Stochastic Actor-Oriented Models for Network Dynamics

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1 Stochastic Actor-Oriented Models: Network Panel Data and Co-evolution

This chapter, about Stochastic Actor-Oriented Models ('SAOMs'), highlights and explicates a statistical model for analyzing network panel data: the data are structured as repeated observations of a network between a given set of nodes that represent political actors of some kind. The set of nodes may be changing now and then, as new actors come in and existing ones drop out, or as actors may combine or split up. At the minimum there are two waves of data, but there might be many more.

The basic idea of stochastic actor-oriented models (Snijders, 2001) is that the panel data are regarded as repeated snapshots of a process that is evolving in continuous time, and this process is a Markov process: probabilities of tie changes are determined by the current state; further, the change is represented as a sequence of changes of single ties. Since each tie change modifies the state of the network, and the later changes will build on this new state, because of path dependence it will change the entire future. The assumption that between the panel observations many tie changes can sequentially take place, each acting on the state that is the result of earlier tie changes, leads to a rich dependence structure for the ties in the observed networks. The name 'actor-oriented' reflects that the tie changes are modeled as being determined by the actors. For the case of directed networks there is no assumption of coordination of tie changes by different actors, so that group-wise changes cannot be represented. The Markov chain assumption can be mitigated in two ways: by including covariates from earlier times; or by extending the outcome space from a single network to multiple networks —thus leading to the analysis of a multivariate network, as in Snijders et al. (2013)—, or with actor-level (i.e., nodal) variables (Steglich et al., 2010). Such an extended outcome space leads to *co-evolution models*, representing the interdependent dynamics of several dependent variables. The co-evolving network may be a two-mode network, i.e., affiliations between the actors who constitute the first mode with some other node set. This offers the possibility of modeling the interdependence of relations between actors and their memberships in organisations, such as the relations between countries through common memberships in NGOs or between individuals through common memberships in civic organizations.

This chapter has four main parts. First, we give a brief overview of applications of this model in political science published until now. Subsequently we present the model for analyzing dynamics of directed network with a brief sketch of the associated estimation methods, implemented in the software package *RSiena* ('Simulation Investigation for Empirical Network Analysis'; Ripley et al., 2016) of the R statistical system (R Core Team, 2016). A tie change in a directed network requires only the decision of one actor in a dyad, such as the decision of one country to direct hostility at another. Third, this model is extended to non-directed networks, i.e., networks in which the ties are by their nature non-directional (such as trade agreements), which is a type of network often encountered in political science. For non-directed networks it is necessary to take into account the joint decision making by both actors involved in a given tie. Several models are considered for how these actors coordinate. The definition of this model has not been published before (although some published applications do exist.) Fourth, the approach to co-evolution is sketched. The paper finishes with a discussion of some aspects of this model. For the positioning of this model in the further array of statistical network models, see Snijders (2011) and Desmarais and Cranmer (2016).

2 Applications

The stochastic actor-oriented model (SAOM) for network dynamics has been applied in a variety of social science disciplines, including various studies in the political sciences. Other approaches common in political science for the analysis of longitudinal network data include temporal exponential random graph models

(Almquist and Butts, 2013) and dynamic latent space models (Cao and Ward, 2014; Dorff and Ward, 2016).

The use of SAOMs in political science is currently largely limited to the subfields of policy (Berardo and Scholz, 2010; Ingold and Fischer, 2014; Giuliani, 2013), international relations (Kinne, 2013; Rhue and Sundararajan, 2014), and international political economy (Manger and Pickup, 2016; Manger et al., 2012). At least one study has applied SAOMS to political behaviour (Liang, 2014). As more and more network data is collected at the level of the individual voter, we might expect an increase in the application of SAOMs to studying political behaviour. After all, the social network has been of central importance to the study of political behaviour since the subfield's inception (Berelson et al., 1954; Campbell et al., 1960). Further, the application of SAOMs to online networks (well-established outside political science) can be expected to grow within political science, as researchers become increasingly interested in the effects of online social networks on political behaviour.

With respect to formal characteristics of the applications, SAOMs in political science have been applied to directed (Rhue and Sundararajan, 2014; Ingold and Fischer, 2014; Liang, 2014; Giuliani, 2013; Fischer et al., 2012; Berardo and Scholz, 2010) as well as non-directed (Manger and Pickup, 2016; Manger et al., 2012; Kinne, 2013) networks. Applications have included networks with anywhere between 2 and 11 waves, and between 23 and 1178 nodes. A few have included co-evolution models with behavioural dependent variables (Manger and Pickup, 2016; Rhue and Sundararajan, 2014; Berardo and Scholz, 2010), and even fewer have studied co-evolution with multiple networks, two-mode networks (Liang, 2014), or multiple behavioural dependent variables (Rhue and Sundararajan, 2014).

Across all subfields within political science, there are methodological innovations that we might increasingly expect researchers to incorporate in their analysis. We can expect an increase in the use of SAOMs to analyse multiple networks as interdependent structures, investigating how changes in one network relate to other networks. This will become increasingly viable as more and more network data is collected. We might expect a greater application of SAOMs to two-mode networks as political scientists apply network analysis to the ties individuals and countries have through organizations. Examples are the ties countries have through treaties, the ties that individuals have through shared media consumption (traditional or online), or shared concept networks (Liang, 2014). SAOMs as they are evaluated in RSiena allows for arbitrary time lags between observations, facilitating the analysis of longitudinal data that does not have evenly-spaced observations in time. RSiena also allows for different rate functions across nodes in the network. This would allow the researcher to account for the differing relevance of the network for the individual or organization. For example, if the network represents social activities or groups, these groups may be more or less salient to different individuals, and those for whom the group is more salient may also change their ties more frequently.

3 Stochastic Actor-Based Models for Network Dynamics

The Stochastic Actor-Based Model is a statistical model for longitudinal data collected in a network panel design, where network observations are available for a given set of nodes, for two or more consecutive waves. Some turnover of the nodes is allowed, which is helpful for representing the creation and disappearance of organizations or countries. It represents a network process in which, as time goes by, ties can be added as well as deleted. The model aims to obtain a statistical representation of the influences determining creation and termination of ties; turnover of nodes, if any, is considered as an exogenous influence. First the fundamental description of the probability model is given, followed by possible ingredients for its detailed specification. Finally, procedures for estimation are briefly described.

