Network Dynamics*

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A DYNAMIC APPROACH TO NETWORK ANALYSIS

Dynamic ideas have been pursued in much of Social Network Analysis. Network dynamics is important for domains ranging from friendship networks (e.g., Pearson and West, 2003; Burk, et al., 2007) to, for example, inter-organizational networks (see the review articles Borgatti and Foster, 2003; Brass et al., 2004). However, formal models of analysis, both in the tradition of discrete mathematics and in the tradition of statistical inference, have for a long time focused mainly on single (i.e., cross-sectional) methods of analysis.

Some history: empirical research

Important early longitudinal network studies were those by Nordlie (1958) and Newcomb (1961) who studied friendships in a college fraternity based on the empirical data collected; Coleman's (1961) *Adolescent Society* study with friendship data in 10 schools and 9,702 individuals; Kapferer's (1972) study of observed interactions in a tailor shop in Zambia (then Northern Rhodesia) over a period of ten months, in a period of industrial conflict; Sampson's (1969) Ph.D. dissertation on the developments of the relations in a group of 18 monks in a monastery; and the study by Hallinan with seven

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waves (see Hallinan, 1974, 1979 and Sørensen and Hallinan, 1976). However, attention before about 1990 was mostly on single observations of networks. Of the twenty data sets distributed with the *Ucinet* package (Borgatti et al., 1998), only three provide longitudinal data: Kapferer's Taylor Shop, Newcomb's Fraternity, and Sampson's Monastery. The leading textbook on Social Network Analysis, Wasserman and Faust (1994), has a section of half a page on *Dynamic and Longitudinal Network Models*. The limited amount of attention paid to explicit longitudinal treatment of Social Network Analysis may be understood from the difficulties of collecting network data which are multiplied when a researcher wishes to collect them longitudinally; and from the difficulties in explicitly modelling the dynamics of social networks.

Starting in the 1980s, network panel data started to be collected more widely. Panel data are data collections where the researchers collected data on a given group of social actors at two or more consecutive moments, called the 'panel waves'. Examples are Bauman's study of friendship networks in five schools, collected in the course of a study focusing on dynamics of cigarette smoking (Bauman et al., 1984), with 954 complete questionnaires in a two-wave study; and the *Teenage Friends and Lifestyle Study* in Scotland with three waves (West and Sweeting 1995, Michell and Amos, 1997, Pearson and West, 2003). The currently most well-known study probably is the *Add Health* study in the USA with three waves (Harris et al. 2003, Udry 2003). Christakis and Fowler (2007) discovered interesting network data in the Framingham Heart Study, a longitudinal study not originally intended to contain a network component. Official records and directories also have been used as sources of longitudinal network data. Some examples of such studies are Gulati and Gargiulo (2000), Powell, White, Koput, and Owen-Smith (2005), and the review by Hagedoorn (2002).

Some history: statistical models

A probabilistic model for network dynamics requires to specify the simultaneous probability distribution of $\{X(t) \mid t \in \mathcal{T}\}$, where t is the time parameter which assumes values in a set $\mathcal{T}$ of time points, and $X(t)$ is the network at time t. In probability theory, this is called a stochastic process, where the outcome space is a space of networks. It will be convenient to think of the network as a directed graph (digraph), although depending on the situation at hand it might be a different structure – e.g., undirected, a valued network, etc. For a directed graph, the network $X(t)$ is composed of directed tie variables $X_{ij}(t)$ indicating by the value 1 that there exists an arc $i \rightarrow j$ at time t, and by 0 that no such tie exists. In all cases we assume that there are no self-loops, i.e., always $X_{ii}(t) = 0$. We shall focus on situations where the node set is fixed and denoted by $\{1, \ldots, n\}$. Thus, the network is comprised of n actors. This is usually meaningful for network panel data, if we allow some flexibility for nodes representing actors who entered after the start of data collection or left before the end. It should be noted that there are models also for growing networks, with nodes entering the network, often with the additional assumption that ties do not change once they are established, and the network change is determined by the links created by the newly created nodes. This is a classical approach in the mathematical theory of random graphs (e.g., Bollobás, 1985).

