



## Sample from Population of Networks

Suppose we have a sample indexed by j = 1, ..., N from a population of networks on disjoint node sets, where the networks are 'replications' of each other in the following sense:

they all are regarded as realizations of processes obeying the same model, but having different parameters  $\theta_1, \ldots, \theta_i, \ldots, \theta_N$ .

Each disjoint network is called a **group**. We assume we have network panel data for each group, and wish to analyze these by fitting Stochastic Actor-Oriented Models ('*SAOMs*', RSiena).

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	Multilevel analysis		
Such data sets have	e multilevel structure:		
all caveats and cons	siderations from usual multilevel an	alveis annlyl	
an caveats and cons		aryois appry:	
	(1 11 1 0010)		
See Snijders & Bosi	ker (textbook 2012);		
T.A.B. Snijders, 'The	Multiple Flavours of Multilevel Issu	ues for	
Networks',			
in Lazega & Sniider	s (2016).		
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Multilevel designs, generalization, and the risks of having few groups

For some basic intuitive understanding of the consequences of multilevel designs, consider the simplest case: estimation of the population mean  $\mu$  of some variable.

Left one-sided testing.

We have N = 5 randomly drawn groups each of size n = 20. Data range from -2.5 to +3.5, as in the histogram (next page). Is there evidence that the population mean is positive?

```
Test H_0: \mu = 0 against H_1: \mu > 0.
```









M	ultilevel analysis		
The conclusion for both the multilevel model tells	data sets is not only, that us $\mu$ is not significantly p	oositive.	
There is also significant $\mu$ is not zero in all groups (this can be concluded a	heterogeneity between th s Iready from an OLS analy	e groups: ysis).	
And one of the groups has that the significance even	as such a small <i>p</i> , n survives multiple testing	g with Bonferroni.	
If we wish to generalize a there is no evidence of a but there is evidence tha at least one of these gro and that the mean of <u>the</u>	to the population of all gro positive mean; tt u <u>ps</u> has a positive mean, <u>se</u> five groups is positive.	oups,	
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Multilevel analysis
Message
The message of the preceding is:
When you have a small number of groups, especially if they are heterogeneous, assuming a random effects model is risky.
It is important to look at results for individual groups.
If groups are large enough to be analysed individually, do so.

Multilevel analysis			
After this sensitization, on with the four approaches:			
1. Multi-group analysis:			
assume all parameters are identical.			
2. Integrated hierarchical approach:			
Assumption: population of networks, normal distribution,			
A. mixed effects,			
some parameters varying, others constant across groups;			
B. random effects, all parameters varying.			
3. Meta analysis:			
Assumption: population of networks,			
no distributional assumptions:			
two-stage meta analysis.			
4. Meta analysis: no population assumption:			
Fisher combination of independent tests.			
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Multi-group analysis		
all parameters are identical.		
<i>possible but risky</i> Gives a global impression, risk of underestimated standard-errors.		
Try to find relatively homogeneous sets of groups for do	ing this.	
sienaTimeTest() can be used to test homogeneity a	ssumption.	
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Integrated hierarchical random effects models		
Hierarchical multilevel analysis		
Assumption: population of networks;		
$\theta_j \sim$ multivariate normal distribution,		
perhaps conditionally on network-level covariates.		
1. mixed effects:		
some parameters varying, others constant across groups:		
restricted integrated hierarchical approach.		
2. random effects:		
all parameters varying across groups:		
integrated hierarchical approach.		
multiSiena: multi-level option, sienaBayes()		
available from Siena website. © Tom A.B. Snijders sienaBayes	June 2023	16 / 66

Integrated hierarchical random effects models

Integrated procedure:

Estimate the <u>distribution</u> of  $\theta_j$  and consider the 'posterior' distribution of  $\theta_j$  given the data.

Advantage:

The analysis of the separate networks draws strength from the total sample of networks by regression to the mean.

Useful especially for many rather small networks.

(For large networks, where the model of interest can be estimated without any problems, a two-stage procedure may be better: (1) estimation by group, (2) meta-analysis.)