3.1 Notation

The dependent variable in this section is a sequence of directed networks on a given node set $\{1, \ldots, n\}$. Nodes represent political or other social actors (countries, NGOs, political leaders, voters, etc.). The existence of a tie from node i to node j is indicated by the *tie indicator variable* X_{ij}, having the value 1 or 0 depending on whether there is a tie $i \rightarrow j$. For the tie $i \rightarrow j$, actor i is called the *sender* and j the *receiver*. Self-ties are not considered, so that always $X_{ii} = 0$, for all i. The matrix with elements X_{ij} is the adjacency matrix of a directed graph, or digraph; the adjacency matrix as well as the digraph will be denoted by X. Outcomes (i.e., particular realizations) of digraphs will be denoted by lower case x. Replacing an index by a plus sign denotes summation over that index: thus, the number of outgoing ties of actor i (the *out-degree* of i) is denoted $X_{i+} = \sum_j X_{ij}$, while the *in-degree*, the number of incoming ties, is $X_{+i} = \sum_j X_{ji}$. For the data structure, it is assumed that there are two or more repeated observations of the network. Observation moments are indicated by $t_1, t_2, \ldots, t_M$ with $M \geq 2$. Beside the dependent network, there may be explanatory variables measured at the level of the actors (*monadic* or *actor covariates* or on pairs of actors (*dyadic covariates*).

3.2 Actor-based Models

One of the issues for network analysis in the social sciences is the fact that networks by their nature are dyadic, i.e., refer to pairs of actors, whereas the natural theoretical unit is the actor. This issue is discussed more generally by Emirbayer and Goodwin (1994). For modeling network dynamics, a natural combination of network structure and individual agency is possible by basing the model on the postulate that creation and termination of ties are initiated by the actors. In this section the model is presented for binary directed networks, and we postulate that it is meaningful to regard ties as resulting from choices made by the actor sending the tie; in Section 4, we consider non-directed networks, assumed to be based on choices made by both involved actors. The model is explained more fully in Snijders (2001) and Snijders et al. (2010b).

The probability model for network dynamics is based, like other statistical models, on a number of simplifying assumptions.

1. Between observation moments t_1, t_2, etc., time runs on, and changes in the network can and will take place without being directly observed. Thus, while the observation schedule is in discrete time, an unobserved underlying process of network evolution is assumed to take place with a continuous time parameter $t \in [t_1, t_M]$.

2. At any given time point $t \in [t_1, t_M]$ when the network changes, not more than one tie variable X_{ij} can change. In other words, either one tie is created, or one tie is dissolved. The observed change is the net result of all these unobserved changes of single ties.

3. The probability that a time t a particular variable X_{ij} changes depends on the current state $X(t)$ of the network, and not on earlier preceding states.

Assumptions 1 and 3 are expressed mathematically by saying that the network model is a continuous-time Markov process. Assumption 2 simplifies the elements of change to the smallest possible constituent: the creation or termination of a single tie. These assumptions rule out instantaneous coordination or negotiation between actors. They were proposed as basic simplifying postulates by Holland and Leinhardt (1977). In future model developments it may be interesting to allow coordination between actors, but the postulates used here can be regarded as a natural first step to modeling network dynamics.

These three assumptions imply that actors make changes in reaction to each others' changes in between observations. This has strong intuitive validity for many panel observations of networks. Exceptions are situation where collections of ties are created groupwise, e.g., in multilateral alliances. The model is described totally by probabilities of single tie changes, depending on the state of the network. The probability model says nothing about the timing of the observations, and therefore the parameter values are not affected by the frequency of the observations, or the time delays between them. It is assumed that the probability function of tie changes, conditional on the current state of the network and covariates, is constant in time; the probability function that a tie exists at any given time may, however, be changing.

The model is actor-based in the sense that tie changes are modeled as the result of choices made by the actor sending the tie. The tie change model is split into two components: timing and choice. The timing component is defined in terms of *opportunities for change*, not in terms of actual change. This is to allow the possibility that an actor leaves the current situation unchanged, e.g., because s/he is satisfied with it.

4. Consider a given current time point t, $t_m \leq t < t_{m+1}$, and denote the current state of the network by $x = X(t)$. Each actor i has a *rate of change*, denoted $\lambda_i(x; \rho)$, where ρ is a statistical parameter, which may depend on m. The rate of change can depend on actor covariates and on their degrees.

5. The waiting time until the next opportunity for change by any actor has the exponential distribution,

$$\begin{aligned} &\mathrm{P}\{\text{Next opportunity for change is before } t + \Delta t \mid \text{ current time is } t\} \\ &\quad = 1 - \exp(-\lambda \, \Delta t) \ , \end{aligned} \tag{1}$$

   with parameter $\lambda = \lambda_+(x; \rho)$.

6. The probability that the next opportunity for change is for actor i is given by

$$\mathrm{P}\{\text{Next opportunity for change is by actor } i\} = \frac{\lambda_i(x; \rho)}{\lambda_+(x; \rho)} \ . \tag{2}$$

   This formula is consistent with a 'first past the post' model, where all actors have stochastic waiting times as in (5.), the first one gets the opportunity to make a change, and then everything starts all over again but in a new state.

7. For the choice component, each actor i has an *objective function* $f_i(x^{(0)}, x; \beta)$ defined on the set of all pair of networks $x^{(0)}$ and x such that $x^{(0)}$ and x differ

in no more than one tie variable. The current network is $x^{(0)}$ and the objective function determines the probability of the next tie change by this actor, brings state $x^{(0)}$ into x; β is a statistical parameter. In a utility interpretation, the objective function may be regarded as the net utility that the actor gains from moving from $x^{(0)}$ to x. Since this is the short-term utility from one tie change, it should be regarded as a proximate, not ultimate utility; e.g., expressing the advantageous network position that the actor is striving after as a means to obtain further goals.

8. To define this probability, the following notation is used. For a digraph x and $i \neq j$, by $x^{(\pm ij)}$ we define the graph which is identical to x in all tie variables except those for the ordered pair (i, j), and for which the tie variable $i \to j$ is toggled, $x_{ij}^{(\pm ij)} = 1 - x_{ij}$. Further, we define $x^{(\pm ii)} = x$ (just as a convenient formal definition).
   Assume that, at the moment of time $t + \Delta t$ (see point 5.) with current network $X(t) = x$, actor i has the opportunity for change. Then the probability that the tie variable changed is X_{ij}, so that the network x changes into $x^{(\pm ij)}$, is given by

$$\frac{\exp\big(f_i(x, x^{(\pm ij)}; \beta)\big)}{\sum_{h=1}^{n} \exp\big(f_i(x, x^{(\pm ih)}; \beta)\big)} = \frac{\exp\big(f_i(x, x^{(\pm ij)}; \beta) - f_i(x, x; \beta)\big)}{\sum_{h=1}^{n} \exp\big(f_i(x, x^{(\pm ih)}; \beta) - f_i(x, x; \beta)\big)} \ . \tag{3}$$

Expression (3) is a multinomial logit form. This can be obtained when it is assumed that i chooses the best j in the set $\{1, \ldots, n\}$ (where $j = i$ formally means 'no change', see above) where the aim is to toggle the variable X_{ij} that maximizes the objective function of the resulting state plus a random residual,

$$f_i(x, x^{(\pm ij)}; \beta) + R_j \ ,$$

where the variables R_j are independent and have a standard Gumbel distribution (for a proof, see Maddala, 1983). Thus, this model can be regarded as being obtainable as the result of *myopic stochastic optimization*. Game-theoretical models of network formation often use myopic optimization, e.g., Bala and Goyal (2000). It should be noted, however, that what we assume is the vector of choice probabilities (3), and not the myopic optimization – the latter being merely one of the ways in which this expression can be obtained; and for the optimization interpretation it should be kept in mind, as suggested above, that the objective functions represents proximate rather than ultimate goals.

