# The co-evolution of one-mode and two-mode networks

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# One-mode and two-mode networks

The combined consideration of one-mode and two-mode networks
is very fruitful because it allows to consider
the mutual dependencies between (one-mode) relational networks
and (two-mode) activities and/or memberships and/or cognitions
and/or internal structure and/or details of behavioral tendencies and/or ....

These slides are about the co-evolution of one-mode and two-mode networks
according to the Stochastic Actor-oriented Model.

In principle, this works just like co-evolution of two one-mode networks.

What is special for one-mode – two-mode co-evolution?

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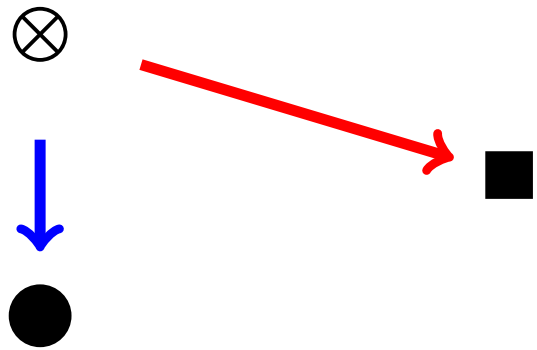

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$\Rightarrow$ activity

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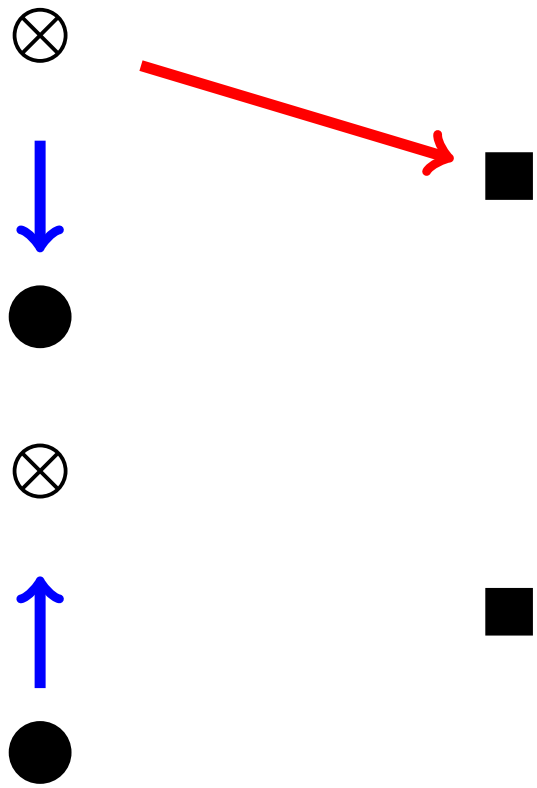

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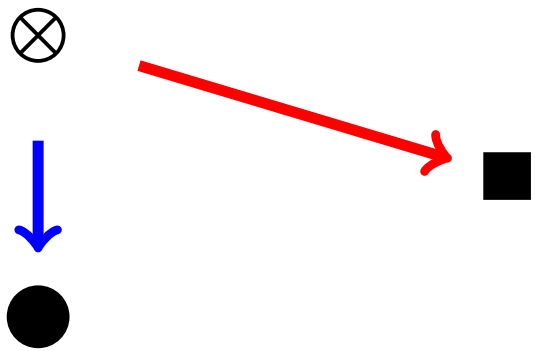