Integrated hierarchical random effects models				
Bayesian approach				
The approach taken here is that of <i>Bayesian statistics</i> , as distinct from the more usual <i>frequentist statistics</i> .				
Bayesian statistics postulates a probability distribution not only for the data, but also for the parameters.				
The summary is that the data Y have a conditional distribution given the parameters $\theta$ , which is the statistical model; and the parameters $\theta$ have a prior distribution				
reflecting what the researcher knows about them in advance,				
the statistical analysis then leads to the posterior distribution.				
which is the conditional distribution of the parameters given the data.				
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Integrated hierarchical random effects models				
Bayesian approach (2)				

In symbols:

Before the data we have the model  $p(x \mid \theta)$ and the prior  $\pi(\theta)$ ,

then given the data we have the posterior  $p(\theta \mid x)$  which is defined, according to Bayes' rule, as

$$p(\theta \mid x) = \frac{\text{joint p.d. of }(x, \theta)}{\text{p.d. of } x} = \frac{p(x \mid \theta) \times \pi(\theta)}{\int (p(x \mid \theta') \times \pi(\theta')) \, d\theta'}$$

Integrated hierarchical random effects models	
Bayesian approach (3)	
The parameter can then be estimated by its posterior mean	
$E\{\theta \mid x\}$	
and its uncertainty is expressed by the posterior covariance matrix	
$\mathrm{cov}\{ heta\mid x\}$ ;	
the posterior standard deviations are the square roots of its diagonal elements, and have the role of standard errors.	
This is all very nice; the catch is that the researcher has to define a prior.	
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For sienaBayes, however, a large number of random parameters seems to be less of a problem.

But note the important distinction:

For fixed parameters ( $\eta$ ), there is much more information than for the means and variances of random parameters ( $\mu$ ,  $\Sigma$ ).

Therefore, the posterior distribution will be more concentrated (smaller variance) for the  $\eta$  parameters than for  $\mu$ .

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Integrated hierarchical random effects models Fixed effect k: Testing  $\eta_k = 0$  means testing the null hypothesis that the effect  $\theta_{ik} = 0$  in every group *j*, under the auxiliary assumption that  $\theta_{ik}$  is the same for all *j*, i.e.,  $\theta_{ik} = \eta_k$ . random effect k: Testing  $\mu_k = 0$  means testing the null hypothesis that the population average of the effects  $\theta_{ik}$  is 0. The auxiliary assumption leads to a smaller posterior standard deviation for  $\eta_k$  than for  $\mu_k$ , unless the prior for  $\mu_k$  has a very small variance. @ Tom A B. Sniiders sienaBaves June 2023 23/66

Integrated hierarchical random effect	ts models			
Group-level variables				
It is possible to use group-lev These should be defined as a with the same value for all ac	el variables. constant actor covaria tors.	<b>ates (</b> coCovar),		
It is helpful for convergence to center the group-level variables. I'm not sure that the centering in the definition of coCovar survives the operation of sienaGroupCreate in a good way.				
Therefore the recommendation is to center such variables "by hand", by subtracting their mean (or approximate mean) before executing coCovar.				
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Integrated hierarchical random effects models

## Varying group sizes

Parameters of the ERGM and SAOM are not well comparable across group sizes (number of actors in the networks), because if the number of actors is larger while further the network evolution is similar, there are more ties that should <u>not</u> be created.

For ERGMs this was shown in research by Krivitsky, Kolaczyk, and Butts.

The main culprit for the SAOM is the density parameter.

To take account of the different group sizes, it is advisable to use the group-level covariate with the value  $\log(n)$ , where  $\log(\cdot)$  is the natural logarithm and *n* group size.



both the  $\theta_{j}$  referring to the individual sampled networks,

and  $\eta,\,\mu$  and  $\Sigma$  referring to the population of networks.

The inference is based on these sampled posteriors.

Three kinds of plot are important:

- multidimensional scaling plots of the group-wise posterior means, indicating possible outliers;
- trace plots, representing successive draws from the posterior distribution (after *thinning*);
- density plots, representing the plausible values of the parameters, given the observed data.