For extensions of this model where different mechanisms or different parameter values may apply for creating new ties and maintaining existing ties, see the treatment in Snijders et al. (2010b) and Ripley et al. (2016) of the endowment function.

3.2.1 Transition rates

The two model components, rate function and objective function, can be put together by considering the so-called transition rates. These give the basic definitions of the continuous-time Markov process resulting from the assumptions formulated above (cf. Norris, 1997, or other textbooks on continuous-time Markov processes), and may be helpful to some for a further understanding. Given that the only permitted transitions between networks are toggles of a single tie variable, the transition rates can be defined as

$$q_{ij}(x) = \lim_{\Delta t \downarrow 0} \frac{\mathrm{P}\{X(t+\Delta t) = x^{(\pm ij)} \mid X(t) = x\}}{\Delta t} \tag{4}$$

for $i \neq j$. Note that this definition implies that the probabilities of toggling a particular tie variable X_{ij} in a short time interval are approximated by

$$\mathrm{P}\{X(t+\Delta t) = x^{(\pm ij)} \mid X(t) = x\} \approx q_{ij}(x)\, \Delta t \, .$$

The transition rate can be computed from the assumptions using the basic rules of probability, and is given by

$$q_{ij}(x) = \lambda_i(x;\rho)\, p_{ij}(x,\beta) \ . \tag{5}$$

3.3 Specification of the Actor-based Model

The specification of the actor-based model amounts to the choice of the rate function $\lambda_i(x;\rho)$ and the objective function $f_i(x;\beta)$. This choice will be based on theoretical considerations, knowledge of the subject matter, and the hypotheses to be investigated. The focus of modeling normally is on the objective function, reflecting the choice part of the model.

In many cases, a simple specification of the rate function suffices:

$$\lambda_i(x;\rho) = \rho_m \ , \tag{6}$$

where m is the index of the observation t_m such that the current time point t is between t_m and t_{m+1}. Including the parameter ρ_m allows to fit exactly the observed number of changes between t_m and t_{m+1}. In other cases, the rate of change may also depend on actor covariates or on positional characteristics such as degrees.

The more important part of the model specification is the objective function. Like in generalized linear modeling, a linear combination is used,

$$f_i(x^{(0)},x;\beta) \ = \ \sum_{k=1}^{K} \beta_k \, s_{ki}(x^{(0)},x) \ , \tag{7}$$

where the $s_{ki}(x^{(0)},x)$ are functions of the network, as seen from the point of view of actor i. These functions are called *effects*. When parameter β_k is positive, tie changes will have a higher probability when they lead to x for which $s_{ki}(x^{(0)},x)$ is higher – and conversely for negative β_k.

The R package RSiena (Ripley et al., 2016) offers a large variety of effects, some of which are the following. First we present some effects depending on the network only, which are important for modeling the dependence between network ties. In most cases the effect $s_{ki}(x^{(0)},x)$ depends only on the new state x, not on the old state $x^{(0)}$. This means that the old state plays a role in determining the option set (i.e., which new states are possible), but not the relative evaluation of the various possible new states. To keep notation simple, we shall write $s_{ki}(x)$ meaning $s_{ki}(x^{(0)},x)$.

1. A basic component is the outdegree, $s_{1i}(x) = \sum_j x_{ij}$. This effect is analogous to a constant term in regression models, and will practically always be included. It balances between creation and termination of ties, which can be understood as follows. Equation (3) shows that it is the change in the objective function that determines the probability. Given the preceding state $x^{(0)}$, the next state x either has one tie more, or one tie less, than $x^{(0)}$; or the two are identical. If $s_{1i}(x)$ has coefficient β_1, for creating a tie the contribution to (7) is β_1; for dissolving a tie the contribution is $-\beta_1$. Therefore the role of the outdegree effect in the model is the contribution of $2\beta_1$ in favor of tie creation vs. tie termination. Usually, networks are sparse, so that there are many more opportunities for creating than for terminating ties. Accordingly, in a more or less stable situation, the parameter β_1 will be negative to keep the network sparse (unless this is already determined by other model components).

2. Reciprocation of choice is a fundamental aspect of almost all directed social networks, because there is almost always some kind of exchange or other reciprocal dependence. This is reflected by the reciprocated degree, $s_{2i}(x) = \sum_j x_{ij}\, x_{ji}$, the number of reciprocal ties in which actor i is involved.

3. The local structure of networks is determined by triads, i.e., subgraphs on three nodes (Holland and Leinhardt, 1976). A first type of triadic dependency is transitivity, in which the indirect connection of the pattern $i \to j \to h$ tends to imply the direct tie $i \to h$. This tendency is captured by $s_{3i}(x) = \sum_{j,h} x_{ij}\, x_{jh}\, x_{ih}$, the number of transitive triplets originating from actor i.

   Theoretical arguments for this effect were formulated by Simmel (1950), who discussed the consequences of triadic embeddedness on bargaining power of the social actors and on the possibilities of conflicts. Coleman (1988) stressed the importance of triadic closure for social control, where actor i, who has access to j as well as h, has the potential to sanction them in case j behaves opportunistically with respect to h. There is also empirical confirmation of

this effect for networks of alliances between firms, e.g., by Gulati and Gargiulo (1999).
Instead of using triad counts, one may represent tendencies toward transitive closure by weighted counts of structures such as those employed in Exponential Random Graph Models, e.g., the Geometrically Weighted Shared Partner ('GWESP') statistic (Snijders et al., 2006; Handcock and Hunter, 2006).

In- and out-degrees are fundamental aspects of individual network centrality (Freeman, 1979). They reflect access to other actors and often are linked quite directly to opportunities as well as costs of the network position of the actors. Degrees may be indicators for influence potential, success (de Solla Price, 1976), prestige (Hafner-Burton and Montgomery, 2006), search potential (Scholz et al., 2008), etc., depending on the context. Accordingly, probabilities of tie creation and dissolution may depend on the degrees of the actors involved. This is expressed by *degree-related effects*, such as the following.

