

The co-evolution of one-mode and two-mode networks

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One-mode and two-mode networks

The combined consideration of one-mode and two-mode networks is very fruitful because it allows to consider the mutual dependencies between (one-mode) relational networks and (two-mode) activities and/or memberships and/or cognitions and/or internal structure and/or details of behavioral tendencies and/or

These slides are about the co-evolution of one-mode and two-mode networks according to the Stochastic Actor-oriented Model.

In principle, this works just like co-evolution of two one-mode networks.

What is special for one-mode – two-mode co-evolution?

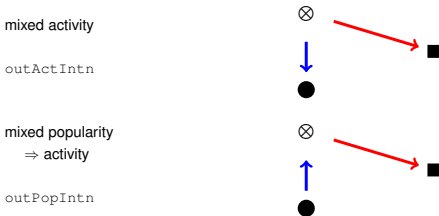


One-mode – two-mode dependencies

Two-mode networks have less structure, so that there are fewer effects.

Within-dyad dependencies are undefined.

Actor-level dependencies are meaningful.



Closed triads are impossible in bipartite networks;
but they are possible as mixed patterns.

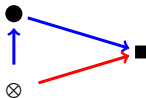
One-with-two-mode triads.

One-mode tie ⇒

two-mode agreement

'I go to places where my friends are'

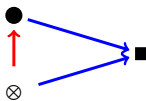
association-based affiliation closure
closure



Two-mode agreement ⇒ one-mode tie

'Those who go to the same places
become friends'

affiliation-based focal closure
to



The two different ways in which this mixed triadic closure can occur implies that, analogous to the distinction influence \leftrightarrow selection in network-behavior co-evolution, in the co-evolution of a one-mode and a two-mode network there is the distinction between *focal closure* and *affiliation closure*, also called *affiliation-based closure* and *association-based closure*.

(One-mode: *association*;
two-mode: *affiliation, focus*).

E.g., Easley and Kleinberg (2010, Section 4.3); Lomi and Stadtfeld (2014).



Example 1: Glasgow friends and pastimes

Example:

West of Scotland 11-16 Study; West et al. (1996 and later).

One school year group from a Scottish secondary school starting at age 12-13 years, monitored over more than 2 years; total of 160 pupils, sociometric & behavior questionnaires at three moments, at appr. 1 year intervals.

First network: friendship;
second network (two-mode): activities.

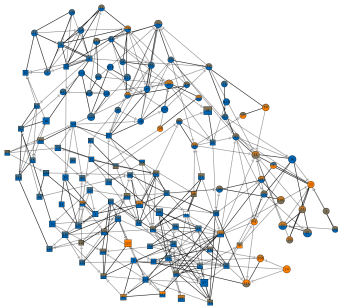
covariates:
gender, smoking of parents and siblings (binary),
money available (range 0–40 pounds/week).



wave 1 girls: circles
boys: squares

node size: pocket money

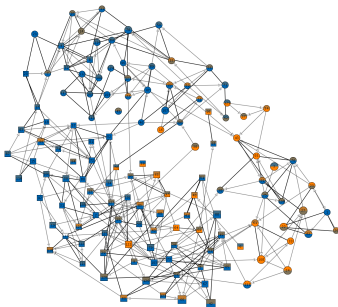
color: top = drinking
bottom = smoking
(orange = high)



wave 2 girls: circles
boys: squares

node size: pocket money

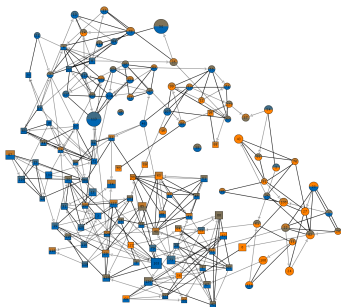
color: top = drinking
bottom = smoking
(orange = high)



wave 3 girls: circles
 boys: squares

node size: pocket money

color: top = drinking
 bottom = smoking
 (orange = high)



Descriptives for friendship

Three waves ~ two periods.

Average degrees 3.7; 3.5; 3.6.

Amount of stability in network ties measured by Jaccard coefficient

$$J = \frac{N_{11}}{N_{01} + N_{10} + N_{11}}$$

where N_{hk} = number of tie variables
 with value h at one wave and value k at the next.

