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Beyond homophily: Incorporating actor variables in statistical network models

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Abstract

We consider the specification of effects of numerical actor attributes, having an interval level of measurement, in statistical models for directed social networks. A fundamental mechanism is homophily or assortativity, where actors have a higher likelihood to be tied with others having similar values of the variable under study. But there are other mechanisms that may also play a role in how the attribute values of two actors influence the likelihood of a tie between them. We discuss three additional mechanisms: aspiration, the tendency to send more ties to others having high values; attachment conformity, sending more ties to others whose values are close to the "social norm"; and sociability, where those having higher values will tend to send more ties generally. These mechanisms may operate jointly, and then their effects will be confounded. We present a specification representing these effects simultaneously by a four-parameter quadratic function of the values of sender and receiver. Flexibility can be increased by a five-parameter extension. We argue that for numerical actor attributes having important effects on directed networks, these specifications may provide an improvement. An illustration is given of dependence of advice ties on academic grades, analyzed by the Stochastic Actor-oriented Model.

Keywords: actor covariate; directed networks; homophily; assortativity; aspiration; conformity; sociability; stochastic actor-oriented model; quadratic model; academic performance

1. Introduction and overview

Representing dependence between tie variables is of paramount importance for the specification of statistical network models. This is done by using so-called structural effects. For the use of such models in empirical research, however, representing the effects of nodal variables is also essential. The importance of nodal variables, also known as (monadic) attributes or actor covariates, was already recognized by Fienberg & Wasserman (1981), who showed how to use categorical attributes in log-linear models for network data. The ability to combine structural and covariate effects is an important feature also for more recent network models such as the Exponential Random Graph Model (Wasserman & Pattison, 1996; Lusher et al., 2013), the stochastic actor-oriented model (SAOM) (Snijders, 2001), and Latent Space Models (Hoff et al., 2002). What precisely is represented by covariate effects in the network literature has been varying, but the main focus has been on homophily, the tendency for actors to relate to others who are similar in terms of a limited number of contextually salient dimensions (Lazarsfeld & Merton, 1954; McPherson et al., 2001; Azoulay et al., 2017). This is also called assortativity; we use the term "homophily." Methodological discussions about how to model homophily mostly focus on similarity based on binary or other categorical variables. This has informed many empirical studies. Examples of binary and categorical variables frequently used to specify homophily include gender, occupation, and membership in ethnic, religious, or other social categories (McPherson, 2004). Some examples of continuous and ordinal variables considered for homophily are age, education, and various attitudinal scales.

However, the importance of nodal variables goes well beyond homophily effects. In this paper, we elaborate this specifically for statistical models for binary social networks where the set of network ties constitutes the dependent variable, and for numerical actor covariates that satisfy an interval level of measurement. Such variables might be truly continuous, such as length or monetary values, but also discrete, such as sum totals of psychological multi-item scales. The set of actor variables considered also includes discrete ordinal variables provided that their numerical values are interpreted as having an interval level of measurement. Whether this is acceptable is a matter of choice and depends on the interpretation in the case at hand. Sometimes even the well-known Likert scale with values, e.g., 1–5, may be interpreted as having approximately interval-level scale properties. A requirement then is that it is reasonable to consider the values as being equidistant in how they are interpreted.

As a scope condition for the network, we only consider directed relations where a tie from sender i to receiver j can be interpreted as the result of a positive choice, in some sense, originating from i to the target j.

In the next section, we propose a basic set of four mechanisms according to which such variables might affect dyadic probabilities of tie existence, creation, and/or termination. These are similarity/homophily, attachment conformity, aspiration, and sociability. Their definitions follow below. We focus on directed networks because the asymmetry between senders and receivers of ties permits a clear distinction between these mechanisms. These four patterns can be represented jointly by a quadratic function of the values of senders and receivers, having a total of four statistical parameters. This model is proposed in Section 3, followed by a five-parameter extension that adds flexibility but decreases parsimony. The model is applied in an example in Section 4, in a longitudinal study of an advice relation in an educational setting, employing the Stochastic Actor-oriented Model. A discussion section concludes the paper. The main conclusion is that researchers interested in modeling actor covariates in statistical network models should go beyond the automatism of considering only homophily and stopping there, and rather consider a wider array of covariate effects that merit consideration, such as the quadratic model proposed here.

2. Homophily and other principles of bonding

Homophily, the tendency to have or form ties to others with the same or similar characteristics, is a dominant principle of dyadic bonding between social actors (McPherson et al., 2001), but it is not the only one. Discussing bonding in the context of network modeling, Stokman (2004) (published as Stokman & Vieth, 2004) distinguishes three types of dimensions influencing interpersonal attraction: similarity, aspiration, and complementarity. What is called similarity-attraction may have several aspects, depending on which units are being compared for the assessment of similarity. The tendency toward *homophily* means that sender and receiver are compared, and ties become more likely as their similarity increases along relevant dimensions. But social actors making choices about sending ties may also compare potential recipients of the tie with a reference group, i.e., with what is socially considered appropriate; a positive tie then will be more likely when the recipient is more similar to the norm describing what is appropriate (Sherif, 1936; Homans, 1974; Cohen, 1977; Abrams et al., 1990). We define *attachment conformity* as the tendency that ties are more likely when the recipient's characteristics are closer to one particular value, common to all; this value then is called the "social norm," but we do not go further into its substantive interpretation because this will depend also on the rest of the model. To avoid confusion, note that conformity has two faces: the high value put on others who display normative behavior—which is what is treated here—and the adjustment of behavior toward normative values—part of the same mechanism, but not considered here. Another closely related concept is cumulative advantage or preferential attachment (de Solla Price, 1976; Barabási & Albert, 1999), defined as the tendency to send ties preferably to those who have high degrees already. This concept is outside the scope of our discussion, because it is not directly related to an exogenous actor variable; but it is clear that attachment conformity for actor variables may have consequences that are similar to preferential attachment.

An *aspiration* dimension is an attribute for which high values are generally found attractive.[1] For a negative aspiration dimension, low values are generally found attractive; since this is just a mirror image obtainable by changing the sign of the variable, we discuss only positive aspiration. This means that the attribute is seen as being positively related to the quality or competence of the receiving actor, for purposes that are directly or indirectly associated with the relation under consideration. Aspiration is a concept used more generally in psychological theories of goal setting (Lewin et al., 1944; Knudsen, 2008), and aspiration dimensions are quite basic in interpersonal attraction. Robins (2009) coins the word "capacities" for individual factors, such as skills, expertise, information or knowledge, that may "bear on social actions" (Robins, 2009) and lead to a higher number of ties for actors commanding them. Already Lott & Lott (1965) reviewed studies finding that high-status and warm individuals are more likely to be considered attractive. Social status is often an important summary signal of quality (Sauder et al., 2012) and, accordingly, status variables may be expected to often have an aspiration aspect. Selfhout-Van Zalk et al. (2010) studied how friendship dynamics is influenced by personality characteristics, as defined by the "Big Five" (McCrae & John, 1992). They found strong evidence for attraction toward persons high on agreeableness, which is defined as the extent to which a person is cooperative, kind, and trusting. The aspirational dimension of social selection becomes especially evident in studies of status and reputation where differences in social or material resources controlled by social actors (directly or through their connections with others) produce systematic differences in the attractiveness of potential partners, as can be seen in Kilduff & Krackhardt (1994) and Stuart et al. (1999).

