The co-evolution of one-mode and two-mode networks

Tom A.B. Snijders

University of Oxford
University of Groningen

May 2010

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We skip the literature review.

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a set $\mathcal{N}$ of actors (the 'actor mode') and
a set $\mathcal{M}$ of groupings (the 'group mode');
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Structural patterns of the evolution of
the one-mode network are not discussed now.

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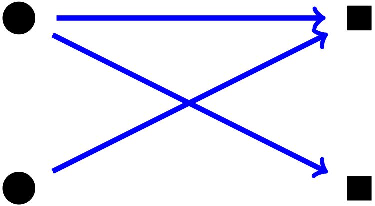

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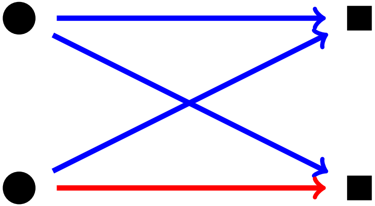

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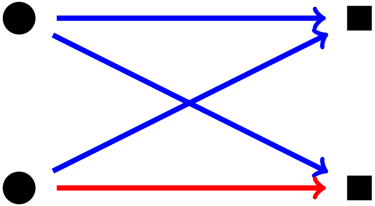

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Robins and Alexander (2004):
transitivity in bipartite networks expressed by 4-cycles.

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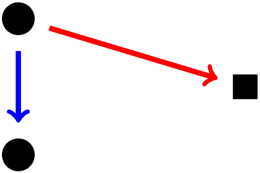

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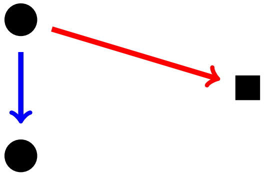

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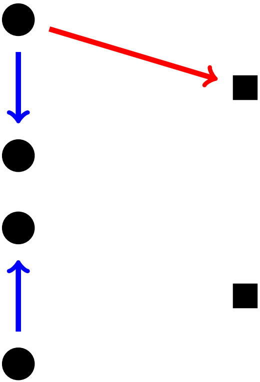

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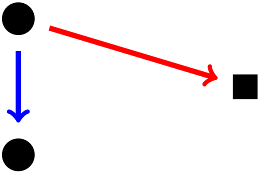

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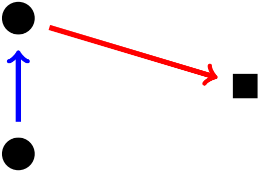

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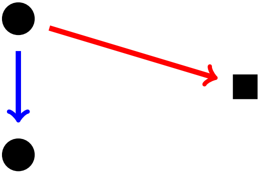

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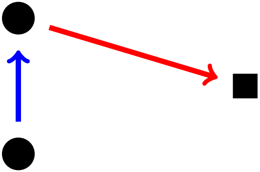

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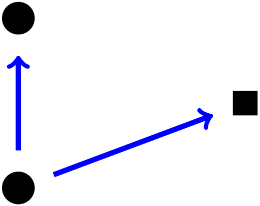
but they are possible as mixed patterns. Examples:

One-with-two-mode triads.

One-mode tie ⇒
two-mode agreement



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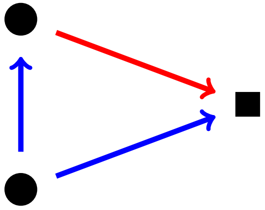

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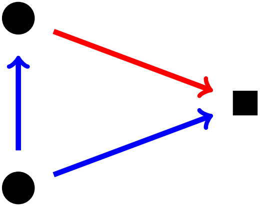

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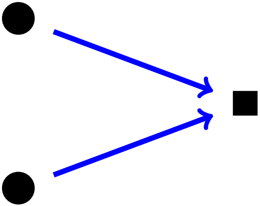

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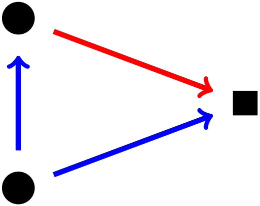