$J = 0.30; 0.35$ for the two periods.



Descriptives for leisure activities

Three waves ~ two periods.

Average degrees 4.7; 4.0; 3.9.

Amount of stability in activities also measured by Jaccard coefficient

$$J = \frac{N_{11}}{N_{01} + N_{10} + N_{11}}$$

where N_{hk} = number of tie variables
with value h at one wave and value k at the next.

$J = 0.51$ for both periods.



Second mode: Leisure time activities

	daily	weekly	monthly	less
I listen to tapes or CDs	388	23	5	16
I look around in the shops	65	290	48	30
I read comics, mags or books	186	121	65	60
I go to sport matches	30	113	90	200
I take part in sports	218	117	30	68
I hang round in the streets	216	64	26	125
I play computer games	157	109	45	122
I spend time on hobby (e.g. art, instrument)	114	113	36	170
I go to something like B.B., Guides or Scouts	36	81	1	314
I go to cinema	11	81	269	71
I go to pop concerts, gigs	7	6	92	326
I go to church, mosque or temple	2	52	10	368
I look after a pet animal	197	25	6	203
I go to dance clubs or raves	15	44	104	266
I do nothing much (am bored)	37	39	24	331

Number of students participating in each of a list of activities, summed over three waves, for Glasgow data.
Bold-faced are categories counted as a tie.



Results

The table of results is distributed over 4 pages:

- ▶ friendship: the basis
- ▶ friendship: effects of leisure activities
- ▶ leisure: the basis
- ▶ leisure: sex-related specializations.



Friendship: basic

Effect	par.	(s.e.)	shortName
rate period 1	12.383	(1.217)	Rate
rate period 2	9.870	(1.132)	Rate

Friendship: endogenous effects

outdegree (density)	-3.633***	(0.258)	density
reciprocity	3.337***	(0.311)	recip
GWESPFF: creation ($\alpha = 0.69$)	3.350***	(0.301)	gwapFF
GWESPFF: maintenance ($\alpha = 0.69$)	0.273	(0.385)	gwapFF
indegree - popularity	-0.079***	(0.020)	inPop
outdegree - activity	0.121***	(0.036)	outAct
reciprocated degree - activity	-0.303***	(0.071)	reciAct
indegree - activity	0.001	(0.056)	inAct

Covariate effects

girls alter	-0.124	(0.085)	altX
girls ego	0.032	(0.086)	egoX
same gender	0.446***	(0.082)	sameX



Friendship: effects of leisure activities

Effect	par.	(s.e.)	shortName
<i>Friendship: effects of leisure</i>			
leisure outdegree popularity	-0.046	(0.037)	outPopIntn
leisure outdegree activity	-0.087*	(0.037)	outActIntn
affiliation-based closure	0.213**	(0.073)	from



Leisure: basic

Effect	par.	(s.e.)	shortName
<i>Activities</i>			
rate period 1	4.386	(0.293)	Rate
rate period 2	4.254	(0.313)	Rate
<i>Endogenous effects of activities</i>			
outdegree (density)	-2.149* **	(0.333)	density
4-cycles	0.0272***	(0.0073)	cycle4
indegree - popularity	0.0269**	(0.0084)	inPop
outdegree - activity	0.389***	(0.086)	outAct
out-in degree assortativity	-0.0128***	(0.0027)	outInAss
<i>Effects of friendship on activities</i>			
friendship indegree activity	0.001	(0.039)	inActIntn
friendship outdegree activity	-0.148*	(0.073)	outActIntn
association-based closure	0.351****	(0.062)	to



Leisure: two-mode sex homophily

Homophily in two-mode networks is treated in

https://www.stats.ox.ac.uk/~snijders/siena/Twomode_s.pdf

Effect	par.	(s.e.)	shortName
<i>Effects of sex on activities</i>			
girls ego	-0.870**	(0.313)	egoX
4-cycles among girls	0.0027	(0.0065)	sameXCycle4
girls \times outdegree - activity	0.066*	(0.029)	egoX \times outAct
indegree - popularity within girls	0.0242*	(0.0098)	egoX \times totInDist2
indegree - popularity within boys	0.0091	(0.0103)	egoX \times totInDist2

Leisure homophily only for girls!

The leisure-only model did show leisure homophily also for boys. This is 'explained away' here by association-based closure.