Dynamic network models have to represent the feedback processes that are characteristic of networks. As examples, consider some of the processes of tie creation that are traditional in social network analysis: reciprocation (Moreno, 1934), transitive closure (Rapoport, 1953 a,b; Davis, 1970), and the Matthew effect ('unto him that hath is given and unto him that hath not is taken away, even that which he hath'; Merton 1957; de Solla Price 1965, 1976; called 'preferential attachment' by Barabási and Albert, 1999). If at some moment t the tie $i \rightarrow j$ does not exist, then at some later moment it might be created by reciprocation if currently there is a tie $j \rightarrow i$; it might be created by transitive closure if there are two ties arranged in a two-path $i \rightarrow h \rightarrow j$ — i.e., there currently is an indirect connection from i to j; and it might be created by the Matthew effect if there are many other actors h for which there is a tie $h \rightarrow j$ — i.e., currently actor j is highly popular in the sense of having a high in-degree. These examples illustrate that statistical models for network dynamics have to express dependence between ties as well as dependence across time.

Dependence across time

For modeling dependence across time, the great majority of published models seem to have used some variation of the Markov property. Loosely defined, this is a property, defined for stochastic processes, which expresses that the future depends on the past via the present. A more formal definition (although still slightly incomplete) is that, for time points $t_1 < t_2 < t_3$, $X(t_3)$, conditional on $X(t_2)$, is independent of $X(t_1)$. The earliest proposed models postulated that, if the panel data are $X(t_1), X(t_2), \ldots X(t_n)$, then these n consecutive observations constitute a Markov process. This was assumed, e.g., by Katz and Proctor (1959), Wasserman (1987), Wasserman and Iacobucci (1988), and Robins and Pattison (2001). Since the observations are finite in number, this is called a *discrete-time Markov process*.

However, the feedback processes mentioned above may be assumed to operate, unobserved, between the observations. For example, in a group in which the Matthew effect operates, if at time t_1 some node i has a low in-degree and at the next observation t_2 it has a very high in-degree then it is likely that this has come about by the gradual accumulation of ties directed toward i; the first of these may have been chance occurrences, but once the in-degree was relatively high, it became a self-reinforcing process. Such a model presupposes that there were changes occurring between the observation moments t_1 and t_2. The most elegant and mathematically tractable way of modeling this is to postulate a *continuous-time Markov process* $\{X(t) \mid t \in t_1 \leq t \leq t_n\}$, in other words to let the set of time points of the process $\mathcal{T}$ be the entire interval $[t_1, t_n]$, while still sticking to the panel design for the observed networks: thus it is postulated that the process of network change goes on, unobserved, between the moments of data collection. This was proposed by Sørensen and Hallinan (1976) and Holland and Leinhardt (1977). These authors also proposed that in this change process, at any instance of time t no more than one tie variable $X_{ij}(t)$ can change. This decomposes the change process into its smallest possible constituents and rules out coordination in the form of the simultaneous creation of a set of ties, as in mutual love at first sight, or the spontaneous creation of a group of friends. This is a reasonable requirement which greatly reduces the complexity of modeling. The model of

Sørensen and Hallinan (1976) focused on the dynamics of the triad census (Holland and Leinhardt 1975), and had the set of vectors defining the outcomes of the triad census as the outcome space. This model was incomplete, however, as it did not elaborate the dependence between the triads in a network. A similar but simpler model was presented by Hallinan (1979), focusing on the dyad census. General models representing the dynamics of networks as continuous-time Markov processes where ties change one by one, were proposed by Holland and Leinhardt (1977). They did not, however, elaborate ways to specify the dependence of ties in the network.