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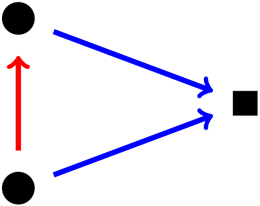

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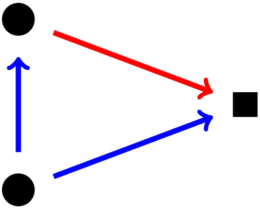

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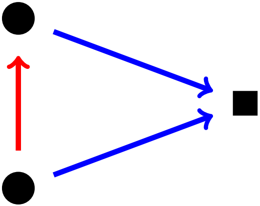

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3. The ties have inertia: they are *states* rather than *events*.
4. The multiple relations together develop stochastically according to a Markov process.
5. At any single moment in time, only one tie variable may change: no coordination.

6. Changes in each network are modeled as choices by actors in their outgoing ties, with probabilities depending on '*objective functions*' of the network state that would obtain after this change.

These objective ('goal') functions are specified separately for each of the R networks.

Notation

Denote tie variable for r^{th} relation from i to j by

$$X_{ij}^{(r)} = \begin{cases} 1 & \text{if} \quad i \stackrel{r}{\to} j \\ 0 & \text{if} \quad \text{not } i \stackrel{r}{\to} j, \end{cases}$$

where this depends on time t.

By X we denote the collection of all R relations:
array $\left(X_{ij}^{(r)}\right)$ for $r = 1, \ldots, R;\ i = 1, \ldots, n;\ j = 1, \ldots, m_r$
($m_r = n$ if the r^{th} relation is one-mode).

The statistical model is a *process model*:

an agent-based simulation model,
which simulates the development of the multiple networks
from one observation to the next;

statistical modeling consists of fitting such a simulation model
to the observed network data, and testing
which model components are required to give a good fit.

The model is defined by its smallest possible steps,
the 'microsteps', which consist of a change in one tie variable:

extend one new tie / withdraw one existing tie.

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Rate of change of relation r is $\lambda_+^{(r)} = \sum_i \lambda_i^{(r)}$;

total rate of change is $\lambda_+^{(+)} = \sum_r \lambda_+^{(r)}$.

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2. The probability that this is an event where actor i may change an r-tie is

$$\frac{\lambda_i^{(r)}}{\lambda_+^{(+)}} \; .$$

   Step: Choose r, i with this probability.

Outline of algorithm – continued

3. For this r and i, actor i may change one outgoing r-tie, or leave all outgoing tie variables $X_{ij}^{(r)}$ unchanged.
The probability of changing toward any new situation x (x differs only in one tie variable from current situation!) is proportional to
$$\exp\left(f_i^{(r)}(x)\right) .$$

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$$\frac{\exp\left(f_i^{(r)}(x \text{ changed in } x_{ij}^{(r)})\right)}{\sum_h \exp\left(f_i^{(r)}(x \text{ changed in } x_{ih}^{(r)})\right)} \; .$$

Model specification

The objective function can be conveniently modeled as a weighted sum (cf. generalized linear modeling),

$$f_i^{(r)}(\beta, x) = \sum_{k=1}^{L} \beta_k^{(r)} \, s_{ik}^{(r)}(x) \, ,$$

where $s_{ik}^{(r)}(x)$ are 'effects' and $\beta_k^{(r)}$ their weights,
which jointly drive the dynamics for relation r,
given the current state of this *and all other* relations.

These effects will represent the 'internal' dynamics
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Estimation

Method of moments, maximum likelihood, Bayesian; straightforward (sometimes tedious) elaboration of these methods for the case of dynamics of a single network (Snijders, 2001; Koskinen & Snijders, 2007; Snijders, Koskinen & Schweinberger, 2010.)

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Score tests useful for testing model extensions where estimation becomes unstable.

