girls and for boys, stronger for girls ($p = 0.01$). (Tested with `testSame.RSiena`)

Discussion

Social networks are mostly accompanied by shared activities, shared cognitions, and other aspects of social organization.

Taking these into consideration is scientifically important, and can be of great help to 'understand' what is happening in the network dynamics.

This can be investigated using models of one-mode – two-mode network coevolution.