Dependence across ties

The Markov chain model of Katz and Proctor (1959) assumed independent tie variables that could change according to a Markov chain at each next observation. Independence of ties is, of course, no more than a straw man assumption as it goes against basic ideas of social network analysis. A first relaxation of this assumption is to postulate independence of dyads, i.e., pairs of tie variables of the type $\big(X_{ij}(t), X_{ji}(t)\big)$. Such an assumption was made, for longitudinal models, by Wasserman (1977, 1979 and other publications), Hallinan (1979), and Leenders (1995 and other publications) for continuous-time Markov processes; and Wasserman (1987) and Wasserman and Iacobucci (1988) for discrete-time Markov processes.

The assumption of independent dyads breaks apart the stochastic process into $n(n-1)/2$ independent sub-processes. This helps for tractability, but of the three basic component processes mentioned above as examples: reciprocity, transitivity, and Matthew effect, it represents only the first. Wasserman (1980) proposed the so-called popularity model which may be said to represent the Matthew effect, but without the reciprocity process – in this model the rows of the random adjacency matrix $\big(X_{ij}(t)\big)$ are independent, which again gives a simplification of the model to make it tractable.

Stochastic models that allow triadic and other higher-order dependencies were proposed for data in the form of rankings – as the Newcomb-Nordlie data – by Snijders (1996), and for data in the form of digraphs by Snijders and Van Duijn (1997) and Snijders (2001). The latter model is described in detail later in this chapter.

Scale-free Networks

De Solla Price (1976), Barabási and Albert (1999), and Dorogovtsev et al. (2000) proposed models where new nodes are added to an existing network and each new node links to m existing nodes with probabilities which depend linearly on the degrees of the existing nodes. This leads to so-called scale-free networks where the distribution of degrees has a power distribution. For most types of networks between human individuals this does not seem realistic because various constraints will limit the frequency of occurrence of very high degrees.

STOCHASTIC MODELS FOR NETWORK DYNAMICS

One of the reasons why stochastic models for network dynamics did not take off before the 1990s is that the dependence structures that characterize networks are so complicated that plausible models for network dynamics can be implemented only (at least, so it seems in the current state of knowledge...) as computer simulation models, like in agent based models, and do not permit the exact calculations that were used in data analysis in the pre-computer era.

In this section we first present tie-based dynamic models, and then actor-based models. The former are simpler, the latter closer to most theories in social science. Both should be regarded as *process models*, which can be defined by probabilistic rules that give a representation of how the network might have evolved from one observation to the next. Technically speaking, all models presented are Markov processes on the space of digraphs. These are continuous-time models, which means that time increases gradually in an infinitesimal fashion, and now and then, at random moments, a change takes place. To keep the model relatively simple, the assumption is followed which first was made by Holland and Leinhardt (1977), that at any given moment ('in any split second'), only one tie can change. This decomposes the dynamics of the network in the smallest possible steps. It assumes away the possibility of simultaneous coordination by actors: actors are dependent because they react to each other (cf. Zeggelink, 1994), not because they coordinate.

Tie-based Models

The simplest approach to construct dynamic network models with quite general dependence structures is by formulating a model where a random pair (i, j) is chosen, and with some probability it is decided to change the value of tie variable X_{ij}: create a new tie (change the value 0 to 1), or terminate an existing tie (change 1 to 0). The probability of change can depend on various function of the network, thus representing the combination of several 'mechanisms', theories, constraints, etc. Technically this is based on the combination of ideas about exponential random graph models with ideas about Markov processes and Gibbs sampling.

Let us first consider an example with four components of the theory, or mechanisms, driving the network dynamics: the tendency to a given average degree, toward reciprocation, transitivity, and the Matthew effect. The Matthew effect is interpreted here as self-reinforcing popularity, contributing to the dispersion of the in-degrees. All of these are understood as stochastic, not deterministic tendencies. These four components will be reflected by the following network statistics:

$$L(X) = \sum_{i,j} X_{ij} \qquad \text{number of ties} \qquad (1)$$

$$M(X) = \sum_{i<j} X_{ij}\, X_{ji} \qquad \text{number of reciprocal dyads} \qquad (2)$$

$$T(X) = \frac{1}{6} \sum_{i,j,h} X_{ij}\, X_{jh}\, X_{ih} \qquad \text{number of transitive triplets} \qquad (3)$$

$$V_{\text{in}}(X) = \frac{1}{n} \sum_{i} (X_{+i} - \bar{X}_{+.})^2 \qquad \text{in-degree variance} \qquad (4)$$

where (5)

$$X_{+i} = \sum_{j} X_{ji} \qquad \text{in-degree of } i \qquad (6)$$

$$\bar{X}_{+.} = \frac{1}{n} \sum_{j} X_{+i} \qquad \text{average degree.} \qquad (7)$$