Aspiration may be regarded as a boundary case of attachment conformity, where the social norm corresponds to a very high value of the attribute. While recognizing this relation, we nevertheless mention them distinctly; in the mathematical implementation, this issue will reappear.

Complementarity, or heterophily (Rivera et al., 2010), is a social selection mechanism in which relations are more likely to be observed between actors who have different attributes, and the combination of attributes is especially valuable. Complementarity plays a role especially in exclusive dyadic relations and also at the level of the personal network, where a focal actor may wish to have a diverse network composition with, e.g., *at least one* person who has a desired complementary, hence different, attribute. It often involves the combination of several variables (Rivera et al., 2010). We do not consider complementarity further.

The three dimensions of homophily, attachment conformity, and aspiration are naturally treated as principles of attraction, for actors having the distinct roles of senders and receivers in a directed network, the sender being attracted to potential receivers to a lower or higher degree. Homophily is about the combination of the characteristics of sender and receiver, while attachment conformity and aspiration are about the receiver's characteristic. A dimension at the side of the sender of ties is *sociability*, also referred to as gregariousness, activity, or outgoingness. A sociability dimension is a characteristic for which high values are associated with sending many ties. Thereby it is a mirror image of aspiration. Variables indicating high resources will be expected to be sociability dimensions, because they help overcome the costs of sending ties. For friendship, Selfhout-Van Zalk et al. (2010) found that—not surprisingly—extraversion is associated with sociability.

The main point we wish to make in this paper is that ordinal and numerical actor variables will often have a combination of similarity, attachment conformity, aspiration, and sociability aspects for choices about dyadic relations. The next section shows how this may be represented in statistical network models. How for a given variable the combination of similarity, attachment conformity, aspiration, and sociability works out will depend on the variable, the actor set and its context, and the relation under consideration.

Some examples of their combinations are the following. For the role of health-related lifestyle variables such as smoking and drinking habits in friendship relations, homophily may be the most important mechanism, but attachment conformity and aspiration may also play a role. For example, in some groups drinking may confer high status and therefore be associated with aspiration (Osgood et al., 2013), whereas in other groups drinking moderately may be the norm so that drinking habits will be associated with attachment conformity. Further, individuals who drink more might make more friends—sociability.

For relations that involve cooperation toward some goal, e.g., collaboration or advice, it is possible that individuals are prone to seek contacts with those who have high values on variables signaling good performance, such as expertise. Therefore, performance-signaling attributes may be associated with the mechanism of aspiration; but similarity and normative behavior decrease uncertainty and facilitate cooperation, and therefore the mechanisms of homophily and attachment conformity could be at play here, too. An example is Brouwer et al. (2018).

For a wide variety of social relations, high-status others may be desirable interaction partners, but ties crossing large-status gaps might be uneasy to manage or violate social norms; the former would be in line with aspiration, the latter with homophily but also with attachment conformity. Podolny (1994) argues that actors in markets prefer ties to others with higher status; since this is everybody's preference, the ties that form will be between actors of similar status. This is to be expected particularly in social settings that are hierarchically structured such as formal organizations, or when status is interpreted a signal for underlying qualities that are not directly observable (Sauder et al., 2012).

Of course, it is an abstraction to focus on only one attribute, and in real life there will be a multiplicity of attributes at work, confounded, interacting, and/or endogenously influencing each other. Homophily interactions for multiple actor variables are discussed by Block & Grund (2014); some other recent examples of studies carefully considering the interplay of multiple actor variables are Schaefer (2018) and Gremmen et al. (2018). In this paper, we focus on modeling a single attribute. Our ideas can be used also in studies with multiple attributes.

The considerations presented above apply to ordinal variables generally, and the requirement of numerical variables comes into play only when formulas are specified for the mathematical specification. The following parts of the paper are specific for numerical variables with an interval level of measurement (permitting some give and take with respect to this requirement, as mentioned in Section 1). Dichotomous variables fall outside the scope because for them, attachment conformity is perfectly confounded with aspiration; this can be positive or negative aspiration, depending on whether the social norm corresponds to the higher or the lower category. For dichotomous actor variables, the combination of sender and receiver effects entails three degrees of freedom, and these are completely represented by using a sender, receiver, and their interaction effect (the latter could equivalently be replaced by a "same," "absolute difference," or "similarity" effect).

3. Representing effects of actor attributes

In this section, we discuss how homophily, attachment conformity, aspiration, and sociability can be expressed in statistical models for networks. The numerical actor attribute that is the variable under consideration will be denoted by V. It is assumed to be one-dimensional. The value of V for actor i is denoted v_i. The network will be represented by tie variables x_{ij} such that $x_{ij} = 1$ indicates the presence of a tie $i \to j$ from sender i to receiver j, and $x_{ij} = 0$ its absence.

In most statistical network models, the probabilities of ties—or tie changes—depend on a linear predictor such as used in generalized linear models. The linear predictor is a function of the entire network x. The part of the linear predictor depending on V will here be called the *social selection function*, with the caveat that it does not represent preference or attraction *per se*: as is generally the case in statistical models with correlated variables, it models just probabilities, and the representation of how V influences tie choices is not determined totally by this social selection function but depends also on other correlated effects. For linear statistical models we have the machinery of partial and semi-partial correlations and coefficients to analyze this, but in network models the dependence structure is more complicated and partialing approaches have not yet been developed. Lacking a more precise set of tools, we shall just keep in mind that the social selection function is somewhat similar in interpretation, but not identical, to a preference function. To avoid cumbersome language, we nevertheless shall sometimes use the word "attraction" meaning something like "sending a tie to this actor with a higher probability if all other circumstances are equal."

A basic issue for the representation of actor attributes in statistical network models is that this representation links the monadic level of individual actors to the dyadic level of network ties, illustrating the fundamental multilevel nature of network analysis (Snijders, 2016). For the tie directed from i to j, the two actors involved have values v_i and v_j, respectively, for variable V. We only consider social selection functions of the form

$$\sum_{i,j} x_{ij}\, a(v_j \mid v_i) \tag{1}$$

the summation extending over all network members. This means that $a(v_j \mid v_i)$ represents the influence of V on the probability of the existence, creation, or maintenance of the tie $i \to j$. With a slight misuse of terminology, we shall also refer to $a(v_j \mid v_i)$ as the social selection function.