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Prior distributions		
Prior distributions		
For a smallish number of groups, the prior is conseq It is given to sienaBayes as priorMu, priorSign priorKappa, priorMeanEta, priorSigEta.	<b>uential</b> . ma,priorDf,	
For rate parameters, signaBayes uses a data-dependent prior (depending on the option chosen).		
The six prior parameters are explained in the following	ng pages.	
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	Prior distributions			
<i>Model</i> : for all groups <i>j</i> ir	ndependently, $ heta_j^{(1)} \sim \mathcal{N}(\mu)$	,Σ).		
Prior. the prior distributi inverse Wishart distribut and, conditional on $\Sigma$ , for mean $\mu_0$ and covariance	on for $(\mu, \Sigma)$ is the tion for $\Sigma$ ; or $\mu$ a multivariate normal e matrix $\Sigma/\kappa_0$ . In formula	distribution with		
$\Sigma \sim \text{InvWish}$ $\mu \mid \Sigma \sim \mathcal{N}_{p_1}($	$\operatorname{art}_{p_1}(\Lambda_0^{-1}, \mathrm{df})$ , and condition $(\mu_0, \Sigma/\kappa_0)$ .	onally on $\Sigma$		
Here $\mu_0$ = priorMu, $\Lambda_0$ df = priorDf, $\kappa_0$ = pri	= priorDf × priorSio orKappa.	gma,		
The 'central tendency' of this inverse Wishart distribution is approximately priorSigma.				
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Prior d	distributions		
priorMu			
<ol> <li>priorMu is your prior g priorSigma is your pr for their between-group</li> </ol>	juess for the mean of $\theta_j^0$ ior guess s variance.	<u>1</u> ),	
<ol> <li>For priorMu you norm perhaps priorMu (out priorMu (reciprocit and its other coordinate</li> </ol>	ally do not want to spec degree) a value like - ty) a value like +1 or + s 0.	sify much; -1 or –2, 2,	
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Prior distributions
priorKappa Your ignorance about the mean $\mu$ of $\theta_j^{(1)}$ is expressed by priorKappa <sup>-1</sup> × priorSigma.
Note that this entails a double use of priorSigma: it is the prior between-groups covariance matrix, and also the ignorance about the prior mean times a constant (viz., 1/priorKappa). This is perhaps unfortunate but a mathematical convenience.
The consequence is that you should choose priorKappa in view of the diagonal elements of priorSigma, so that priorSigma/priorKappa represents a reasonable degree of uncertainty.
E.g., diag(priorSigma) between 0.05 and 0.6 and priorKappa=0.1. © Tom A.B. Sniders sienaBayes June 2023 32/66



Prior distributions	
priorMeanEta <b>and</b> priorSigEta	
<i>Model</i> : $\eta$ ('eta') is the vector of non-varying param	neters.
Mostly, there will be a lot of information about the and it is not necessary to specify a prior distributi technically, they can get a <i>constant improper prior</i>	ım, on; or.
However, some effects could be group-level effect e.g., effects of group variables such as the propo or log group size (represented by its egox effect)	rtion of males
For such variables, sienaBayes has the option to specify a normal distribution with mean given b the variance is given by priorSigEta.	<b>Dy</b> priorMeanEta;
The elements of priorMeanEta and priorSig will specify the constant prior.	Eta which are NA

Prior distributions	
Practical advice for prior specification In most cases: use defaults for priorRatesFromData, priorDf, priorSigEta;	
for priorMu, e.g. use –2 for the density and +2 for the reciprocity parameter;	
if p is the number of randomly varying effects (including rate parameters), as reported from print (myeff, includeRandoms=TRUE, dropRates=TRUE) if there are two waves and one dependent variable:	
Sig <- matrix(0,p,p) diag(Sig) <- 0.1 Sig[2,2] <- 0.6 # density parameter Sig[3,3] <- 0.6 # reciprocity parameter  ans <- sienaBayes(, priorMu=Mu, priorSigma=Sig, priorKappa=0.1,	. )
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Prior distributions		
The determination of these prior param that it is advisable to do a sensitivity an try out several plausible values and investigate the importance of the differe	eters is so full of uncertainties, alysis: ences in results.	
Unfortunately, this takes a lot of time		
Perhaps it is more important to try out several specifications of what are the randomly varying parameters.		
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The function sienaBayes		
sienaBayes (multiSiena)	R D	ocumentation
A function for fitting Bayesian models		
Description		
A function to fit hierarchical Bayesian models random effects to sienaGroup data objects. Uses the function	naxlikec for the S.	AOM part,
the Bayesian part is performed in R.		
Usage		
sienaBayes(data, effects, algo, saveFreq=100,		
initgainGlobal=0.1, initgainGroupwise = 0.02, initfgain=0.2, gamma=0.05,		
initML-FALSE, priorMeanEta-NULL, priorSigEta-NULL,		
priorMu=NULL, priorSigma=NULL, priorDf=NULL, priorKappa=NULL,		
priorRatesFromData=2,		
frequentist=FALSE, incidentalBasicRates=FALSE,		
reductionFactor=0.5, delta=1e-04,		
nprewarm=50, nwarm=50, nmain=250, nrunMHBatches=20,		
nSampVarying=1, nSampConst=1, nSampRates=0,		
nImproveMH=100, targetMHProb=0.25,		
<pre>lengthPhase1=round(nmain/5), lengthPhase3=round(nmain/5),</pre>		
storeScores = FALSE,		
storeAll-FALSE, prevAns-NULL, usePrevOnly-TRUE,		
<pre>prevBayes=NULL, newProposalFromPrev=(prevBayes\$nwarm &gt;= 1),</pre>		
silentstart = TRUE,		
<pre>nbrNodes=1, clusterType=c("PSOCK", "FORK"),</pre>		
getDocumentation=FALSE)		
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The function sienal	Bayes		
The function sienaBayes	s: initialization		
The initialization consists, first, for the whole data set as a mult Assumption: all parameters equal across gro	of a brief MoM estimation li-group model by siena0 bups except for basic rates	7.	
After this, a brief MoM estimation as taking the earlier estimation as	on for each group, the starting point.		
These estimations do not need they are allowed to be quite rou	to converge (they use nsu igh.	ıb=2 <b>)</b> ,	
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The function sienaBayes
Multiplication factor
Subsequent simulations (in the nrunMH steps) are correlated. The autocorrelations of the estimation statistics <sup>1</sup> should preferably be less than 0.4. In Section 6.11 of the manual, this is mentioned as a requirement for ML estimation. For sienaBayes this is not really a requirement; but it might nevertheless be helpful to satisfy it also here. This autocorrelation is influenced by the <i>multiplication factor</i> .
See the manual for the description of what the multiplication factor does.
To get a lower autocorrelation for a given group/period, the multiplication factor can be increased, which then increases the number of MCMC steps (given as ans\$nrunMH).
<sup>1</sup> For likelihood estimation used in signaBayes, these are the score functions