If the network dynamics has a tendency to favor changes that increase the value of these four statistics, respectively, then this will steer the network process into a direction, respectively, of higher density, more reciprocity, stronger transitivity, or larger in-degree (popularity) differences. This can be achieved by a model in the following way. First, let us rewrite the in-degree variance $V_{\text{in}}(X)$ as follows:

$$\begin{aligned} V_{\text{in}}(X) &= \frac{1}{n}\sum_i X_{+i}^2 - \bar{X}_{+.}^2 \\ &= \frac{1}{n}\sum_i X_{+i}(X_{+i}-1) + \bar{X}_{+.} - \bar{X}_{+.}^2 \\ &= \frac{S_2(X)}{n} - \bar{X}_{+.}(\bar{X}_{+.}-1)\ , \end{aligned}$$

where $S_2(X)$ is the number of two-in-stars in the digraph X, i.e., the number of configurations i, j, k with $j \to i, k \to i$ and $j \neq k$. This shows that, for a fixed average degree $\bar{X}_{+.}$, having a large in-degree variance $V_{\text{in}}(X)$ is just the same as having a large number of two-in-stars $S_2(X)$. We shall henceforth be working with two-in-stars instead of the in-degree variance to express the Matthew effect.

For allowing differential strengths for the tendency toward the four theoretical components, define the linear combination

$$f(x;\beta) \;=\; \beta_1\, L(x) \;+\; \beta_2\, M(x) \;+\; \beta_3\, T(x) \;+\; \beta_4\, S_2(x)\ , \tag{8}$$

where the values of the parameters β_k determine the strength of these four tendencies, and x is an arbitrary digraph. A change process for networks now will be defined that operates by changing ('toggling') single tie variables $X_{ij}(t)$ and that favors changes in the statistics L, M, T, and S_2 depending on the values of the coefficients β_k. This is achieved by the following algorithm, which shows how to transform the current graph $X(t)$ to the next graph, and when this change occurs.

Algorithm 1 . *Tie-based network dynamics* .

For digraphs x, define $x^{(ij+)}$ and $x^{(ij-)}$ as the graphs which are identical to x in all tie variables except those for the ordered pair (i, j), and for which $x^{(ij+)}$ does have a tie $i \to j$, while $x^{(ij-)}$ does not have this tie. In other words, $x_{ij}^{(ij+)} = 1$ and $x_{ij}^{(ij-)} = 0$.

1. *Choose a random pair (i, j) with equal probabilities, given that $i \neq j$.*

2. *Define $x = X(t)$.*

3. *Define*

$$\pi_{ij} = \frac{\exp\big(f(x^{(ij+)};\beta)\big)}{\exp\big(f(x^{(ij+)};\beta)\big) + \exp\big(f(x^{(ij-)};\beta)\big)} \ . \tag{9}$$

   With probability π_{ij}, choose the next network to be $x^{(ij+)}$;

   with probability $1 - \pi_{ij}$, choose the next network to be $x^{(ij-)}$.