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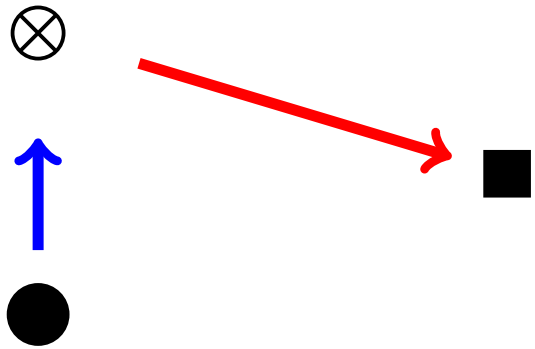
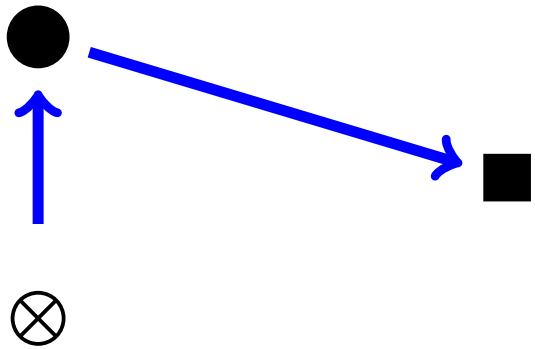


Closed triads are impossible in bipartite networks;
but they are possible as mixed patterns.

*One-with-two-mode triads.*

One-mode tie $\Rightarrow$
two-mode agreement
'I go to places



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'I go to places where my friends are'

*association-based affiliation closure*

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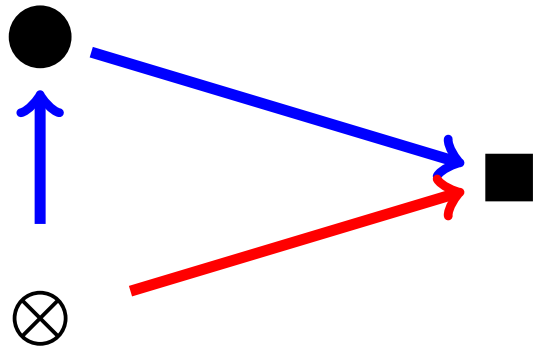

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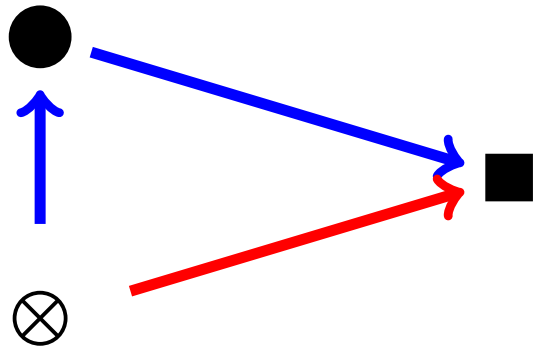
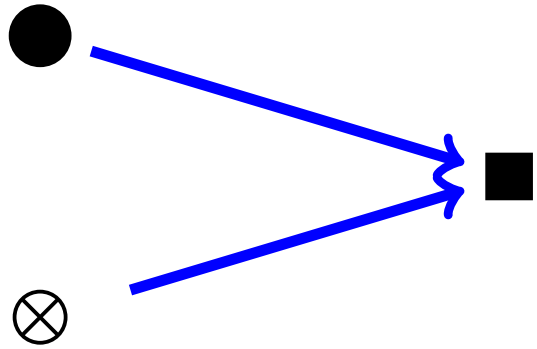


Two-mode agreement $\Rightarrow$ one-mode tie

'Those who go to the same places



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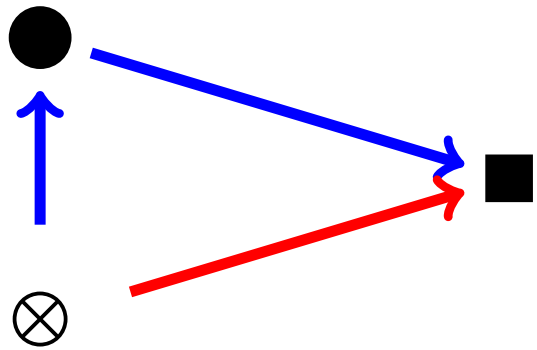
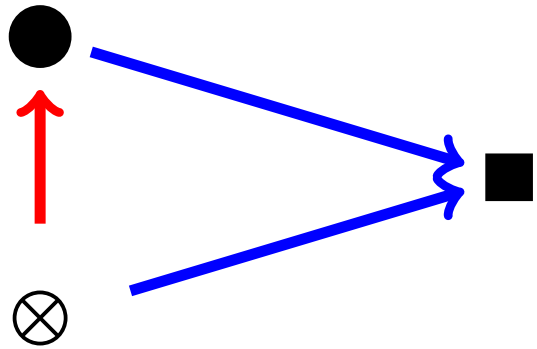