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The function sienaE	Bayes		
Setting the multiplication	n factor		
The multiplication factor can be or a vector with length nGroup. (the number of basic rate param referring to the group × period	given as one number, s × nPeriods neters for one dependent v combinations.	ariable),	
Computation time is roughly pro so it should not be set unneces It may be best to set it for each	oportional to the multiplicat sarily high. group/wave combination s	ion factor	, /.
Adequate values of the multiplication factor do not depend strongly on the model specification, and therefore can be determined from results for ML estimation using an empty model.			
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just to have an idea of their sizes.



## Prolonging sienaBayes

If you decide that your sienaBayes run was too short, you can prolong it giving it as the prevBayes parameter to a new run of sienaBayes.

This should have the same prior distribution. The initialization then is skipped, and the new runs start at the end of the earlier runs.

If newProposalFromPrev=TRUE, the proposal distribution will be determined from the results of the prevBayes object;

if newProposalFromPrev=FALSE, the proposal distribution will be identical to the one of the prevBayes object.

Advice: use newProposalFromPrev=TRUE if the trace plots of the prevBayes object are not very stable.

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sienaBayes

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The function sienaBayes			
Main parameters (5): less important			
incidentalBasicRates: the value TRUE can be tried out if convergence is difficult mainly because of the basic rate parameters. The basic rate parameters then are taken out of the Bayesian estimation and estimated by Maximum Likelihood. Experimental. (Target density of the MCMC process then is profile likelihood, maximized over rate parameters.)			
<ul> <li>nImproveMH: Number of iterations per improveMH step.</li> <li>Can be chosen smaller if faster computations are required. Small values will lead to less precise tuning.</li> </ul>			
<ul> <li>targetMHProb: the desired proportion of acceptances in MCMC steps of updating θ<sub>j</sub><sup>(1)</sup>, μ, η, Σ.</li> <li>Theoretical optimal value 0.25, tried to be reached by improveMH. Experimental.</li> </ul>			
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The function sienaBayes		
Combining several sienaBayes objects		
When you have several signaBayes objects for the same d model specification (including choice of random effects), e.g. use of prevBayes, they can be combined by	lata an . throug	d gh
<ul> <li>glueBayes: produces one new sienaBayes object by concatenating two sienaBayes objects;</li> </ul>	/	
<ul> <li>extractSienaBayes: extracts simulated parameters f of sienaBayes objects;</li> </ul>	from a	list
this can be used with a list of one such object, then think of using		
drop(extractSienaBayes(		
to get rid of the superfluous dimension.		
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The function sienaBayes	
print.sienaBayesFit {multiSiena}	
Methods and functions for processing sienaBayes objects	
Description	
print and summary methods for sienaBayesFit objects, and further functions for interpretation of results.	
Usage	
## S3 method for class 'sienaBayesFit'	
<pre>print(x, nfirst=NULL,)</pre>	
## S3 method for class 'sienaBayesFit'	
<pre>summary(object, nfirst=NULL, allGroups=FALSE,)</pre>	
## S3 method for class 'summary.sienaBayesFit'	
<pre>print(x, nfirst=NULL, allGroups=FALSE,)</pre>	
$shortBayesResults(x, \ nfirst=NULL)$	
<pre>plotPostMeansMDS(x=NULL, pm=NULL, pmonly=1, dim=2, method=1, excludeRates=TRUE, nfirst=NULL,)</pre>	
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	The function sienaBayes					