Given our interpretation of the tie $i \to j$ as resulting from a choice by i of actor j, we consider the social selection function $a(v_j \mid v_i)$ for given v_i as a function of v_j, and explore which families of functions are useful as social selection functions. For this purpose, we require that—depending on the values of the parameters in this family—the family of functions can represent combinations of tendencies toward homophily, toward attachment conformity, toward aspiration, and toward sociability. The *optimum* of the function is defined as the highest value of $a(v_j \mid v_i)$ as a function of v_j for a given v_i.

Dyadic attraction as influenced by numerical actor variables, seen in the perspective of a sender choosing a receiver, is often formulated in terms of *ideal points* (Coombs, 1964; Jones, 1983). Given a preference function of an actor, an ideal point is an argument value where an optimum is assumed; if the preference function is unimodal, this will be a unique point. Although the social selection function $a(v_j \mid v_i)$ is not strictly interpreted as a preference function, the parallel with preference functions still can teach us some things. For preference functions depending on actor attributes, for an attribute leading exclusively to homophily, the ideal point is the actor's own value; for an attribute representing aspiration in its strongest sense, where the attraction to others becomes higher when the attribute gets larger, it is the highest possible value of the covariate (or infinity); for an attribute representing pure attachment conformity, it is the value corresponding to the social norm, common to all actors and hence independent of v_i. In a suitable family of social selection functions, depending on its parameters any of these various points should be possible as the location of its optimum. Furthermore, to represent sociability, depending on the parameters the function should be able to be generally increasing in v_i on the whole range of V.

3.1 Usual representations

Homophily with respect to a numerical actor variable is usually represented in statistical network models by making the probability of existence or change of a tie depend on the absolute difference between v_i and v_j,

$$a_1(v_j \mid v_i) = \beta_1 \mid v_i - v_j \mid \tag{2}$$

For $\beta_1 < 0$, this is a function with its optimum in $v_j = v_i$. This representation tends to be used almost without reflection; see Snijders (2001) and Snijders et al. (2010) for actor-oriented models, Goodreau (2007) and Lusher et al. (2013) for exponential random graph models, and, among many other examples, van Duijn et al. (2004) and Louch (2000) for various other models. This is a very parsimonious representation, requiring only one parameter, but inflexible because the optimum can only be assumed in $v_j = v_i$, and therefore this can represent only pure homophily, not aspiration or attachment conformity. Sometimes the main effects of the sender's and receiver's values are added,

$$a_1(v_j \mid v_i) = \beta_1 \, v_i + \beta_2 \, v_j + \beta_3 \, \left| v_i - v_j \right| \tag{3}$$

and then β_1 is meant to represent sociability, β_2 to represent aspiration, and $-\beta_3$ homophily. The parameters can only be interpreted together, however. The optimum is assumed in $v_j = v_i$ if and only if $\beta_3 \le -|\beta_2|$, and there is generally aspiration to high values of v_j if and only if $\beta_2 > |\beta_3|$. This means that homophily and aspiration are not readily combined in this model, and attachment conformity cannot be represented.

As an alternative, sometimes the ego-by-alter product interaction is proposed to represent homophily (e.g., Snijders et al., 2010). The main effects of sender and receiver then also have to be included, leading to

$$a_2(v_j \mid v_i) = \beta_1 \, v_i + \beta_2 \, v_j + \beta_3 \, v_i \, v_j \tag{4}$$

This expresses, for $\beta_3 > 0$, that senders with higher v_i have a higher tendency to connect to receivers with high v_j. Considering (4) as a function of v_j, for given v_i, shows that this function can be linearly decreasing or increasing, switching this behavior at $v_i = -\beta_2/\beta_3$. Thus, it represents not homophily but differential aspiration; assuming that $\beta_3 > 0$, attraction is toward low values of V for egos with $v_i < -\beta_2/\beta_3$, and toward high values for i with $v_i > -\beta_2/\beta_3$. Thus, the two functions $a_1(v_j \mid v_i)$ and $a_2(v_j \mid v_i)$ have fundamentally different properties.

This shows that the most commonly used models for representing effects of numerical actor variables on tie creation and change are, respectively, a model representing only pure homophily, a model combining homophily, aspiration, and sociability in a rather inflexible way, and another model representing pure differential aspiration. In practice, however, actor variables may be associated with homophily, attachment conformity, aspiration, as well as sociability, and any combination of these mechanisms; and researchers hardly ever have enough strong theoretical knowledge to be able to *a priori* exclude some of these possibilities and make a confident bet on only one or two of them.

3.2 Quadratic representations

For a combination of the four mechanisms potentially associated with the attribute V, we need a parametric family of functions that can represent unimodal as well as monotone functions; with the property that—for the unimodal type—the location of the optimum can be close to ego's value to represent homophily, can be drawn toward a common (normative) value to represent attachment conformity, and can be higher or lower to represent aspiration.

Absolute differences such as used in Equations (2) and (3) are inconvenient for this purpose, because extending these to a class of functions with a variable mode would lead to functions depending nonlinearly on the parameters, which is technically complicated for statistical inference. For functions that are quadratic in v_j, adding a constant to the argument v_j does not take the function outside the class of quadratic functions; in other words, horizontal translations can be represented by linear parameters. Therefore, quadratic functions are more useful for our purpose. It should be noted that quadratic functions are quite common as representations of choice functions with ideal points that themselves are explained by other attributes (Jones, 1983).

We survey families of quadratic functions with the aim to be able to represent homophily, attachment conformity, aspiration, as well as sociability. A model representing pure homophily, the direct analogue to Equation (2), is

$$a_3(v_j \mid v_i) = \beta_1 (v_j - v_i)^2 \tag{5}$$

with $\beta_1 < 0$. This is too restricted because it forces the optimum value of v_j to be equal to ego's value v_i, just like Equation (2). A more general model proposes separate parameters for each of the four mechanisms under consideration. In the following formula, we give two equivalent expressions: the first shows the linear parametrization and the second indicates explicitly the location of the "social norm":

$$a_4(v_j \mid v_i) = \theta_1 (v_j - v_i)^2 + \theta_2 v_j^2 + \theta_3 v_j + \theta_4 v_i \tag{6a}$$

$$\sim \theta_1 (v_j - v_i)^2 + \theta_2 \left(v_j + \frac{\theta_3}{2\theta_2}\right)^2 + \theta_4 v_i \tag{6b}$$

where the $\sim$ symbol means that the functions differ by only a constant term, which will be absorbed by the intercept.[2]

The interpretation can best be based on the three terms in expression (6b), and is most straightforward in the case that θ_1 as well as θ_2 have negative values. The first term reflects an attraction with a weight $-\theta_1$ toward i's own value, expressing homophily. The second term reflects an attraction with a weight $-\theta_2$ toward the value

$$V^{\mathrm{norm}} = -\frac{\theta_3}{2\,\theta_2} \tag{7}$$

expressing attachment conformity. If this value is within the range of V, it may be regarded as a normative value, and this is the terminology we shall use. The third term allows to express a smaller or higher extent of sociability. In the next section, we describe properties of this social selection function, and the way in which it can express the four mechanisms.