4. *Increment the time variable t by the amount Δt, being a random variable with the exponential distribution with parameter ρ.*

This is a model for network dynamics closely related to the exponential random graph model developed by Frank and Strauss (1986), Frank (1991) and Wasserman and Pattison (1996). To elucidate the link to this model, the basic issue is that (9) is the conditional probability for the existence of the tie $i \to j$, given that we know the entire network x except whether this particular tie exists, under the exponential random graph distribution defined by the probability function

$$\mathrm{P}\{X = x\} \ = \ \frac{\exp\big(f(x;\beta)\big)}{C} \tag{10}$$

where C is the normalizing constant

$$C = \sum_x \exp\big(f(x;\beta)\big) \ ,$$

the summation extending over all digraphs x. Thus, the dynamic algorithm above selects whether or not the tie $i \to j$ exists using the conditional probability of this tie under model (10), the condition being the total network configuration outside of the existence of this tie. From general theorems about Markov processes, or more specifically about Gibbs sampling (Geman and Geman 1983), it follows that when this algorithm is repeated indefinitely, the distribution of $X(t)$ (where repeating indefinitely means that t tends to infinity) tends to the distribution with probability function (10). This dynamic algorithm is one of the standard algorithms to obtain random draws from this model, see Snijders (2002) and Robins et al. (2005).

By choosing the parameters β_k in (10), one can choose different models with different strength of the tendencies toward density, reciprocation, transitivity, and self-reinforcing popularity. For example, for $\beta_2 = \beta_3 = \beta_4 = 0$ one obtains a random ('Erdös - Rényi', 'Bernoulli') graph. For $\beta_3 = \beta_4 = 0$ this is a special case of the reciprocity model of Wasserman (1977, 1979), with independent dyads. This independence between dyads is broken when $\beta_3 \neq 0$ or $\beta_4 \neq 0$. For $\beta_2 = \beta_3 = 0$ one obtains the popularity model of Wasserman (1977, 1980). The possibility of having positive values of β_3 as well as β_4, allows to have a model that expresses a tendency towards transitivity as well as the Matthew effect.

Actor-based Models

One of the challenges of network analysis is to incorporate agency in a network model. This was formulated forcefully by Emirbayer and Goodwin (1994) – who likewise stressed the importance of culture, which has to be left aside in this chapter. A natural way to combine agency and structure in a statistical model is to use a model for network dynamics where changes of ties are initiated by actors. Such a model can be a good vehicle for expressing and testing social science theories in which the actors have a central role, cf. Udehn (2002) and Hedström (2005). Actor-based models were proposed by Snijders (1996) for ranked network data, and by Snijders and van Duijn (1997) for binary network data. Here the presentation of Snijders (2001) will be followed. A tutorial introduction to these models, also including practical advice on how to employ and specify them, is given by Snijders et al. (2010).

The actor-based nature of the model means that the model is formulated as if the actors have control over their outgoing ties – under constraints that in the continuous-time representation ties are changed only one at a time, and that the probabilities of changes take into account the total current network configuration. The model specification employs the so-called *rate function* $\lambda_i(x; \alpha)$, depending on actors i and the current network state x, which indicates the frequency per unit of time with which actor i gets the opportunity to change an outgoing tie; and the *objective function* $f_i(x; \beta)$ which can be interpreted as a measure of how attractive the network state x is for actor i. Formulated more neutrally, the objective function is such that, when making a change, actors have a higher probability to move toward networks x for which the objective function $f_i(x;\beta)$ is higher. The statistical parameters α and β are used to reflect the strengths of the various different components included in the rate and objective functions. (For extensions of this model without antisymmetry between creating a new tie and terminating an existing tie, see the discussion in the mentioned literature about gratification or endowment functions.)

The algorithm is formulated in terms of probability distributions only, but it can be interpreted as representing actors embedded in a network, being each others' changing environment (cf. Zeggelink, 1994), who make changes in their outgoing ties each at a rate $\lambda_i(x;\alpha)$ (which could be constant, but which will be changing if the rate function is a non-constant function of x) so as to optimize the value of the objective function that will obtain after their change is made, given that random disturbances are added to the objective function. This may be called *myopic stochastic optimization* of the objective function, and is often used in game-theoretical models of network formation (e.g., Bala and Goyal, 2000).