4. In-degree popularity, indicating the extent to which those with currently high in-degrees are more popular as receivers of new ties. This can be expressed by $s_{5i}(x) = \sum_j x_{ij}\, x_{+j}$, the sum of the in-degrees of those to whom i has a tie. When in-degrees are seen as success indicators, this can model the Matthew effect of Merton (1968), which was used by de Solla Price (1976) in his network model of cumulative advantage, rediscovered by Barabási and Albert (1999) in their 'scalefree model'. This is an example of an effect with emergent (micro-macro) consequences: if individual actors have a preference for being linked to popular (high-indegree) actors, the result is a network with a high dispersion of in-degrees.
Since degrees may often have diminishing returns, as argued by Hicklin et al. (2008), alternative specification of this effect could be considered, e.g. $s'_{5i}(x) = \sum_j x_{ij} \sqrt{x_{+j}}$.
5. Similarly for out-degrees and for combinations of in- and out-degrees (e.g.,

'assortativity': are the outgoing ties of actors with high degrees directed disproportionately toward other actors with high degrees), effects can be defined, linear and non-linear; see Snijders et al. (2010b).

Depending on the research questions and the type of network under study a lot of other effects may be considered and many are available in the software, see Ripley et al. (2016).

In addition to these effects based on the network structure itself, research questions will naturally lead to effects depending on attributes of the actors – indicators of goals, constraints, and resources, etc., defined externally to the network. Since network ties involve two actors, a monadic actor variable v_i will lead to potentially several effects for the network dynamics, such as the following. Here the word 'ego' is used for the focal actor, or sender of the tie; while 'alter' is used for the potential candidate for receiving the tie.

6. The *ego effect* $s_{10i}(x) = \sum_j x_{ij}\, v_i = x_{i+} v_i$, reflecting the effect of this variable on the propensity to send ties, and leading to a correlation between v_i and out-degrees.

7. The *alter effect* $s_{11i}(x) = \sum_j x_{ij}\, v_j$, reflecting the effect of this variable on the popularity of the actor for receiving ties, and leading to a correlation between v_i and in-degrees.

8. The *similarity (homophily) effect*, which implies that actors who are similar on salient characteristics have a larger probability to become and stay connected, as reviewed in general terms by McPherson, Smith-Lovin, and Cook (2001). An example is the finding by Huckfeldt (2001) that people tend to select political discussion partners who are perceived to have expertise and who are perceived to have similar views; this would be reflected by an alter and a similarity effect with respect to (perceived) expertise. Another example is the finding (Manger and Pickup, 2016) that democracies are more likely to form trade agreements with other democracies. Similarity can be

represented by the effect

$$s_{12i}(x) = \sum_j x_{ij} \left(1 - \frac{|v_i - v_j|}{\mathrm{Range}(v)} \right) \ ,$$

where $\mathrm{Range}(v) = \max_i(v_i) - \min_i(v_i)$.

9. The *ego-alter interaction effect*, represented like a product interaction, $s_{13i}(x) = \sum_j x_{ij}\, v_i v_j$, which is a different way to represent how the combination of the values on the covariate of the sender and the receiver of the potential tie may influence tie creation and maintenance.

Further, it is possible to include attributes of pairs of actors – of which one example is how they are related in a different network. Such *dyadic covariates* can express, e.g., meeting opportunities (e.g., Huckfeldt, 2009), spatial propinquity (e.g., Baybeck and Huckfeldt, 2002), military alliances, institutional relatedness, competing for the same resources or scarce outcomes, etc.

10. The *dyadic covariate effect* of a covariate w_{ij} is defined as $s_{14i}(x) = \sum_j x_{ij}\, w_{ij}$.

Many other effects that may be used for model specification are mentioned in the RSiena manual (Ripley et al., 2016). Further differences between actors in their objective functions may be represented by interaction effects. Time heterogeneity in the formation probabilities can be incorporated by using time-changing covariates.

3.4 Parameter Estimation

If a continuous time record is available from the network evolution process as described above, so that for each tie the exact starting and ending times within the observation period are known, and these starting and ending times are all distinct, then the model can be framed as a generalized linear model and maximum likelihood estimation is possible, in principle, in a straightforward way. It is more usual, however, that only panel data ('snapshots') are available. Sometimes this is at given intervals (e.g., yearly), sometimes at irregular times depending on what is convenient for data collection. The definition of the model implies that irregular observation times present no problem at all; these will be absorbed in the panel wave-specific parameters ρ_m (see (6)) without affecting the other parameters. For panel data this is a generalized linear model with a lot of missing data (viz., the unobserved timings of the tie changes). Estimation in this case is possible by a variety of simulation-based methods.

A method of moments estimator was proposed by Snijders (2001). The principle of the method of moments operates by selecting a vector of statistics, one for each parameter coordinate to be estimated, and determining the parameter estimate as the parameter value for which the expected value of this vector of statistics equals the observed value at each observation (wave). For the Stochastic Actor-Oriented Model, the required expected values cannot be calculated analytically but they can be approximated by Monte Carlo simulations. These are used in the stochastic approximation developed by Snijders (2001) and implemented in the RSiena package (Ripley et al., 2016). The method simulates the network dynamics many times with trial parameter values, updating them, until the averages of a suitable set of network descriptives, reflecting the estimated parameters, are close enough to the observed values. This method can be called an MCMC method but it is frequentist in nature, not Bayesian, and accordingly does not require the specification of a prior distribution. It is important to check convergence of the algorithm, as discussed in the RSiena manual; sometimes it is necessary to repeat the estimation, using earlier obtained parameters as the new initial values.

The Method of Moments estimator has proven to be quite reliable and efficient. More recently, other and potentially more efficient estimators were developed: a Bayesian estimator by Koskinen and Snijders (2007), a Maximum Likelihood estimator by Snijders et al. (2010a), and a Generalized Method of Moments estimator by Amati et al. (2015).

3.5 Changing actor sets

In networks of organizations, voters, countries, or other political actors, changes in composition of the node set are not an exception. Actors may be created or disappear, they may enter or be dropped from the delineation of the data set, and organizations or countries may also merge or split. Often it is reasonable to regard these changes as exogenous. There are several ways in which they can be accommodated. One is to include actors as nodes in the data set for all time points, but for the panel waves where they are absent specify their incoming and outgoing ties as structural zeros, signifying that these ties are impossible and their absence has no information content. A second way is to use the implementation as a simulation model, and specify for some actors one or more time intervals —which may begin and end at or anywhere between the moments determined by the panel waves— where there cannot be any ties to or from these actors; see Huisman and Snijders (2003). This permits, e.g., that simulations for new actors start at some specific time point between the panel waves, if such information is available. Third, one can construct the pairs (t_m, t_{m+1}) of consecutive panel waves each as a separate transition; if the data set is large enough these may be analyzed separately, but if the data set is not so large, in order to obtain an adequate amount of data, several or all of these pairs can be analyzed simultaneously under the assumption of constant parameter values. This is the 'multiple group' option of the package (Ripley et al., 2016). If the assumption of constant parameters is doubtful, interactions with time trends (e.g., linear, polynomial, or with dummy variables) may be added. In cases of reconfiguration of the actor set, the third option may be especially useful.