The social selection function (6) is a quadratic function of the two variables v_i and v_j, and a linear function of $(v_j - v_i)^2$, v_j^2, v_j, and v_i. The sender's value v_i and receiver's value v_j are treated differently. As an empirical safety valve, it may be advisable to check whether also an additional free parameter for v_i^2 should be included, not directly related to the homophily term; this leads to an unrestricted quadratic dependence on v_j and v_i,

$$a_5(v_j \mid v_i) = \theta_1 (v_j - v_i)^2 + \theta_2 v_j^2 + \theta_3 v_j + \theta_4 v_i + \theta_5 v_i^2 \tag{8}$$

The interpretations above remain, except that now the tendency to sociability is expressed by the term $\theta_4 v_i + \theta_5 v_i^2$.

3.3 Properties of the quadratic representation

We study some properties of the quadratic functions (6) and (8), and elaborate how the four mechanisms are associated with the four, or five, parameters.

Location of the optimum. If $\theta_1 + \theta_2 < 0$, functions (6) and (8) are unimodal, the optimum being located at

$$v_i^{\mathrm{opt}}(\theta) = \frac{\theta_1\, v_i \,-\, \theta_3/2}{\theta_1 \,+\, \theta_2} = \frac{\theta_1\, v_i \,+\, \theta_2\, V^{\mathrm{norm}}}{\theta_1 \,+\, \theta_2} \tag{9}$$

which is a weighted mean of i's own value and the socially normative value. This could be called the point of attraction, or ideal point, for an actor with value v_i; it is further interpreted below. If this value is outside the range of V, the location of the optimum must be truncated and will be assumed at the minimum or maximum value of the range.

Homophily. Homophily is expressed directly by the first term in (6b) and by parameter $-\theta_1$. The weight for homophily is $\theta_1/(\theta_1 \,+\, \theta_2)$, as shown in (9).

Attachment conformity. Attachment conformity is expressed directly by the second term in (6b) and its strength by parameter $-\theta_2$. This term includes two parameters, θ_2 and θ_3. The weight for attachment conformity in (9) is $\theta_2/(\theta_1 \,+\, \theta_2)$. The social norm is located at the value (7), if this is within the range of V; if it is not in this range the attachment conformity has the nature of aspiration, as discussed below.

The value of (7) can be estimated by plugging in the estimate $\hat{\theta}$, depending on the further statistical model used. Standard errors for $V^{\mathrm{norm}}(\hat{\theta})$ can then be calculated using the delta method (Wasserman, 2004); see Appendix B.

Aspiration. The value of the social norm (7) can be regarded as a parameter expressing the extent of aspiration.[3] When could one say that variable V has an aspiration aspect? This may be defined in more than one way, because of the confounding with homophily. We propose the following three definitions.

1. The strongest definition is that, although there may be an element of homophily, aspiration trumps homophily for everybody, in the sense that the selection function is increasing on the entire range of V, for every value of v_i. This condition depends on the range of V. Denote the minimum value of V by V^- and its maximum by V^+. For the selection function to be an increasing function of v_j for all v_i in the range of V, given that $\theta_1 < 0,\ \theta_2 < 0$, the location of the optimum $v_i^{\mathrm{opt}}(\theta)$ in (9) should be equal to or larger than V^+ even for senders i with $v_i = V^-$. This can be expressed as

$$V^{\mathrm{norm}} \;\geq\; V^+ \,+\, \frac{\theta_1}{\theta_2}\big(V^+ \,-\, V^-\big) \tag{10}$$

   This can be tested by a right one-sided test of the linear combination

$$\theta_3 \,+\, 2\theta_2\, V^+ \,+\, 2\theta_1\big(\, V^+ - V^-\big)$$

   It should be noted that this situation is impossible if V is unbounded with $V^+ = \infty$; any quadratic function with $\theta_1 + \theta_2 < 0$ tends to minus infinity for $v_j \to \infty$. Therefore, the quadratic family proposed here may be less suitable for attributes with unbounded range; one possibility to handle this is to first transform such attributes to a variable with finite range.
2. A weaker definition of aspiration is that the contribution to the social selection function of the terms for the social norm, $\theta_2\, v_j^2 + \theta_3\, v_j$, increases in v_j. This is equivalent to the condition that the location (9) of the optimum is greater than or equal to the own value v_i for all actors. For negative θ_2, this second definition is equivalent to the location of the social

norm being at least as large as the maximum value of V, i.e., $V^{\text{norm}} \geq V^{+}$. For positive θ_2, it is equivalent to

$$-\frac{\theta_3}{2\,\theta_2} \leq V^{-}$$

This can be tested by a right one-sided test of the linear combination

$$\theta_3 \,+\, 2\theta_2\, V^{+}.$$

3. The weakest definition, in the case that $\theta_2 < 0$, is that the location of the norm (7) is larger than the mean of V. This can be tested by a right one-sided test of the linear combination

$$\theta_3 \,+\, 2\theta_2\, \bar{V}$$

where $\bar{V}$ is the mean of V. Note that, if V is a centered variable, this is equivalent to testing θ_3.

We see that, if $\theta_1 < 0$ and $\theta_2 < 0$, the three definitions of aspiration, from strong to weak, are expressed by progressively weaker lower bounds for θ_3, which depend on the distribution of V.

Sociability. Variable V is associated with sociability if higher values of v_i tend to imply that actor i, as a sender, has the inclination to make more tie choices. This means that the social selection function tends to be higher for higher values of v_i. We propose two definitions.

1. A strong definition is that the social selection function increases as a function of v_i for all receivers' values v_j. The derivative of Equation (8) is

$$\frac{\partial a(v_j \mid v_i)}{\partial v_i} = 2\,(\theta_1 \,+\, \theta_5)\, v_i \,-\, 2\theta_1\, v_j \,+\, \theta_4 \tag{11}$$

If this value is non-negative for all values v_i, v_j, the strong definition is satisfied. Depending on the signs of the coefficients, only one of the combinations of V^{-}, V^{+} needs to be checked. In the case that $\theta_1 \leq 0,\ \theta_1 + \theta_5 \leq 0$, the condition is

$$2\,(\theta_1 \,+\, \theta_5)\, V^{+} \,-\, 2\theta_1\, V^{-} \,+\, \theta_4 \geq 0$$

2. A weak definition for V to have a sociability dimension is that the optimum value of the social selection function for given v_i,

$$a^{\text{opt}}(v_i) \,=\, \max_{v_j} a(v_j \mid v_i) \tag{12}$$

increases with v_i. If $\theta_1 + \theta_2 < 0$, the optimum is assumed for $v_j = v_i^{\text{opt}}(\theta)$ given in (9). Some calculations show that the value of the optimum is

$$a^{\text{opt}}(v_i) \,=\, \frac{2\,\theta_1\,\theta_2}{\theta_1 \,+\, \theta_2}\big(v_i - V^{\text{norm}}\big)^2 \,+\, \theta_4\, v_i \,+\, \theta_5\, v_i^2 \tag{13}$$

This is an increasing function of v_i if

$$\frac{4\,\theta_1\,\theta_2}{(\theta_1 \,+\, \theta_2)^2}\big(v_i - V^{\text{norm}}\big) \,+\, \theta_4 \,+\, 2\theta_5\, v_i \geq 0 \tag{14}$$

Since the latter function is linear in v_i, it needs to be checked only for the extremes $v_i = V^{-}, V^{+}$.