Two-mode agreement $\Rightarrow$ one-mode tie

'Those who go to the same places
become friends'

*affiliation-based focal closure*

`to`



The two different ways in which this mixed triadic closure can occur
implies that, analogous to the distinction influence $\leftrightarrow$ selection
in network-behavior co-evolution,
in the co-evolution of a one-mode and a two-mode network
there is the distinction between
*focal closure* and *affiliation closure*,
also called *affiliation-based closure* and *association-based closure*.

(One-mode: *association*;
two-mode: *affiliation, focus*).

E.g., Easley and Kleinberg (2010, Section 4.3); Lomi and Stadtfeld (2014).

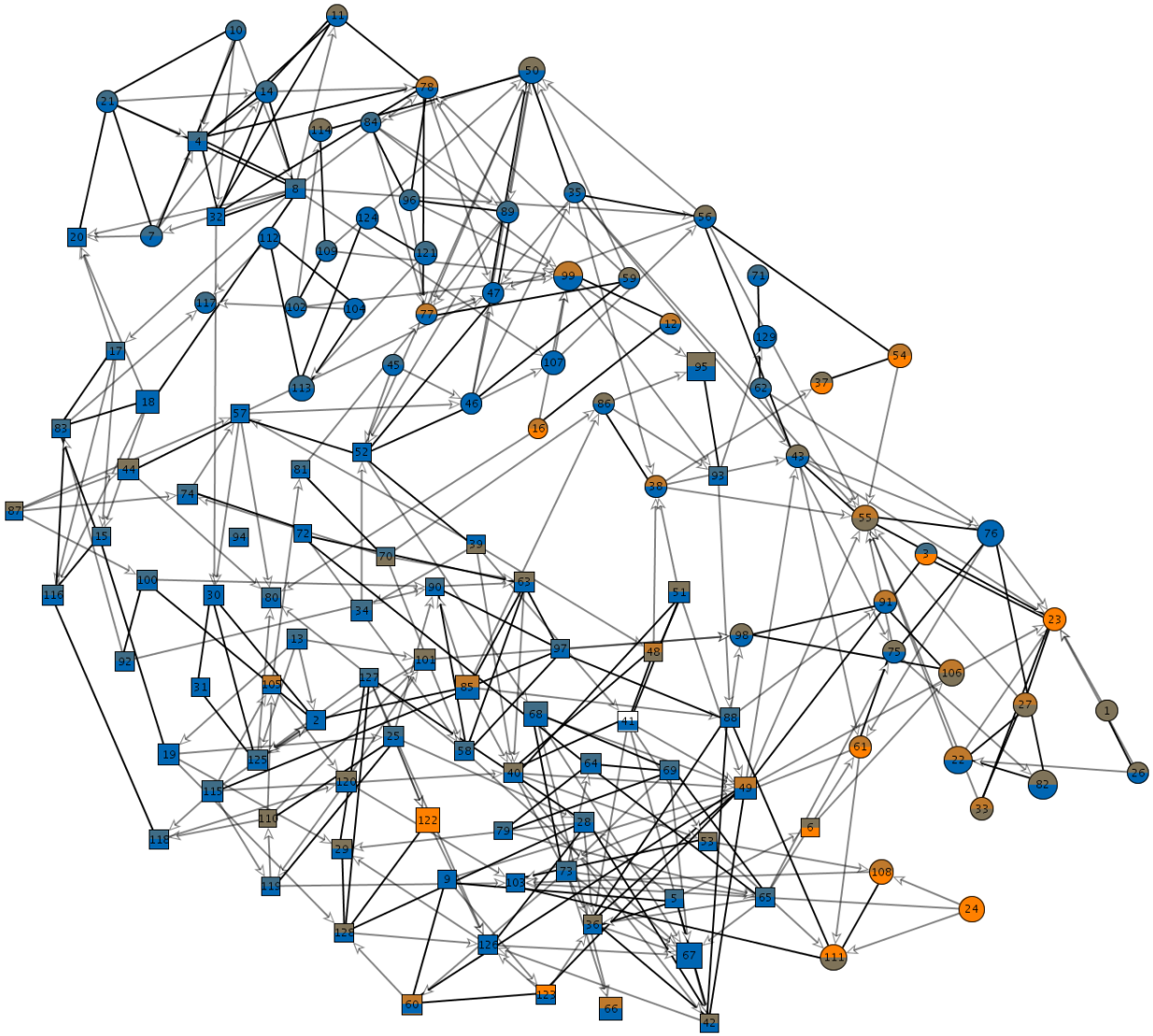
# Example 1: Glasgow friends and pastimes