4 Dynamics of Non-directed Networks

The Stochastic Actor-Oriented Model as explained above is defined for directed networks, assuming that the actor sending the tie determines its existence. In this section we extend the model to non-directed networks. Such networks occur often in political and organizational studies. There the two actors at either side of the tie have *a priori* a say in its existence, and assumptions must be made about the negotiation or coordination between the two actors involved in tie creation and termination.

In game-theoretic models of networks, it is usually assumed that for a tie to exist, the consent of both actors is involved. This is the basis of the definition of pairwise stability proposed by Jackson and Wolinsky (1996): a network is pairwise stable if no pair of actors can both gain from creation of a new tie between them, and if no single actor can gain from termination of one of the ties in which this actor is involved. In our statistical approach such a stability concept has no place, but the basic idea that both actors should benefit from the tie is translated to our probabilistic framework. Several models are presented here, all based on a two-step process of opportunity and choice, and making different assumptions concerning the combination of choices between the two actors involved in a tie.

4.1 Two-sided Choices

It now is assumed that the network is non-directed, i.e., ties have no directionality: $X_{ij} = X_{ji}$ holds by necessity, and the tie variables X_{ij} and X_{ji} are treated as being one and the same variable. Ties now are indicated by $i \leftrightarrow j$.

For the opportunity, or timing, process, two options are considered.

1. *One-sided initiative*: One actor i is selected and gets the opportunity to make a change. This is a multinomial choice about changing one of the ties from i to another actor.

2. *Two-sided opportunity*: An ordered pair of actors (i, j) (with $i \neq j$) is selected and gets the opportunity to make a new decision about the existence of a tie between them. This is a binary choice about the existence of the tie $i \leftrightarrow j$.

The choice process is modeled as one of three options.

D. *Dictatorial*: One actor can impose a decision about a tie on the other.

M. *Mutual*: Both actors have to agree for a tie between them to exist, in line with Jackson and Wolinsky (1996).

C. *Compensatory*: The two actors decide on the basis of their combined objective function, which can represent coming to a joint agreement. The combination with one-sided initiative seems somewhat artificial here, and we only elaborate this option for the two-sided initiative.

Model M.1, one-sided initiative with reciprocal confirmation, is in most cases the most appealing simple representation of the coordination required to create and maintain non-directed ties. An example about preferential trade agreements is given by Manger and Pickup (2016); an example about trade markets in cultural goods by Shore (2015). Models D.1 and D.2 have potential for the modelling of military conflicts, and model C.2 may be of value in a joint bargaining situation.

4.2 Mathematical elaboration

We give a brief elaboration of the formulae involved in these five options, treating first the opportunity element and then the choice process.

1. *One-sided initiative*:

   For the opportunity to make a change, assumptions (1.-6.) mentioned above for the directional case still are in place.

2. *Two-sided opportunity*:

   For the selection of an ordered pair of actors (i, j) $(i \neq j)$ assumptions (1.-3.) above are maintained, but (4.-6.) are replaced (in abbreviated description) as follows.

   4.2. Each ordered pair of actors (i, j) has a *rate of change*, denoted $\lambda_{ij}(x; \rho)$.

5.2. The waiting time until the next opportunity for change by any pair of actors has the exponential distribution with parameter $\lambda_{\text{tot}}(x;\rho) = \sum_{i \neq j} \lambda_{ij}(x;\rho)$.

6.2. The probability that the next opportunity for change is for pair (i, j) is given by

$$\mathrm{P}\{\text{Next opportunity for change is for pair } (i,j)\} = \frac{\lambda_{ij}(x;\rho)}{\lambda_{\text{tot}}(x;\rho)} \ . \quad (8)$$

The choice process has the three options D(ictatorial), M(utual) and C(ompensatory). In all cases assumption (7.) as defined for the directed case is retained, and assumption (8.) is replaced as follows.

Dictatorial.

Like in the directed case, actor i selects the change of the single tie variable X_{ij} given the objective function $f_i(x^{(0)}, x; \beta)$ using (3), and actor j just has to accept. Combined with the two opportunity options, this yields the following cases.

8.D.1. For one-sided initiative, the probability that the tie variable changed is X_{ij}, so that the network x changes into $x^{(\pm ij)}$, is given by

$$p_{ij}(x, \beta) = \frac{\exp\big(f_i(x, x^{(\pm ij)}; \beta)\big)}{\sum_{h=1}^{n} \exp\big(f_i(x, x^{(\pm ih)}; \beta)\big)} \ , \quad (9)$$

just like in the model for directed relations.

8.D.2. For two-sided initiative, actor i makes the binary choice about whether or not tie $i \leftrightarrow j$ should exist. The probability that network x changes into $x^{(\pm ij)}$, is given by

$$p_{ij}(x, \beta) = \frac{\exp\big(f_i(x, x^{(\pm ij)}; \beta)\big)}{\exp\big(f_i(x, x; \beta)\big) + \exp\big(f_i(x, x^{(\pm ij)}; \beta)\big)} \ . \quad (10)$$

Mutual.

8.M.1. In the case of one-sided initiative, actor i selects the tie variable to be changed with probabilities (9) according to i's objective function. If currently $x_{ij} = 0$ so that the change would mean creation of a new tie $i \leftrightarrow j$, this is proposed to actor j, who then accepts according to a binary choice based on j's objective function, with acceptance probability

$$\mathrm{P}\{j \text{ accepts tie proposal}\} = \frac{\exp\big(f_j(x, x^{(\pm ij)}; \beta)\big)}{\exp\big(f_j(x, x; \beta)\big) + \exp\big(f_j(x, x^{(\pm ij)}; \beta)\big)} \; .$$

If the choice by i means termination of an existing tie, the proposal is always put into effect. Jointly these rules lead to the following probability that the current network x changes into $x^{(\pm ij)}$:

$$p_{ij}(x, \beta) = \frac{\exp\big(f_i(x, x^{(\pm ij)}; \beta)\big)}{\sum_{h=1}^{n} \exp\big(f_i(x, x^{(\pm ih)}; \beta)\big)} \left(\frac{\exp\big(f_j(x, x^{(\pm ij)}; \beta)\big)}{\exp\big(f_j(x, x; \beta)\big) + \exp\big(f_j(x, x^{(\pm ij)}; \beta)\big)}\right)^{1-x_{ij}} . \tag{11}$$

