A.6 Life insurance: Premiums and reserves

1. Show that the present value at the rate of interest i of an annuity payable in advance of £1 pa accumulating during the life x until the year of his death at the (compound) rate j is given by

$$\frac{1+j}{j}\,(A_x^* - A_x)$$

where A_x^* is calculated at the rate of interest $J = (j-i)/(1+j)$.

2. Show that the product of

- the reserve after t years for an annual premium n-year pure endowment issued to a life aged x

and

- the annual premium for pure endowment of like amount issued at the same age but maturing in t years

is constant for all values of t. (Ignore expenses.)

3. Let ${}_tV_x$ be the reserve at time t on an insurance policy issued to a life aged x that pays a benefit of $S_x(t)$ on death at time t, under which premiums are payable of $P(t)$ at time t.

(i) Show by general reasoning that

$$\frac{\partial}{\partial t_t}\,V_x = P(t) + \delta_t V_x - \mu_{x+t}(S_x(t) -_t V_x)$$

where δ is the force of interest.

(ii) Using the above, show that for a continuously paid constant premium whole life assurance with sum assured 1 payable at the point of death

$${}_tV_x = 1 - \frac{\bar{a}_{x+t}}{\bar{a}_x}$$

4. Thiele's equation for the policy value at duration t, ${}_t\bar{V}_x$, of an immediate life annuity payable continuously at a rate of £1 per annum from age x is:

$$\frac{\partial}{\partial t_t}\,\bar{V}_x = \mu_{x+t}\;{}_t\bar{V}_x - 1 + \delta\;{}_t\bar{V}_x.$$

(i) Derive this result algebraically showing all the steps in your argument.

(ii) Explain this result by general reasoning.

5. Show an expression for the premium (in standard actuarial functions) for a 25 year endowment assurance on a life aged 40. The initial expenses are £2 per £100 sum insured, renewal expenses are 5% of each premium and 30p each year per £100 sum insured, and there is an initial commission of 50% of the first year's gross premium.

6. Some time ago, a life office issued an assurance policy to a life now aged exactly 55. Premiums are payable annually in advance, and death benefits are paid at the end of the year of death. The office calculates reserves using gross premium policy values. The following information gives the reserve assumptions for the policy year just completed. Expenses are assumed to be incurred at the start of the policy year.

   Reserve brought forward at the start of the policy year: £12,500

   Annual premium: £1,150

   Annual expenses: £75

   Death benefit: £50,000

   Mortality: A1967/70

   Interest 5.5% per annum

   Calculate the reserve at the end of the policy year.

7. A deferred annuity is purchased by 20 annual payments payable by a life aged 40 for a year annuity in advance of £2,500 a year, commencing in 20 years, for life. Find an expression for the premium on the basis of 4% pa interest with expenses of 5% of each premium and £5 at each annuity payment.

8. Suppose that $l_x = 100,000(100 - x)$, where $0 \leq x \leq 100$, and the interest rate is 5%.

   (a) Calculate $A_{50:\overline{10}|}$ and $\ddot{a}_{50:\overline{10}|}$.

   (b) Calculate the net annual premium for a 10 year endowment assurance for £10,000 to someone aged 50 and the policy values of years 3 and 4 using the values above.

   (c) Suppose that expenses are as follows

Commission:	50% of First Premium
	2% of Subsequent Premiums
General Expenses:	£150 Initially
	£10 in each subsequent year.

   Calculate the office premium for the policy in (b).