*Example:*
West of Scotland 11-16 Study; West et al. (1996 and later).

One school year group from a Scottish secondary school
starting at age 12-13 years, monitored over more than 2 years;
total of 160 pupils, sociometric & behavior questionnaires
at three moments, at appr. 1 year intervals.

First network: friendship;
second network (two-mode): activities.

covariates:
gender, smoking of parents and siblings (binary),
money available (range 0–40 pounds/week).



*wave 1*

girls: circles

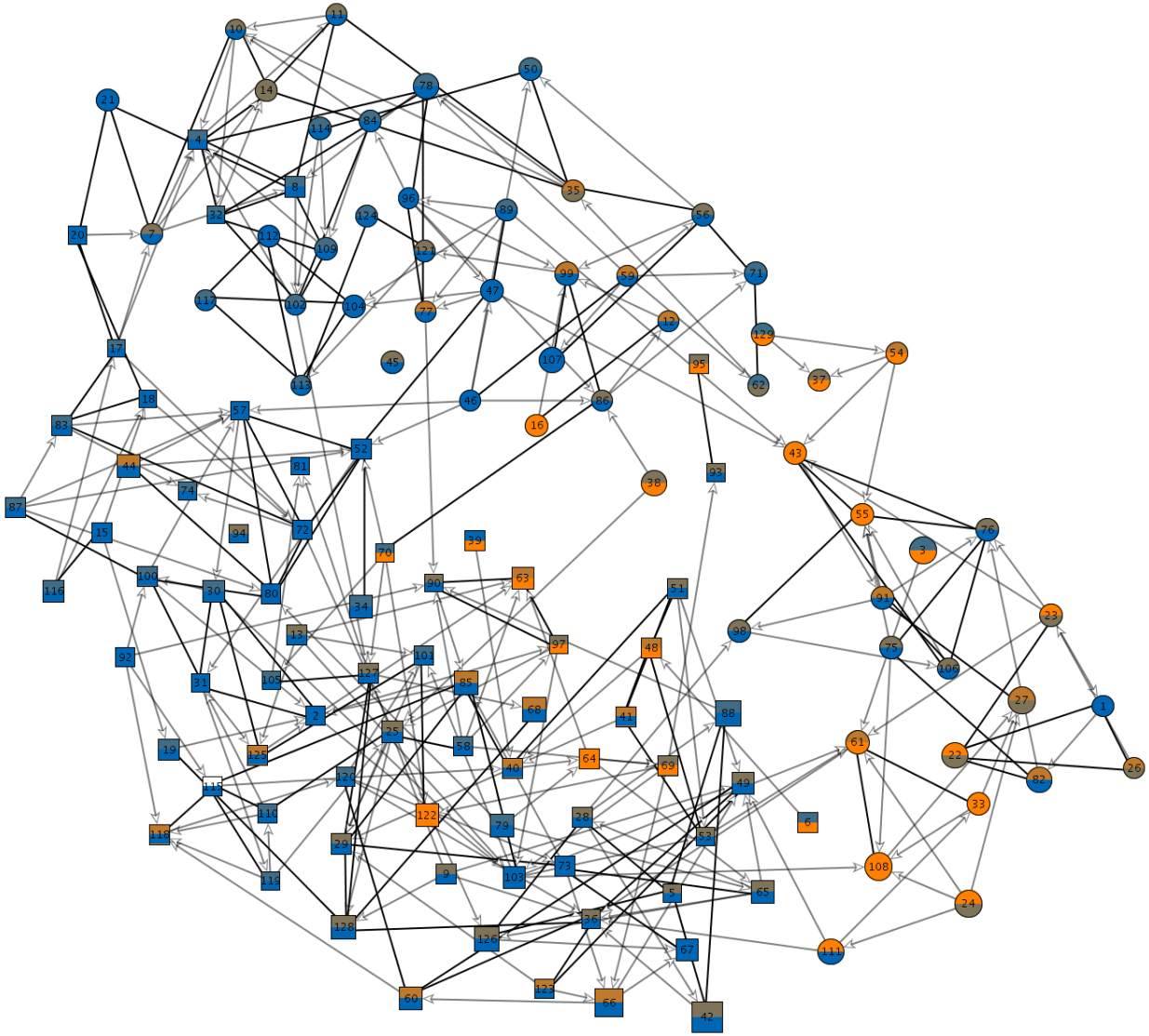
boys: squares

node size: pocket money

color: top = drinking

bottom = smoking

(orange = high)