extract.sien	Bayes (multiSiena)		R Documentation			
	Extraction of posterior samples or posterior	means from sienaBayes results				
Descripti	20					
The first far	The first function extracts posterior samples from a list of <u>signalinestic</u> object to be used, e.g., for assessing convergence.					
The second	uncuon extracts posterior means and standard deviations per group	from a sterarayesent object.				
Usage						
extract.s	ienaBaves(zlist, nfirst-zlist[[1]]\$nwarm+1, extra	ted.				
	sdLoe=TRUE)					
extract.p	extract.posteriorMeans(z, nfirst=z\$nwarm+1, pmonly=1, excludeRates=FALSE,					
	verbose=TRUE)					
Arguments						
zlist	A list of $\underline{signality} esfit}$ objects, further called 'chains', resulting f specification.	om calls to <u>sienallayes</u> with a common da	ita set and model			
z	A sienallayesFit object.					
nfirst	Integer: the first element for the first MCMC chain used for calcu	lating properties of the chain.				
extracted	The parameters for which posterior samples are to be extracted: "all": all parameters; "extes": all groupwise rate parameters;					
"varying": all varying non-rate parameters: global means and standard deviations;						
	"non-varying": all estimated non-varying (and therefore, non-rate	) parameters;				
	"objective": all non-rate parameters.					
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	The function sienaBayes		
bayesTest {	multiSiena}	R Documentation	
	Tests for sienaBayes results with print and plot methods		
Descripti	on		
These functi results of the	ons compute tests based on <u>simaflayesfit</u> objects resulting from <u>simaflayes</u> . Print and plot methods are a multi-parameter test.	wailable for the	
Usage			
simpleBay	<pre>esTest(z, nfirst-z\$mwann+1, tested0-0, probs = c(0.025.0.975), ndigits=4)</pre>		
multiple8	ayesTest(z, testedPar, nfirst=z\$nwarm+1, tested0=0, ndigits=4)		
## S3 met	hod for class 'multipleBayesTest'		
print(x,	descriptives-FALSE,)		
## S3 met	hod for class 'multipleBayesTest'		
plot(x, x	lim-NULL, ylim-NULL, main-NULL,)		
Arguments			
z	A simultayesfit object, resulting from a call to simultayes.		
nfirst	The first element of the MCMC chain used for calculating properties of the chain; this can be the first e assumed that convergence has occurred.	lement for which it is	
tested0	The value to be tested; for simpleBayesTest this must be a number (applied to all coordinates), for mult: be a number (applied to all coordinates) or a vector.	ipleBayesTest if can	
probs © To	A vector of two numbers between $\hat{n}$ and $1$ the credibility limits for the codibility intervals m A.B. Snijders sienaBayes	June 2023 61 /	66





Posterior means can be plotted by the function plotPostMeansMDS.

I use this mainly as a diagnostic for whether there are too many random effects.

If there are too many random effects (in view of what is reasonable given the number of groups) the 'estimation' of the random effects will be trapped randomly by a few groups, and this will show in a pattern with a dense core and a few outliers, where the outlying groups will be different in different runs of sienaBayes (with different random number seeds).

In this case, there probably also will be poor convergence.





## Literature

Johan H. Koskinen and Tom A. B. Snijders (2023), Multilevel Longitudinal Analysis of Social Networks. *Journal of the Royal Statistical Society, Series A*, in press.

DOI: https://doi.org/10.1093/jrsssa/qnac009

An example of the use of sienaBayes is in the script http://www.stats.ox.ac.uk/~snijders/siena/ RscriptsienaBayes\_4.r

An example of the meta-analysis approach (not the integrated random parameter approach) using Chris Baerveldt's data is in the script

RscriptMultipleGroups\_meta.R

on the scripts page of the Siena website.

RSiena manual: Chapter 11.

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