8.M.2. In the case of two-sided opportunity, actors i and j both reconsider the value of the tie variable X_{ij}. Actor i proposes a change (toggle) with probability (10) and actor j similarly. If currently there is no tie, i.e., $x_{ij} = 0$, then the tie is created if this is proposed by both actors, which has probability

$$\begin{aligned} p_{ij}(x, \beta) = {} & \frac{\exp\big(f_i(x, x^{(\pm ij)}; \beta)\big)}{\Big(\exp\big(f_i(x, x; \beta)\big) + \exp\big(f_i(x, x^{(\pm ij)}; \beta)\big)\Big)} \\ & \times \frac{\exp\big(f_j(x, x^{(\pm ij)}; \beta)\big)}{\Big(\exp\big(f_j(x, x; \beta)\big) + \exp\big(f_j(x, x^{(\pm ij)}; \beta)\big)\Big)} \; . \end{aligned} \tag{12a}$$

If currently there is a tie, i.e., $x_{ij} = 1$, then the tie is terminated if one or both actors wish to do this, which has probability

$$\begin{aligned} p_{ij}(x, \beta) = 1 \; - & \left\{ \frac{\exp\big(f_i(x, x; \beta)\big)}{\Big(\exp\big(f_i(x, x; \beta)\big) + \exp\big(f_i(x, x^{(\pm ij)}; \beta)\big)\Big)} \right. \\ & \left. \times \frac{\exp\big(f_j(x, x; \beta)\big)}{\Big(\exp\big(f_j(x, x; \beta)\big) + \exp\big(f_j(x, x^{(\pm ij)}; \beta)\big)\Big)} \right\} . \end{aligned} \tag{12b}$$

Compensatory.

The two actors decide on the basis of their combined objective function, which can represent coming to a joint agreement. The combination with one-sided initiative is somewhat artificial here, and we only elaborate this option for the two-sided initiative.

8.C.2. The binary decision about the existence of the tie $i \leftrightarrow j$ is based on the sum of the objective functions of actors i and j. The probability that network x changes into $x^{(\pm ij)}$, now is given by

$$p_{ij}(x, \beta) = \frac{\exp\big(f_i(x, x^{(\pm ij)}; \beta) + f_j(x, x^{(\pm ij)}; \beta)\big)}{\exp\big(f_i(x, x; \beta) + f_j(x, x; \beta)\big) + \exp\big(f_i(x, x^{(\pm ij)}; \beta) + f_j(x, x^{(\pm ij)}; \beta)\big)} . \tag{13}$$

Putting this together in the transition rates, defined above, gives the following results. Account must be taken of the fact that toggling variable X_{ij} is the same as toggling X_{ji}, and that the rules described above give different roles for the first and the second actor in the pair (i, j). For the models with one-sided initiative, the transition rate is

$$q_{ij}(x) = \lambda_i(x; \rho)\, p_{ij}(x, \beta) + \lambda_j(x; \rho)\, p_{ji}(x, \beta) \ , \tag{14}$$

and for the models with two-sided opportunity

$$q_{ij}(x) = \lambda_{ij}(x; \rho)\, p_{ij}(x, \beta) + \lambda_{ji}(x; \rho)\, p_{ji}(x, \beta) \ , \tag{15}$$

where the functions λ_i, λ_{ij}, and p_{ij} are as defined above.

4.3 Model specification for undirected networks

Again a convenient and flexible class of objective functions can be represented by the linear combination (7). The same effects can be used as for directed networks, but some are redundant, because the ties $i \to j$ and $j \to i$ now are equivalent. For example, the reciprocity effect s_{2i} is the same as the degree effect s_{1i}, for monadic covariates the ego effect s_{10i} is the same as the alter effect s_{11i}, etc.

The choice between the five options has to be made primarily on theoretical knowledge of how the two actors on both sides of a potential tie act together in deciding about the tie. One-sided initiative with reciprocal confirmation (M.1) often may be the most plausible option, in accordance with Jackson and Wolinsky (1996). Studying the correspondence between the five options mentioned above may give some insight into which differences to expect when the same data set is subjected to the different model options. Perhaps the clearest example is the following. If all effects s_{ki} included in a model (7) are such that the contributions of ties are the same for both actors involved (which is the case, for example, for the degree effect s_{1i} and the similarity effect s_{12i}), then the compensatory dyadic model C.2 is identical to the dictatorial dyadic model D.2, except that the parameters β_k are twice as small for C.2 compared to D.2., because of the addition of the two objective functions in (13). For general models this identity will not hold, but in a first-order approximation it still may be expected that the β_k parameters in model C.2 are about twice as small as those in D.2, and the ρ_m parameters are quite similar.

4.4 Estimation and Examples

Method of moment estimators can be obtained for these models in exactly the same way as described in the previous section for models for directed networks. This is because the algorithm for these estimators is based directly on simulation of the network evolution, and the assumptions in this section can be used straightforwardly for simulating the evolution of a non-directed network.

5 Models for Co-evolution of Networks and Nodal Attributes

A major reason for the fruitfulness of a network-oriented research perspective is the entwinement of networks and individual behavior, performance, attitudes, etc., of political actors. The effect of peers on individual political behavior is a well-studied issue, starting from Lazarsfeld et al. (1948); see, e.g., Klofstad (2007). Huckfeldt (2009) argues that, since social interaction leads to influence with respect to political behaviors, the composition of the social context of individuals influences their own attitudes and behaviors, and he draws attention to the endogeneity of the network of interaction partners. Inter-organizational studies have also drawn attention to the importance of networks for organization-level outcomes. Scholz et al. (2008) show that the position of organizations in general contact networks influences their propensity to collaborate and to perceive agreement between stakeholders. Berardo (2009) shows that cooperation between governmental and nongovernmental organizations enhances organizational performance.

Studying the entwinement of networks and actor-level outcomes is difficult because of the endogeneity of both: the network affects the outcomes while the outcomes affect the network. One way to get a handle on this is to model these dynamic dependencies both ways in studies of the co-evolution of networks and nodal attributes. The combination of network and attributes then is viewed as an evolving system, in which the changes in the network are determined probabilistically by the network itself and also by the attributes, while the same holds for the changes in the attributes. A method for modeling this, using panel data of the network and the attributes, was proposed by Steglich et al. (2010), using an elaboration of the Stochastic Actor-Oriented Model. This methodology does not pretend to yield causal conclusions by virtue of the statistical analysis; Shalizi and Thomas (2011) argued cogently that this is impossible in panel studies. What is offered by the method, if applied with a well-specified model, is insight in *time sequentiality*: to what extent is there evidence that changes in attributes depend on the state of the network ('first the network, then changes in the attributes'); and to what extent is there evidence that changes in the network depend on the state of the attributes ('first the attributes, then changes in the network'). The idea is similar to Granger causality, but a mathematical elaboration of this similarity has not been made yet. A more elaborate discussion of the severe difficulties in attempts to establish causality in social networks research is presented in Robins (2015, Chapter 10).