*wave 2*

girls: circles

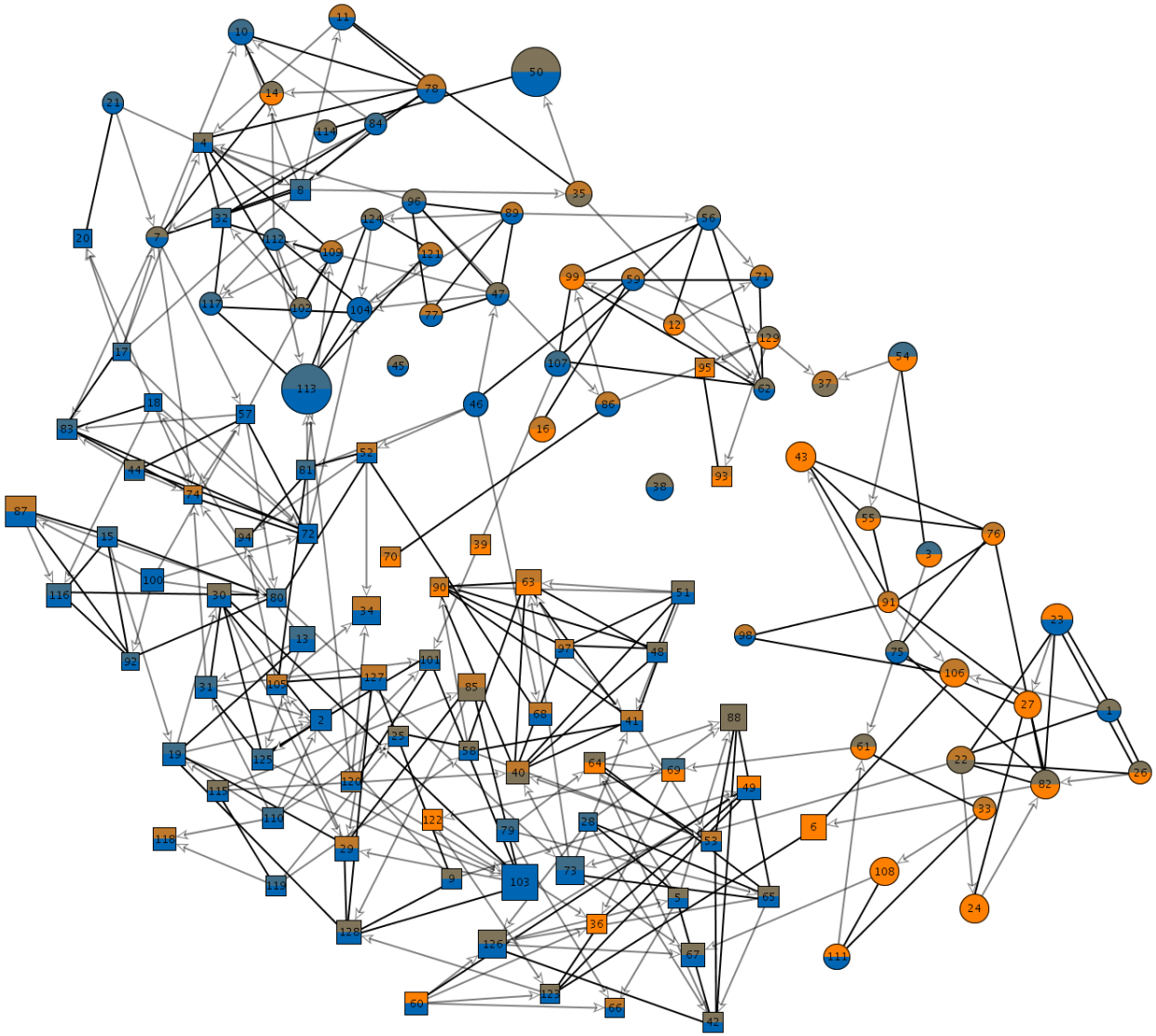
boys: squares

node size: pocket money

color: top = drinking

bottom = smoking

(orange = high)



*wave 3*

girls: circles

boys: squares

node size: pocket money

color: top = drinking

bottom = smoking

(orange = high)

# Descriptives for friendship

Three waves $\sim$ two periods.

Average degrees 3.7; 3.5; 3.6.

Amount of stability in network ties measured by Jaccard coefficient

$$J = \frac{N_{11}}{N_{01} + N_{10} + N_{11}}$$

where $N_{hk} =$ number of tie variables
with value $h$ at one wave and value $k$ at the next.

$J$ = 0.30; 0.35 for the two periods.

# Descriptives for leisure activities

Three waves $\sim$ two periods.

Average degrees 4.7; 4.0; 3.9.

Amount of stability in activities also measured by Jaccard coefficient

$$J = \frac{N_{11}}{N_{01} + N_{10} + N_{11}}$$

where $N_{hk} =$ number of tie variables
with value $h$ at one wave and value $k$ at the next.

$J = 0.51$ for both periods.

# Second mode: Leisure time activities

| | daily | weekly | monthly | less |
|---|---|---|---|---|
| I listen to tapes or CDs | **388** | 23 | 5 | 16 |
| I look around in the shops | **65** | 290 | 48 | 30 |
| I read comics, mags or books | **186** | 121 | 65 | 60 |
| I go to sport matches | **30** | **113** | 90 | 200 |
| I take part in sports | **218** | 117 | 30 | 68 |
| I hang round in the streets | **216** | 64 | 26 | 125 |
| I play computer games | **157** | 109 | 45 | 122 |
| I spend time on hobby (e.g. art, instrument) | **114** | 113 | 36 | 170 |
| I go to something like B.B., Guides or Scouts | **36** | **81** | 1 | 314 |
| I go to cinema | **11** | **81** | 269 | 71 |
| I go to pop concerts, gigs | **7** | **6** | **92** | 326 |
| I go to church, mosque or temple | **2** | **52** | **10** | 368 |
| I look after a pet animal | **197** | 25 | 6 | 203 |
| I go to dance clubs or raves | **15** | **44** | **104** | 266 |
| I do nothing much (am bored) | 37 | 39 | 24 | 331 |

Number of students participating in each of a list of activities, summed over three waves, for Glasgow data.
Bold-faced are categories counted as a tie.