Example: Research with Vanina Torlo and Alessandro Lomi.

International MBA program in Italy;
75 students; 3 waves.

1. *Friendship*
2. *Advice*:
   To whom do you go for help if you missed a class, etc.

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1. *Friendship*
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   To whom do you go for help if you missed a class, etc.
3. Two mode: *organizational preference*:
   in which organizations are you interested
   as potential employer.
   A total of 100 organizations were mentioned.

Results: Friendship, univariate

Effect	par.	(s.e.)
Out-degree	–2.024	(0.237)
Reciprocity	1.604	(0.098)
Transitive triplets	0.183	(0.017)
3-cycles	–0.101	(0.028)
Indegree popularity ($\surd$)	0.218	(0.060)
Outdegree popularity ($\surd$)	–0.343	(0.066)
Outdegree activity ($\surd$)	–0.061	(0.045)
Same nationality	0.253	(0.083)
Sex alter	–0.020	(0.072)
Sex ego	–0.168	(0.074)
Same sex	0.294	(0.067)
Performance alter	–0.021	(0.025)
Performance ego	–0.076	(0.025)
Performance similarity	0.795	(0.200)

Results: Advice, univariate

Effect	par.	(s.e.)
Out-degree	–2.281	(0.334)
Reciprocity	1.329	(0.130)
Transitive triplets	0.317	(0.038)
3-cycles	–0.060	(0.064)
Indegree popularity ($\sqrt{}$)	0.255	(0.056)
Outdegree popularity ($\sqrt{}$)	–0.370	(0.145)
Outdegree activity ($\sqrt{}$)	–0.077	(0.062)
Same nationality	0.460	(0.125)
Sex alter	–0.044	(0.095)
Sex ego	–0.276	(0.101)
Same sex	0.175	(0.091)
Performance alter	0.124	(0.036)
Performance ego	–0.110	(0.036)
Performance similarity	0.746	(0.262)

Results: Organizational Preference, univariate

Effect	par.	(s.e.)
Out-degree	–2.610	(0.102)
Four-cycles	0.056	(0.009)
Indegree popularity ($\sqrt{}$)	0.293	(0.048)

Results: Friendship Co-evolution (1/2)

Effect	par.	(s.e.)
Out-degree	–2.281	(0.256)
Reciprocity	1.288	(0.116)
Transitive triplets	0.158	(0.019)
3-cycles	–0.062	(0.032)
Indegree popularity ($\sqrt{}$)	0.366	(0.066)
Outdegree popularity ($\sqrt{}$)	–0.359	(0.074)
Outdegree activity ($\sqrt{}$)	0.037	(0.044)
Same nationality	0.191	(0.091)
Sex alter	–0.013	(0.076)
Sex ego	–0.140	(0.073)
Same sex	0.229	(0.076)
Performance alter	–0.019	(0.030)
Performance ego	–0.086	(0.028)
Performance similarity	0.760	(0.194)

Results: Friendship Co-evolution (2/2)

Effect	par.	(s.e.)
Advice $\Rightarrow$ Friendship	1.653	(0.223)
‘Incoming’ advice $\Rightarrow$ Friendship	0.669	(0.187)
Indegree advice ($\surd$) $\Rightarrow$ Friendship pop.	–0.157	(0.048)
Outdegree advice ($\surd$) $\Rightarrow$ Friendship act.	–0.185	(0.076)

Results: Advice Co-evolution (1/2)

Effect	par.	(s.e.)
Out-degree	–2.357	(0.409)
Reciprocity	0.551	(0.173)
Transitive triplets	0.251	(0.043)
3-cycles	–0.082	(0.060)
Indegree popularity ($\surd$)	0.328	(0.057)
Outdegree popularity ($\surd$)	0.016	(0.162)
Outdegree activity ($\surd$)	0.049	(0.074)
Same nationality	0.460	(0.125)
Sex alter	0.052	(0.104)
Sex ego	–0.175	(0.109)
Same sex	0.050	(0.099)
Performance alter	0.147	(0.044)
Performance ego	–0.056	(0.040)
Performance similarity	0.478	(0.276)