It is possible that Equation (9) is outside of the range of V for some values v_i. Then for such values, the optimum is assumed at the minimum or maximum value of the range, and the value of the optimum has to be calculated accordingly.

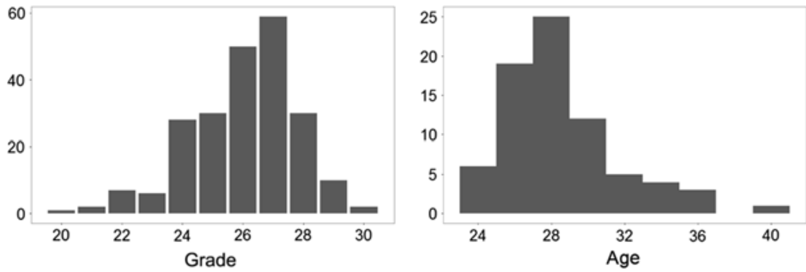

Figure 1. *Left:* bar plot of grades, pooling the three waves. *Right:* histogram of ages. Frequencies on vertical axis.

In many cases, a visual check of the plotted function will easily show whether the weak or strong versions of sociability are satisfied. The formulae show that the strong version depends on parameters θ_1, θ_4, and θ_5. The weak version depends on all five parameters. But always, higher values of θ_4 are conducive to the association of V with sociability.

These interpretations seem reasonable but may not always be compelling. They are based, theoretically, on the assumption that attraction for sending ties, as far as dependent on the values of V, can be expressed as a combination of homophily and attraction to a common normative value (which may be a hypothetical value outside of the range of the possible); and, empirically, on the fit of the quadratic shape of the social selection function in whatever statistical network model is being utilized. Furthermore, they ignore other elements of the model specification, which can depend on variables or network positions that may be associated with V in some way.

Concluding, this reasoning leads to a four-parameter quadratic model in v_i and v_j, which may be extended to a five-parameter model. Quadratic social selection functions have been used occasionally in statistical network modeling. Examples are Robins et al. (2001, Equation (16)), where a quadratic selection function is mentioned as a possibility for nondirected networks, without elaboration or example; and Mercken et al. (2012), using a squared term of alter's smoking habits in a coevolution study of friendship and smoking. Our proposal is to use them more systematically.

4. Example

We demonstrate the empirical value of this approach by analyzing a longitudinal network of advice ties among students enrolled in a master degree program in business administration (MBA), with academic performance and age as the actor attributes under consideration. The data were collected by Vanina Torlò. The network was composed of full-time students in an elite Italian school for professional management education. This network was analyzed earlier in Snijders et al. (2013). Educational settings provide an ideal context for the study of homophily-related network processes because—contrasting with behavior in the context of formal organizations—students' behavior is hardly affected by preassigned roles or by differences in formal hierarchical positions. The cohort consisted of 75 students, providing full response for all variables. The program had a duration of 1 year, and data collection for the three panel waves took place close to examination periods in March, July, and November.

For the advice relation, respondents were asked to indicate the names of other students whom they regularly consulted for help and support on program-related tasks; examples mentioned were asking for class notes, help in solving homework problems, etc. Any number of classmates could be mentioned. Academic performance was measured as the average grade, rounded to integers, on the 10–12 exams in the examination period, calculated from information supplied by the MBA office. The range of academic grades was 20–30, with a mean of 26. Age ranges from 24 to 40 years, with an average of 29 years. The distributions of grades and age are shown in Figure 1. Further information on this data set can be found in Lomi et al. (2011).

Academic grades are important in the context of professional management education, and may be expected to be important for structuring interpersonal advice relations (Lomi et al., 2011). In the extremely competitive context of an MBA class, academic performance, as represented by grades, is treated as a signal of students' commitment, sense of duty, and competence—qualities valued both by potential employers as well as potential business partners. A strong emphasis is therefore placed on performance when it comes to forming a student's network partners for getting advice in academic matters. This suggests that grades could have an aspiration dimension for the advice relation. However, asking advice is much easier in dyads with high mutual social acceptance. The tension between the two objectives of individual achievement and social acceptance has indeed been a central factor in the economics and sociology of schooling at least since the "Coleman Report" (Coleman, 1966) and perhaps even earlier (Coleman, 1961). The best performers may become quite selective and reciprocate ties only to those with a similarly high performance; in anticipation, lower performers may be reluctant to ask their advice. In addition, social acceptance may be higher between students of similar performance. This would lead to grade-related homophily in advice. If social acceptance would generally be higher for students of normative performance (whatever the normative level is, provided it is less than the maximum possible value), there would be an attachment conformity dimension. For the sociability dimension, one may argue in two opposite ways: those with high grades are less in need of advice, so they will ask less for advice, i.e., grades represent a negative sociability mechanism; or those with high grades find academic performance more important, and therefore are more active generally also in advice asking, reflected by positive sociability. This shows that *a priori* all four mechanisms of aspiration, homophily, attachment conformity, and sociability might be associated with grades in their effect on advice asking.

Age is expected to be of lower importance than grades for selection of advisors. Homophily still might be relevant, but rather as a consequence of general social interaction being easier between students of similar age than as a mechanism related to advice specifically. Formulating prior expectations would be quite speculative; therefore, we refrain from doing so. Age is included here to improve the fit of the model and to have a second illustration of a numerical actor variable, with possibly a different, less important role for structuring the advice network.

We estimate the Stochastic Actor-oriented Model (SAOM) for this data set. Explanations and further background to this model are given in Snijders et al. (2010) and Snijders (2017). Single parameters are tested by t-tests (dividing parameter by standard error and testing in a standard normal distribution); multidimensional tests are tested by Wald-type tests with a χ^2 null distribution (Ripley et al., 2018). This is supported not by mathematical proofs but by numerous simulation studies. Significance will be gauged at the conventional level of $\alpha = 0.05$. For the analysis, we used the R package RSiena (Ripley et al., 2018), version 1.2-8. The implementation of the five-parameter model in RSiena is briefly treated in Appendix A.