Example 2: American high school

Other example, based on Fujimoto, Snijders, & Valente (*NWS*, 2018).

US high school, X = friendship, Z = sport activities.



Descriptives

Two waves \sim one period.

$n = 309$ students, $m = 16$ sports,

$X =$ friendship, $Z =$ sport participation in past 12 months.

Average friendship degrees 6.6, 6.2;

Jaccard similarity 0.25.

Average sport out-degrees 1.2, 1.1;

Jaccard similarity 0.44.

Again, four pages of results.



Results: friendship (1)

Effect	par.	(s.e.)
outdegree	-3.519***	(0.413)
reciprocity	2.775***	(0.171)
transitive triplets	0.398***	(0.032)
transitive reciprocated triplets	-0.293***	(0.071)
3-cycles	0.101	(0.064)
transitive ties	0.425***	(0.073)
indegree - popularity	0.022***	(0.005)
outdegree - popularity	-0.065***	(0.009)
outdegree - activity	0.011	(0.023)
outdegree - activity (\surd)	0.154	(0.187)
reciprocal degree - activity	-0.079***	(0.015)
outdegree positive	-0.776	(0.763)
gender (F) alter	-0.035	(0.041)
gender (F) ego	0.093*	(0.042)
same gender	0.363***	(0.047)
same gender \times reciprocity	-0.442**	(0.136)



Results: friendship (2)

Effect	par.	(s.e.)
hispanic alter	0.013	(0.065)
hispanic ego	-0.045	(0.063)
same hispanic	0.144*	(0.064)
grade alter	-0.021	(0.022)
grade ego	-0.026	(0.023)
grade similarity	0.317***	(0.088)
same class	0.564***	(0.091)
same class \times reciprocity	-0.210	(0.154)
same class \times same gender	-0.041	(0.107)



Results: sports

Effect	par.	(s.e.)
outdegree	-2.369***	(0.613)
4-cycles	0.041	(0.030)
indegree - popularity	0.020**	(0.007)
outdegree - activity	-0.029	(0.102)
outdegree positive	-2.116***	(0.592)
gender ego (F)	0.023	(0.184)
two-mode gender similarity	1.750***	(0.416)
4-cycles same gender	-0.085*	(0.039)
hispanic ego	-0.599**	(0.222)
grade ego	0.299**	(0.115)

Strong evidence for homophily!



Results: cross-networks

Effect	par.	(s.e.)
<i>Sports</i> ⇒ <i>Friendship</i>		
outdegree (\surd) sports activity	-0.106**	(0.038)
affiliation-based closure	0.159**	(0.057)
<i>Friendship</i> ⇒ <i>Sports</i>		
friendship outdegree (\surd) activity (eval.)	0.171	(0.468)
friendship outdegree (\surd) activity (maint.)	-1.386	(1.063)
association-based closure (evaluation)	0.442*	(0.187)
association-based closure (maintenance)	0.646	(0.452)

Those mentioning more sports mention fewer friends;
 shared sport activities lead to friendship;
 friendship leads to shared sport activities
 (not different for creating or maintaining activities).



Discussion

- ⇒ See Snijders, Lomi & Torlò in *Social Networks*, 2013
 Fujimoto, Snijders & Valente (*Network Science*, 2018),
 Lomi & Stadtfeld (*KZfSS*, 2014).
- ⇒ It's a multilevel issue (but not nested):
 ties, dyads, actors, triads, subgroups, ...
- ⇒ Testing cross-network dependencies in
 dynamics of multiple networks gives interesting
 new possibilities for hypothesis testing.
- ⇒ Elaborated along the lines of actor-based modeling.
- ⇒ Compared to modeling dynamics of single networks,
 this approach attenuates the Markov assumption
 by extending the state space to a multiple network.



- ⇒ New perspectives possible by combining one-mode and two-mode networks.
- ⇒ The method is available in **RSiena**.
This works for a small number (e.g., 2–6) of networks, and a limited number of actors (up to a few hundred).
- ⇒ If there are implication relations between the networks, e.g., two networks might be mutually exclusive, or one might be a sub-network of the other, then this constraint is observed, noted in the `print01Report`, and respected in the simulations.
This gives possibilities for networks with valued ties by using different dichotomies.