An outline of this model is presented here. When the nodal attributes play the role of dependent variables we use the term 'behavior' as a catch-word that also can represent other outcomes such as performance, attitudes, etc.

5.1 Dynamics of Networks and Behavior

The modeling framework used above for an evolving network $X(t)$ now is extended by considering a simultaneously and interdependently evolving vector of H behavior variables $Z(t) = \big(Z_1(t), \ldots, Z_H(t)\big)$. The value of the h'th variable for the i'th actor is denoted $Z_{ih}(t)$. We assume that all components of the behavior vector $Z(t)$ are *ordinal discrete variables* with values coded as an interval of integers; binary variables are the simplest case.

For modeling the joint dynamics of the network and behavior $\big(X(t), Z(t)\big)$, we follow the same principles as those used to model the development of $X(t)$ alone: time t is a continuous parameter; changes in network and behavior can take place at arbitrary moments between observations; at any single time point, only one variable can change, either a tie variable X_{ij} or a behavior variable Z_{ih}; and the process $\big(X(t), Z(t)\big)$ evolves as a Markov process, i.e., change probabilities depend on the current state of the process, not on earlier states. The principle of decomposing the dynamics in the smallest possible steps is carried further by requiring that a change of a behavior variable at one single moment can only be one step up or down the ladder of ordered values – i.e., by $+1$ or -1, as these variables have integer values.

These principles are elaborated by Snijders, Steglich and Schweinberger (2007) and Steglich, Snijders and Pearson (2010) in a model that has the following basic components.

- For the network changes the *network rate function* $\lambda_i^X(x, z; \rho^X)$ indicates the average frequency with which actor i has the opportunity to make changes in one outgoing network variable.
- For each behavior variable Z_h the *behavior rate function* $\lambda_i^{Zh}(x, z; \rho^{Zh})$ indicates the average frequency with which actor i has the opportunity to make changes in this behavior variable.
- The *network objective function* $f_i^X(x^{(0)}, x; z, \beta^X)$ determines the probability

of the next tie change by actor i, conditional on i having the opportunity to make a network change.

- The *behavior objective function* $f_i^{Zh}(z^{(0)}, z; x, \beta^{Zh})$ for behavior variable Z_h determines the probability of the next behavior change by actor i, conditional on i having the opportunity to make a change in this behavior.

The network dynamics proceeds just as defined above for the network-only case. The behavior dynamics is analogous. Here the option set for the decision of change is different, however, in the following way. For notational simplicity, we give the formulae only for the case of $H = 1$ dependent behavior variable, dropping index h. In a process driven by the rate functions $\lambda_i^Z(x, z; \rho^Z)$, actor i at stochastic moments gets the opportunity to change the value of her behavior Z_i. When this happens, and the current value is denoted $z^{(0)}$, the actor has three options: increase by 1, stay constant, or decrease by 1. If the current value is at the minimum or maximum of the range, one of these options is excluded. The choice probabilities again have a multinomial logit form, the probability of choosing z (with permitted values $z^{(0)} - 1, z^{(0)}, z^{(0)} + 1$) being

$$\frac{\exp\big(f_i^Z(z^{(0)}, z; x, \beta^Z)\big)}{\sum_{d=-1}^{1} \exp\big(f_i^Z(z^{(0)}, z^{(0)} + d; x, \beta^Z)\big)}\ , \tag{16}$$

with obvious modifications in case z is at the boundary of its range. Again, an interpretation of myopic optimization is possible but not necessary.

This model for the co-evolution of networks and behavior permits the expression of both *selection* (e.g., homophilous selection), where the values of Z_{ih} and Z_{jh} influence the probability of creating, or of maintaining, a tie from i to j; and of *influence*, or contagion, where for actor i the probability of changes in Z_{ih} depends on the behaviors Z_{jh} of those actors j with whom i is tied. More generally, changes in the behavior of actor i may depend on i's network position as well as on the composition of the network neighborhood of i.

5.2 Specification of Behavior Dynamics

The main extra component of the model specification regards the objective function for behavior. Here also we use notation just for one single behavior variable Z. Again, a linear combination is considered:

$$f_i^Z(z^{(0)}, z; x, \beta^Z) \; = \; \sum_k \beta_k^Z s_{ki}^Z(z^{(0)}, z; x) \; , \tag{17}$$

where the effects $s_{ki}^Z(z^{(0)}, z; x)$ depend on the network and the behavior. We present effects depending only on the new state z, and accordingly write $s_{ki}^Z(z; x)$ for $s_{ki}^Z(z^{(0)}, z; x)$. A baseline is a quadratic function of the actor's own behavior as the expression of short-term goals and restrictions. With a negative coefficient for the quadratic term this represents a unimodal function that could be regarded as a preference function (again, with a myopic interpretation).

1. This includes the linear term $s_{1i}^Z(z; x) = z_i$, and

2. the quadratic term $s_{2i}^Z(z; x) = (z_i)^2$.

Several statistics could be specified to represent social influence (contagion), such as the following two.

3. The similarity between the behavior of actor i and the actors to whom i is tied, measured just like the analogous effect s_{12i} for the network dynamics,

$$s_{3i}^Z(z; x) = \sum_j x_{ij} \left(1 - \frac{|z_i - z_j|}{\text{Range}(z)} \right) \; .$$

4. The product of the own behavior z_i with the average behavior of the other actors to whom i is tied, $s_{4i}^Z(z; x) = z_i \big(\sum_j x_{ij} \, z_j \big) / \big(\sum_j x_{ij} \big)$ (defined as 0 if this is 0/0). Together with the two terms s_{1i}^Z and s_{2i}^Z, this yields a quadratic function of which (if the coefficient of s_{2i}^Z is negative) the location of the maximum is a linear function of the average behavior in the 'personal network' of i.

The effects s^Z_{3i} and s^Z_{4i} both express the concept of influence, albeit in different mathematical ways. The choice between them can be based on theoretical grounds, if any theoretical preferences exist – else on empirical grounds.

The behavior dynamics can also depend on network position directly, for example, on the degrees of the actor.

5. It can depend, e.g., on the 'popularity' of actor i as measured by the indegree, i.e., the number of incoming ties, $s^Z_{5i}(z;x) = z_i\, x_{+i}$, and/or
6. on the 'activity' of actor i as measured by the outdegree, i.e., the number of outgoing ties, $s^Z_{6i}(z;x) = z_i\, x_{i+}$.