4.1 Results

We present results for a model that includes the five effects (8) of grade and age. Both variables are centered. The structural part of the model is defined in a way that is now more or less standard for SAOMs (Snijders et al., 2010; Ripley et al., 2018) with reciprocity, three degree-related effects to reflect the variances and correlations of degrees, and transitivity implemented by the Geometrically Weighted Edgewise Shared Partners (gwesp) statistic (Snijders et al., 2006; Hunter, 2007). This gives here a better fit than the more traditional specification by a count of transitive triplets. Also an interaction between reciprocity and transitivity is included (cf. Block, 2015); even though not significant, this effect improved the goodness of fit of the model. Additional homophily for categorical actor variables is included in the model for gender and the binary variable nationality (Italian vs. non-Italian).

Table 1. Parameter estimates and standard errors for the advice network between MBA students. Grades and age are centered

Effect	Parameter	(standard error)
Rate period 1	7.939	(0.691)
Rate period 2	5.883	(0.471)
Outdegree	−2.181***	(0.208)
Reciprocity	1.606***	(0.197)
Transitivity gwesp	1.307***	(0.121)
Reciprocity × transitivity gwesp	−0.314	(0.250)
Indegree—popularity	0.0253**	(0.0089)
Outdegree—popularity	−0.101**	(0.033)
Outdegree—activity	−0.0072	(0.0092)
Gender alter (M)	0.027	(0.098)
Gender ego (M)	−0.239*	(0.100)
Same gender	0.130	(0.092)
Same nationality	0.405***	(0.122)
$\hat{\theta}_1^g$ (grades ego minus alter) squared	−0.0288***	(0.0073)
$\hat{\theta}_2^g$ grades squared alter	−0.003	(0.012)
$\hat{\theta}_3^g$ grades alter	0.044	(0.032)
$\hat{\theta}_4^g$ grades ego	−0.095**	(0.031)
$\hat{\theta}_5^g$ grades squared ego	0.026**	(0.010)
$\hat{\theta}_1^a$ (age ego minus alter) squared	−0.0014	(0.0023)
$\hat{\theta}_2^a$ age squared alter	−0.0070	(0.0045)
$\hat{\theta}_3^a$ age alter	0.039*	(0.019)
$\hat{\theta}_4^a$ age ego	0.038*	(0.018)
$\hat{\theta}_5^a$ age squared ego	−0.0071†	(0.0041)

Convergence t ratios all < 0.04; overall maximum convergence ratio 0.09.
Number of decimals presented depends on standard errors.
$^{\dagger}p < 0.10$, $^{*}p < 0.05$, $^{**}p < 0.01$, and $^{***}p < 0.001$ (two-sided).

Parameter estimates are presented in Table 1. The table gives asterisks according to two-sided tests, but for parameters θ_1 and θ_2 we can have in the back of our mind that they are expected to be negative. Goodness of fit for distributions of indegrees, outdegrees, and geodesic distances, as well as for the triad count, was tested by the `sienaGOF` function (Ripley et al., 2018). This operates by simulating networks according to the model with estimated parameters, and comparing the observed values of selected statistics with their distributions in the simulated set of networks. The fit is then assessed by comparing the Mahalanobis distance of the observations to the mean of the simulated values and computing the associated p-value. For all four sets of statistics mentioned, the p-value was between 0.10 and 0.90, which means the fit was good.

Before discussing the effects of grade and age, we give a very brief discussion of the other effects (all under the usual "if everything else is equal" clause). There are the expected strong reciprocity and transitivity effects. The positive indegree-popularity and negative outdegree-popularity effects show that advice is asked with higher probability from those who already give much advice, and those who ask little for it; this corresponds to the nature of advice giving. Males tend to ask for advice less than females, and advice is asked more from students having

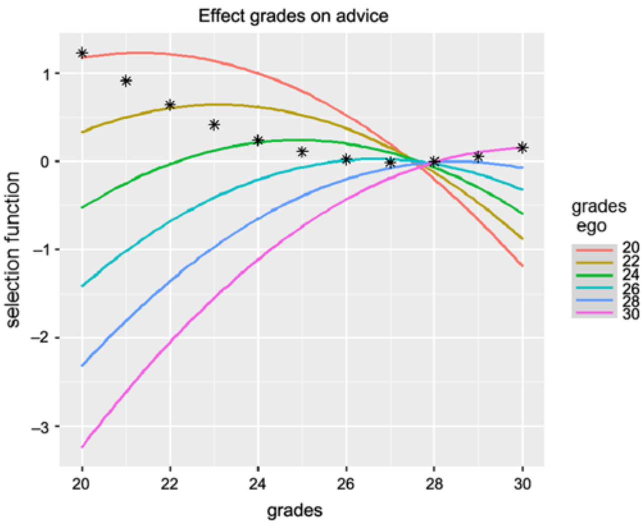

Figure 2. Social selection functions for the effect of grades on advice for the model in Table 1. The continuous curves are the social selection functions, separately for six values of ego's grade from 20 to 30, as a function of alter's grade (horizontal axis). The asterisks indicate the optimum of the social selection function as a function of ego's grade on the horizontal axis.

the same nationality. Thus, there is evidence for homophily with respect to nationality, but the table shows this is not significantly the case for gender.

The effect of grades on advice is important. The joint test of the five parameters yields $\chi^2_5 = 23.3\ p < 0.0005$. The effect of age is also significant, but less strongly so, $\chi^2_5 = 11.9,\ p < 0.05$. Testing whether the quadratic effects are an improvement on model (4) without any quadratic terms, we find that the two quadratic effects of grades are jointly significant ($\chi^2_2 = 6.6,\ p < 0.05$), and those for age likewise ($\chi^2_2 = 10.1,\ p < 0.01$). For both variables, the squared ego term is significant at $p < 0.10$, so model (8) seems indeed to be slightly better than model (6). Summarizing, there is strong evidence for influence of grades on the advice network, as well as evidence for influence of age. This confirms the applicability of the five-parameter model to this data set, for grades and also for age.

Over and above issues of model fit, the five-parameter model also affords interpretation of the effects of grades and age on performance. The social selection function for grades can be interpreted in the following way. The centered grades variable, i.e., grades—26.1, is denoted by V. This ranges from $V^- = -6$ to $V^+ = 4$.

1. There is a clear and strongly significant aspect of homophily, with $\hat{\theta}_1 = -0.0288 < 0$.
2. The coefficient of grades squared, $\hat{\theta}_2 = -0.003$, is negative, so we can elaborate the potential interpretation of an attraction to a socially normative value. However, the parameter is not significantly different from 0, so the interpretation is not very strong.

   The estimated value of the social norm (7) is $\hat{V}^{\mathrm{norm}} = -\hat{\theta}_3/(2\,\hat{\theta}_2) = 6.9$, higher than the maximum value of V. Therefore, the second definition of aspiration is satisfied with respect to the parameter estimates, although there is no statistical significance to support this. The weight for attachment conformity is only $\hat{\theta}_2/(\hat{\theta}_1 + \hat{\theta}_2) = 0.1$, while it is 0.9 for homophily. In other words, homophily dominates attachment conformity.