# Results

The table of results is distributed over 4 pages:

- friendship: the basis
- friendship: effects of leisure activities
- leisure: the basis
- leisure: sex-related specializations.

# Friendship: basic

| Effect | par. | (s.e.) | shortName |
|---|---|---|---|
| rate period 1 | 12.383 | (1.217) | `Rate` |
| rate period 2 | 9.870 | (1.132) | `Rate` |
| *Friendship: endogenous effects* | | | |
| outdegree (density) | –3.633*** | (0.258) | `density` |
| reciprocity | 3.337*** | (0.311) | `recip` |
| GWESPFF: creation ($\alpha = 0.69$) | 3.350*** | (0.301) | `gwespFF` |
| GWESPFF: maintenance ($\alpha = 0.69$) | 0.273 | (0.385) | `gwespFF` |
| indegree - popularity | –0.079*** | (0.020) | `inPop` |
| outdegree - activity | 0.121*** | (0.036) | `outAct` |
| reciprocated degree - activity | –0.303*** | (0.071) | `reciAct` |
| indegree - activity | 0.001 | (0.056) | `inAct` |
| *Covariate effects* | | | |
| girls alter | –0.124 | (0.085) | `altX` |
| girls ego | 0.032 | (0.086) | `egoX` |
| same gender | 0.446*** | (0.082) | `sameX` |

# Friendship: effects of leisure activities

| Effect | par. | (s.e.) | shortName |
|---|---|---|---|
| *Friendship: effects of leisure* | | | |
| leisure outdegree popularity | –0.046 | (0.037) | `outPopIntn` |
| leisure outdegree activity | –0.087* | (0.037) | `outActIntn` |
| affiliation-based closure | 0.213** | (0.073) | `from` |

# Leisure: basic

| Effect | par. | (s.e.) | shortName |
|---|---|---|---|
| *Activities* | | | |
| rate period 1 | 4.386 | (0.293) | `Rate` |
| rate period 2 | 4.254 | (0.313) | `Rate` |
| *Endogenous effects of activities* | | | |
| outdegree (density) | –2.149*** | (0.333) | `density` |
| 4-cycles | 0.0272*** | (0.0073) | `cycle4` |
| indegree - popularity | 0.0269** | (0.0084) | `inPop` |
| outdegree - activity | 0.389*** | (0.086) | `outAct` |
| out-in degree assortativity | –0.0128*** | (0.0027) | `outInAss` |
| *Effects of friendship on activities* | | | |
| friendship indegree activity | 0.001 | (0.039) | `inActIntn` |
| friendship outdegree activity | –0.148* | (0.073) | `outActIntn` |
| association-based closure | 0.351*** | (0.062) | `to` |

# Leisure: two-mode sex homophily

Homophily in two-mode networks is treated in

`https://www.stats.ox.ac.uk/~snijders/siena/Twomode_s.pdf`

| Effect | par. | (s.e.) | shortName |
|---|---|---|---|
| *Effects of sex on activities* | | | |
| girls ego | –0.870** | (0.313) | `egoX` |
| 4-cycles among girls | 0.0027 | (0.0065) | `sameXCycle4` |
| girls × outdegree - activity | 0.066* | (0.029) | `egoX` × `outAct` |
| indegree - popularity within girls | 0.0242* | (0.0098) | `egoX` × `totInDist2` |
| indegree - popularity within boys | 0.0091 | (0.0103) | `egoX` × `totInDist2` |

Leisure homophily only for girls!

The leisure-only model did show leisure homophily also for boys.
This is 'explained away' here by association-based closure.

# Example 2: American high school

Other example, based on Fujimoto, Snijders, & Valente (*NWS*, 2018).

US high school, $X$ = friendship, $Z$ = sport activities.

# Descriptives

Two waves $\sim$ one period.