In addition, it will often be important to include effects of other actor-level and contextual variables on z_i, in accordance with the political theories explaining this dependent variable.

The parameter estimation for this model is treated in Snijders et al. (2007). An application to partner selection in policy networks and its co-evolution with generalized trust is presented by Berardo and Scholz (2010). Manger and Pickup (2016) and Rhue and Sundararajan (2014) apply this model to the diffusion of democracy.

6 Co-evolution of Multiple Networks

The principle of co-evolution can also be applied to the interdependent dynamics of several networks on the same set of actors; one might call this a multivariate network. The elaboration is quite analogous to the co-evolution of networks and behaviour. Each network has its own rate function and objective function, and the objective function (perhaps also the rate function) for each network will depend on the network itself and the other network/s. It should be noted that the dependence is on the current state of the system, not on the last observed state.

The model for co-evolution of multiple networks was first presented by Snijders et al. (2013). This extension also allows the representation of networks with ordered tie values (although the number of values must be small).

Liang (2014) applied this method in a study of the co-evolution of the discussion network and a semantic interpretation network in an internet discussion forum about the 2012 U.S. presidential election.

7 Discussion

Network-related research questions lead to various issues at the interface between theory and methodology – in political as well as other sciences. One issue is how to make the combination of, on the one hand, theories in which individual actors have primacy and which recognize the embeddedness in the social context (cf. DiPrete and Forristal, 1994; Udehn, 2002; Huckfeldt, 2009) and, on the other hand, empirical research with data sets including dyadic as well as monadic variables. Another issue is the fact that hypotheses about dyadic relations between social actors almost by necessity will imply dependence between dyadic tie variables, and also between dyadic and monadic variables, which requires novel statistical methods. This dependence often can be regarded as a consequence of endogeneity, i.e., resulting from interdependent choices by multiple actors: for example, in studies of how actors are influenced by those actors to whom they are tied it is important to recognize that the network may be endogenous, and in studies of homophilous choice of interaction partners the behavior that is the dimension for homophily may be endogenous. Dropping the assumption of independence implies that the dependence between variables has to be specified in a plausible way in order for the statistical analysis to be reliable. However, our theories mostly give only a very incomplete handle on this specification; statistical models representing dependencies between dyadic variables are recent and still in various stages of development; and as yet we know little about the sensitivity of conclusions for the misspecification of such statistical models.

The three main approaches currently available to statistically analyzing network dynamics —which next to the SAOM are temporal Exponential Random Graph Models ('ERGMs') and latent Euclidean space models (for the overview see Desmarais and Cranmer, 2016)— employ quite different means to express network and time dependencies, i.e., the correlation structure between the tie variables observed at the same and at different moments. In the SAOM the flow of time is explicitly present, which implies that varying time lags between observations are easily incorporated. If the model is valid for some data set, dropping an observation, or inserting an additional one, or changing the timing of observations, will keep the probability distribution of the rest intact; this is not the case for the other approaches. The fact that the analysis is simulation-based allows flexible handling of irregularities in the observation design such as exogenous changes in the node set. The simulation setup is limited, however, in the sense that observed changes are considered as the result of a series of changes of single ties, each of which implies a change —usually small— of the network context for all actors. Sometimes this is reasonable, but this representation of network dependencies may be deficient in cases where groups of actors work in concert to form or rupture ties, especially when the composition of these groups changes endogenously. The SAOM as well as the various temporal Exponential Random Graph variants, such as the TERGM (Hanneke et al., 2010) and the StERGM (Krivitsky and Handcock, 2014), are based on explicitly specifying the network dependencies. The latent space models (Sewell and Chen, 2015; Dorff and Ward, 2016), by contrast, assume that network dependencies can be represented by positioning the nodes in a low-dimensional Euclidean space. For some research questions the representation with latent spatial positions will be more natural, for others the representation by differentiated types of network dependencies. For example, for directed networks, the SAOM and ERGM representations include separate parameters for reciprocation and for transitive closure, permitting to test hypotheses about such processes, while in latent space models these are represented together by the spatial configuration. It will be interesting to compare the interpretative value of the spatial representation in latent space models to the value of the dependence aspects represented by the ‘effects’ as explained above for the SAOM and which are used similarly for the ERGM. The interpretative insights provided by these representations of dependence have not yet been much explored. It should be noted that covariates will ‘take out’ part of the dependence, so that the ‘remaining’ representation of dependence may depend strongly on the covariates used.

The SAOM and temporal ERGM approaches tackle issues of dependence for panel data on ‘complete networks’. This means that the network consists of the pattern of ties between all actors in a well-delineated group, and ties of these actors with others outside the group may be ignored: the network boundary problem (Marsden, 2005) is assumed to have been solved in an earlier phase of the research. Within these models the wider context outside of this group, to which every group member is exposed, is therefore kept constant, and its influence is not considered. This implies, in terms of the statement by Huckfeldt (2009, p. 928) that ‘(p)olitical communication networks are created as the complex product of this intersection between human choices and environmentally imposed options’, that the methods treated here focus on the ‘human choice’ component, while the determination of the node set is considered to be ‘environmentally imposed’. The latent space models, on the other hand, are less strict in the assumption of a complete network having been observed; if the data follow a latent space model, after randomly dropping some nodes from the data set, what remains still will follow the latent space model. This is a strength with respect to data sets permitted; but it also signals a potential weakness in the representation of network dependencies, because theoretical network ‘mechanisms’ may be sensitive to arbitrary deletions of parts of the network.

Above we commented on the difficulty of achieving causal interpretations from statistical analyses where networks figure among the dependent variables; briefly stated, the endogeneity of the networks is so strong that statistical methods will be able to establish time sequentiality, but not strict cause-and-effect relationships.

The definition of the SAOM in terms of choices by individual actors means that changing dyadic and monadic variables can be analyzed in a coherent framework according to theories where the analytical primacy is with structurally embedded individual actors, in line with structural individualism (Udehn, 2002). The interpretation of the multinomial logistic model in terms of myopic choices does not exclude strategic considerations, but means that these have to be represented by the short-term goals through which actors attempt to reach their long-term objectives. Theoretical arguments given in the literature for the occurrence of structural effects such as reciprocity and transitivity are, indeed, mostly based on their importance as intermediate goals serving the purpose of ulterior objectives; this is the case, e.g., for the argument proposed by Coleman (1988) that transitivity (triadic closure) gives opportunities for social control and sanctioning, as well as for the theory (Burt, 1992) that structural holes are a means for obtaining positional advantage.

The models presented here are implemented in the program *SIENA*, 'Simulation Investigation for Empirical Network Analysis', which is available as the R package RSiena (Ripley et al., 2016). The associated website `http://www.stats.ox.ac.uk/~snijders/siena/` contains a lot of tutorial and example material.

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