   Because the coefficient $\hat{\theta}_2$ is far from significant, it is not meaningful to calculate a standard error for $\hat{V}^{\mathrm{norm}}$.
3. The social selection function is plotted in Figure 2. For egos with low grades, it is almost equally strongly decreasing as it is increasing for egos with high grades; this is in line with the low weight for attachment conformity.

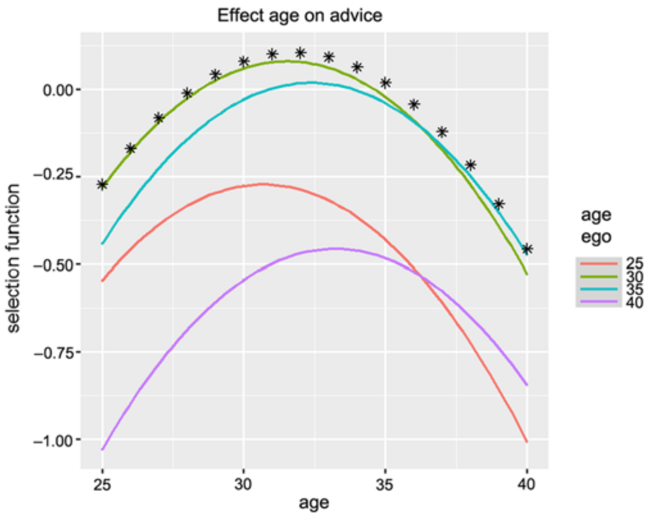

Figure 3. Social selection functions for the effect of age on advice for the model in Table 1. The continuous curves are the social selection functions, separately for four values of ego's age from 25 to 40 years, as a function of alter's age (horizontal axis). The asterisks indicate the maximum of the social selection function as a function of ego's age on the horizontal axis.

4. Sociability, represented by the optimum (12) of the social selection function, is plotted by the asterisks in Figure 2. It is decreasing for the lower half of the range of grades, and approximately constant for the upper half. Although not decreasing uniformly, this plot nevertheless suggests a weakly negative sociability aspect for grades—but weaker even than the weak definition.

For age, similarly, the social selection function can be studied. Now V is age in years, of which the mean of 29 years is subtracted. It ranges from −5 to +11.

1. Both $\hat{\theta}_1$ and $\hat{\theta}_2$ are negative; the former is clearly not significant ($\hat{\theta}_1 = -0.0014$, s.e. $=$ 0.0023), the latter has $p < 0.10$ ($\hat{\theta}_2 = -0.0070$, s.e. $= 0.0045$), and is significant in a one-sided test. This shows that for age, the aspect of homophily is not significant and there is weak support for an aspect of attachment conformity.
2. The estimated value of the social norm (7) is $\hat{V}^{\text{norm}} = -\hat{\theta}_3/(2\,\hat{\theta}_2) = 2.8$, corresponding to 32 years, and higher than the mean age. Therefore, the last definition of aspiration is satisfied. Thus, there is a weak aspiration aspect; this is significant, as $\hat{\theta}_3$ is significantly positive; however, only its weakest definition is satisfied, and the social norm is not much higher than the mean age.
3. The social selection function is plotted in Figure 3. We see that the location of the optimum hardly changes with ego's age. It is noteworthy that the range (i.e., maximum minus minimum) of the selection function is 1.3, much smaller than the range of about 5 of the social selection function for grades, which underscores that age is much less important than grades for the advice relation.
4. The main difference between the social selection functions for different values of ego's age is that it is highest for egos of medium age, and lower for egos who are on the young or on the old side. This is exhibited by the asterisks in Figure 3, giving the value of the optimum depending on ego's age.

Summarizing, for age there is no homophily and a weak attachment conformity dimension, with a slight aspiration aspect, and sociability is highest for medium values of age.

Comparison with other social selection functions for grades. Models with the social selection functions defined by the main effects for ego and for alter with the absolute differences (3) and

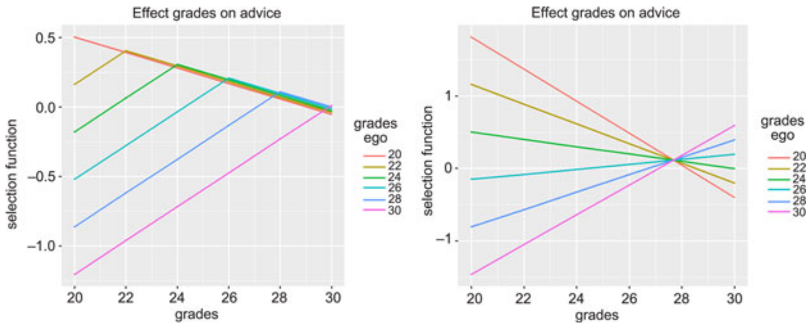

Figure 4. Social selection function for advice depending on ego's and alter' grades. *Left:* similarity specification with main effects (Equation (3)). *Right:* linear interaction specification (Equation (4)).

with ego-by-alter interaction (4) were also estimated. Model (4) fitted less well than model (6). Plots are presented in Figure 4. The purpose of the figures is to demonstrate the consequences of the model assumptions. The differences with Figure 2 show that the representation of this data set by the quadratic model is quite different from the representation using the absolute difference or the product interaction. For model (3), the social selection functions for $v_j > v_i$ are almost on the same line, which is the case because parameters β_1 and β_3 cancel each other almost precisely for $v_j > v_i$. When looking carefully it turns out that the lines in the left-hand side figure are not so strongly different from the curves in Figure 2, and indeed the fit for model (3) is not much worse than the fit for model (6).

5. Summary and discussion

This paper is about how to specify effects of numerical actor attributes, satisfying an interval level of measurement, in statistical models for directed social networks, where the set of network ties constitutes the dependent variable. Effects of actor attributes on networks are not as straightforward as main effects in a generalized linear model, because the dependent variable is defined at the dyadic level: ordered pairs of actors, whereas the attribute is defined at the monadic level: actors. Some transformation from the actor level to the dyadic level is necessary.

This paper considers directed networks where a tie from sender i to receiver j can be interpreted as the result of a positive choice, in some sense, originating from i to the target j. This allows us to interpret the effects of the attribute as a way of structuring attraction between actors. Homophily is a major mechanism of attraction, as discussed by Lazarsfeld & Merton (1954) and many others, cf. McPherson et al. (2001). While homophily is often a mechanism of primary importance, it may be not always the strongest and it can easily be confounded with other rival mechanisms. Our model combines a diversity of mechanisms: homophily, i.e., attraction to similar others (also called assortativity); aspiration, attraction to high values; attachment conformity, attraction to a value common for all in the network, which might be called a normative value; and sociability, the inclination to make many tie choices. Each of these mechanisms could be associated to a larger or smaller extent with the actor attribute in question. Choices by social actors are likely to be steered by multiple considerations; hence, these mechanisms may well be confounded.