Results: Advice Co-evolution (2/2)

Effect	par.	(s.e.)
Friendship $\Rightarrow$ Advice	1.782	(0.269)
'Incoming' friendship $\Rightarrow$ Advice	0.266	(0.203)
Indegree friendship ($\sqrt{}$) $\Rightarrow$ Advice pop.	–0.293	(0.076)
Outdegree friendship ($\sqrt{}$) $\Rightarrow$ Advice act.	–0.318	(0.061)
Org. pref. agreement $\Rightarrow$ Advice	0.232	(0.079)

Results: Organizational Preference Co-evolution

Effect	par.	(s.e.)
Out-degree	–1.984	(0.438)
Four-cycles	$p > 0.20$	
Indegree popularity ($\sqrt{}$)	–0.189	(0.338)
Friendship $\Rightarrow$ Org. pref. agreement	0.294	(0.118)

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The co-evolution between friendship and organizational preference shows that these effects emerge from the influence between friends.

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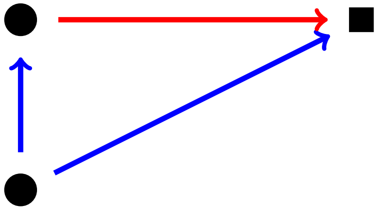

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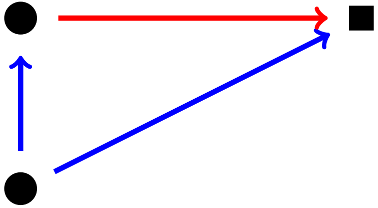

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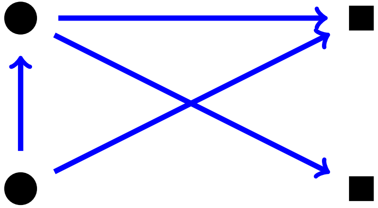

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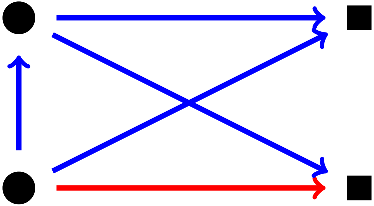

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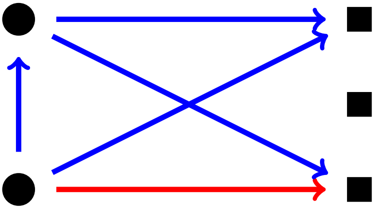
then I will agree with persons about multiple organizations,

The univariate approach yielded a significant 4-cycle effect.
This disappeared in the multivariate approach,
because friendship leads to agreement on org. pref.



If I agree with my friends about organizations ...
and we tend to make multiple choices ...
then I will agree with persons about multiple organizations,
& indegree differences between organizations are reinforced.

Discussion

$\Rightarrow$ Testing cross-network dependencies in dynamics of multiple networks gives interesting new possibilities for hypothesis testing.

Discussion

⇒ Testing cross-network dependencies in dynamics of multiple networks gives interesting new possibilities for hypothesis testing.

⇒ Elaborated along the lines of actor-based modeling.

Discussion

- ⇒ Testing cross-network dependencies in dynamics of multiple networks gives interesting new possibilities for hypothesis testing.
- ⇒ Elaborated along the lines of actor-based modeling.
- ⇒ Compared to modeling dynamics of single networks, this approach attenuates the Markov assumption by extending the state space to a multiple network.

- ⇒ New perspectives possible by combining one-mode and two-mode networks.
- ⇒ The method is being made available in Siena.
  This will work for a small number (e.g., 2–6) of networks, and a limited number of actors (up to a few hundred).
- ⇒ Models for larger networks are under development.