$n = 309$ students, $m = 16$ sports,
$X$ = friendship, $Z$ = sport participation in past 12 months.

Average friendship degrees 6.6, 6.2;
Jaccard similarity 0.25.

Average sport out-degrees 1.2, 1.1;
Jaccard similarity 0.44.

Again, four pages of results.

# Results: friendship (1)

| Effect | par. | (s.e.) |
|---|---|---|
| outdegree | –3.519*** | (0.413) |
| reciprocity | 2.775*** | (0.171) |
| transitive triplets | 0.398*** | (0.032) |
| transitive reciprocated triplets | –0.293*** | (0.071) |
| 3-cycles | 0.101 | (0.064) |
| transitive ties | 0.425*** | (0.073) |
| indegree - popularity | 0.022*** | (0.005) |
| outdegree - popularity | –0.065*** | (0.009) |
| outdegree - activity | 0.011 | (0.023) |
| outdegree - activity ($\sqrt{}$) | 0.154 | (0.187) |
| reciprocal degree - activity | –0.079*** | (0.015) |
| outdegree positive | –0.776 | (0.763) |
| gender (F) alter | –0.035 | (0.041) |
| gender (F) ego | 0.093* | (0.042) |
| same gender | 0.363*** | (0.047) |
| same gender $\times$ reciprocity | –0.442** | (0.136) |

# Results: friendship (2)

| Effect | par. | (s.e.) |
|---|---|---|
| hispanic alter | 0.013 | (0.065) |
| hispanic ego | –0.045 | (0.063) |
| same hispanic | 0.144* | (0.064) |
| grade alter | –0.021 | (0.022) |
| grade ego | –0.026 | (0.023) |
| grade similarity | 0.317*** | (0.088) |
| same class | 0.564*** | (0.091) |
| same class × reciprocity | –0.210 | (0.154) |
| same class × same gender | –0.041 | (0.107) |

# Results: sports

| Effect | par. | (s.e.) |
|---|---|---|
| outdegree | –2.369*** | (0.613) |
| 4-cycles | 0.041 | (0.030) |
| indegree - popularity | 0.020** | (0.007) |
| outdegree - activity | –0.029 | (0.102) |
| outdegree positive | –2.116*** | (0.592) |
| gender ego (F) | 0.023 | (0.184) |
| two-mode gender similarity | 1.750*** | (0.416) |
| 4-cycles same gender | –0.085* | (0.039) |
| hispanic ego | –0.599** | (0.222) |
| grade ego | 0.299** | (0.115) |

Strong evidence for homophily!

# Results: cross-networks

| Effect | par. | (s.e.) |
|---|---|---|
| *Sports* $\Rightarrow$ *Friendship* | | |
| outdegree ($\sqrt{}$) sports activity | –0.106** | (0.038) |
| affiliation-based closure | 0.159** | (0.057) |
| *Friendship* $\Rightarrow$ *Sports* | | |
| friendship outdegree ($\sqrt{}$) activity (eval.) | 0.171 | (0.468) |
| friendship outdegree ($\sqrt{}$) activity (maint.) | –1.386 | (1.063) |
| association-based closure (evaluation) | 0.442* | (0.187) |
| association-based closure (maintenance) | 0.646 | (0.452) |

Those mentioning more sports mention fewer friends;
shared sport activities lead to friendship;
friendship leads to shared sport activities
(not different for creating or maintaining activities).

# Discussion

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⇒ Elaborated along the lines of actor-based modeling.

⇒ Compared to modeling dynamics of single networks,
this approach attenuates the Markov assumption
by extending the state space to a multiple network.

$\Rightarrow$ New perspectives possible
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This works for a small number (e.g., 2–6) of networks,
and a limited number of actors (up to a few hundred).

⇒ If there are implication relations between the networks,
e.g., two networks might be mutually exclusive,
or one might be a sub-network of the other,
then this constraint is observed, noted in the `print01Report`, and
respected in the simulations.
This gives possibilities for networks with valued ties
by using different dichotomies.