The mathematical specification of our model is a quadratic function (6), extendable to (8), of the attribute values of the sender and the receiver of the tie. This function can be used in a linear predictor in any statistical network model; our example was for a SAOM, but our reasoning applies likewise to other statistical models, e.g., the Exponential Random Graph Model (Wasserman & Pattison, 1996; Lusher et al., 2013). For the SAOM, Appendix A mentions the effects that can be used for implementing (6) and (8).

For the interpretation of the model, considering the figure is the best option. The four or five parameters separately are hard to interpret, because their effects are combined. Nevertheless, something can be said about the association between the four mechanisms and the parameters in the statistical model. For homophily, this is straightforward, and it is represented by a single parameter (i.e., by $-\theta_1$). Attachment conformity and aspiration are inseparable in the model because both are associated with the location of the social norm. Aspiration means conformity to the notion that ties should be sent especially to actors with a high value of the attribute. Taken together, these mechanisms are represented by two parameters (i.e., $-\theta_2$ and θ_3). Sociability has the least direct interpretation in terms of model parameters, and to interpret it the figure will be required.

The quadratic family proposed in this paper is only a relatively simple version of the realm of possibilities. Other monadic-to-dyadic transformations could be used to operationalize the combination of the four confounded mechanisms. This choice should be based on considerations of theory and empirical fit. One possibility is to use model (6) but replace the squared difference between ego and alter by the absolute difference. This model can be obtained also as an extension of Equation (3). Whether the kink in the function at $v_j = v_i$ is an advantage or disadvantage will depend on the research in question.

Other possible transformation are cubic and higher-order polynomials, which will yield more flexibility but also increase the number of parameters. Splines or fractional polynomials could also be considered (e.g., Sauerbrei et al., 2007). For example, a cubic function could represent that the selection curves may be wider for high than for low values of V. For actor attributes with an unbounded range, the quadratic transformation and other polynomials may have a less good fit especially at for extreme values, because they tend to positive or negative infinity unless the function is exactly constant; this does not necessarily hold for fractional polynomials.

Other extensions of these models are possible by proposing interactions of these variable-related mechanisms with structural effects such as reciprocity, endogenous popularity, and transitivity. For example, reciprocity could attenuate a tendency toward homophily as argued by Block (2018). There may be arguments, theoretical and/or empirical, for other interactions between the four mechanisms treated in this paper with structural network effects, and this is an interesting topic worth of further study.

With respect to what is outside of our scope conditions the following can be said. For nondirected networks, the arguments based on regarding the tie as a directional choice by the sender do not apply. Quadratic transformations of numerical variables may be useful there, too, but we do not go into discussing interpretations of such models. For dichotomous variables, there are only three degrees of freedom, which are included in models (3) and (4), and the quadratic models are superfluous. For categorical variables, the situation is more complex, and we do not feel able to propose any generalizable ideas in this paper.

An important caveat for interpretations is that, just like in other generalized linear models, these attribute effects are net of the further effects included in the network model, and the other effects may be correlated in complex ways with the attribute effects. This implies that the social selection function cannot be interpreted as something akin to a true preference function, although the terms we have been using may suggest this. For example, we do not regard the normative value V^{norm} as a revealed norm in any real sense.

For the example presented here, in the analysis of the evolution of an advice network in an MBA cohort with academic grades as the salient attribute, there was clear evidence of homophily, and the medium-strength definition of aspiration was satisfied, although not significantly. There were no signs of attachment conformity other than aspiration, or of sociability associated with grades.

To conclude, the four-parameter model (6) is attractive theoretically and can "let the data speak for themselves" about how the elements of homophily, aspiration, and attachment conformity may combine in any specific empirical setting. The five-parameter model (8) is a more flexible version that will sometimes be an improvement. We propose that empirical researchers have these models in mind when estimating statistical models for directed networks with numerical actor attributes, and we expect that in many cases these specifications will be appropriate. This does not imply that we suggest to necessarily use such quadratic models for all numerical actor variables. They are less parsimonious, with four or five parameters instead of only one for the absolute difference model (2) and three for models (3) and (4). For cases where the dependence of the network on the attribute is weaker, it may be found that one of these three models provides a good enough approximation, so that the conclusions reached will be basically the same and the goodness of fit still adequate. Then for the empirical analysis the quadratic model is not necessary, although theoretically it may still be preferable.

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Notes

1 This definition is different from Stokman's (2004) first definition, which is formulated in terms of aspiration to belong to a group to which one currently does not belong, but the further interpretation is similar.
2 In the SAOM or ERGM representation, the intercept corresponds to the outdegree effect in the linear predictor.
3 In this discussion, we only consider positive aspiration; negative aspiration, an attraction to low values of V, can be treated as its opposite, the directionality being downward instead of upward.

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Appendix A. Implementation in RSiena

Models (6) and (8) can be implemented in the RSiena package (Ripley et al., 2018) by the following effects. The "shortNames" are the shorthand codes that can be used to specify the effects in RSiena.

Name	shortName	$s_{ki}(x, v)$
V ego	`egoX`	$\sum_j x_{ij}\, v_i$
V ego minus alter squared	`diffSqX`	$\sum_j x_{ij}\, (v_i - v_j)^2$
V alter	`altX`	$\sum_j x_{ij}\, v_j$
V alter squared	`altSqX`	$\sum_j x_{ij}\, v_j^2$
V ego × alter	`egoXaltX`	$\sum_j x_{ij}\, v_i\, v_j$

All five effects are simple transformations from the monadic to the dyadic level, so that the implementation will be straightforward also for other statistical network models and other software, using calculated dyadic covariates.

Appendix B. Standard errors

The location (7) of the social norm, $V^{\text{norm}}(\theta)$, is a nonlinear function of the parameter vector θ. When θ is estimated by some estimator $\hat\theta$, the standard error of $V^{\text{norm}}(\hat\theta)$ can be obtained from the delta method (Wasserman, 2004). According to this method, an approximation to the covariance matrix of a function $f(Z)$ of a random vector Z is given by

$$\text{Cov}\big(f(Z)\big) \approx D' \, \text{Cov}(Z)\, D \tag{15}$$

where D is the gradient

$$D = \left.\frac{\partial f(z)}{\partial z}\right|_{z = \text{E}(Z)} .$$

This is applied to $Z = \big(\hat\theta_2, \hat\theta_3\big)$ with the function (7)

$$f(\theta_2, \theta_3) = \frac{\theta_3}{-2\theta_2}$$

and

$$D = \left(\frac{\theta_3}{2\theta_2^2},\ \frac{-1}{2\theta_2} \right)$$

filling in the estimate for the expected value of $\hat\theta$. For Method of Moments estimation in the SAOM, $\text{Cov}(\hat\theta)$ is obtained as in Snijders (2001).

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