Algorithm 2 . *Actor-based network dynamics*.
For digraphs x, define $x^{(ij\pm)}$ as the graph which is identical to x in all tie variables except those for the ordered pair (i,j), and for which the tie variable $i \to j$ in $x^{(ij\pm)}$ is just the opposite of this tie variable in x, in the sense that $x_{ij}^{(ij\pm)} = 1 - x_{ij}$.
Define $x^{(ii\pm)} = x$ (as a convenient formal definition without ulterior meaning).

1. *Define $x = X(t)$.*

2. *For $i \in \{1,...,n\}$, define*

$$\tau_i = \frac{\lambda_i(x;\alpha)}{\sum_{h=1}^{n}\lambda_h(x;\alpha)} \ . \tag{11}$$

   Choose actor i with probability τ_i.

3. *For $j \in \{1,...,n\}$, define*

$$\pi_{ij} = \frac{\exp\big(f_i(x^{(ij\pm)};\beta)\big)}{\sum_{h=1}^{n}\exp\big(f_i(x^{(ih\pm)};\beta)\big)} \ . \tag{12}$$

   With probability π_{ij}, choose the next network to be $x^{(ij\pm)}$.

4. *Increment the time variable t by the amount Δt, being a random variable with the exponential distribution with parameter $\sum_{h=1}^{n} \lambda_h(x; \alpha)$.*

The properties of the exponential function imply that equation (12) can be rewritten as

$$\pi_{ij} = \frac{\exp\big(f_i(x^{(ij\pm)}; \beta) - f_i(x; \beta)\big)}{\sum_{h=1}^{n} \exp\big(f_i(x^{(ih\pm)}; \beta) - f_i(x; \beta)\big)} \ , \tag{13}$$

i.e., the probability of a given change depends monotonically on the *increase* in objective function that would be generated by this change. This shows that an actor i for whom the current state x of the network is near the optimum of the objective function $f_i(x; \beta)$, is rather likely to make *no* change, because the probability π_{ii} of choosing to keep the current state $x^{(ii\pm)} = x$ as the next network then is relatively high.

Model specification

In the tie-based as well as in the actor-based model, the researcher has to specify the function $f(x; \beta)$ or $f_i(x; \beta)$, respectively, to specify the model (and in the actor-based model also the rates of change $\lambda_i(x; \alpha)$). This choice should be based on knowledge of the subject matter, theoretical considerations, and the hypotheses to be investigated. We discuss here only the actor-based case.

Like in generalized linear modeling, a convenient class of functions is offered by linear combinations

$$f_i(x; \beta) \ = \ \sum_k \beta_k s_{ki}(x) \ , \tag{14}$$

where the $s_{ki}(x)$ are functions of the network, as seen from the point of view of actor i – in many cases, functions of the personal network of i. An analogue of (8), but now defined for the actor-based model, is

$$f_i(x; \beta) \ = \ \beta_1 \sum_j x_{ij} \ + \ \beta_2 \sum_j x_{ij} x_{ji} \ + \ \beta_3 \sum_{j,h} x_{ij}\, x_{jh}\, x_{ih} \ + \ \beta_4 \sum_{j,h} x_{ij}\, x_{hj} \ . \tag{15}$$

Just like the four terms in (8), but now seen from the point of view of actor i, these four statistics represent, respectively, the number of ties, number of reciprocated ties, number of transitive triplets $\{i \to j \to h, i \to h\}$, and the added in-degrees $\sum_h x_{hj}$ of the actors

j toward whom i has an outgoing tie. The tie-based model with specification (8) and the actor-based model with specification (15) define very similar but nevertheless different probability distributions for the network dynamics; the choice between the tie-based and actor-based specifications will have to based on theoretical preferences, or on empirical fit if any differences in fit can be discerned.

This model specification just serves here as an example of how these models can be used to represent, by the four parameters β_1 to β_4, tendencies toward a given value for the mean degree, toward reciprocity, transitive closure, and preference for already popular actors. It should be noted that these four statistics are highly correlated, which implies that although the parameters β_2, β_3, and β_4 can be used to test the respective tendencies, these parameters all collaborate in their implications for the probability distributions of the statistics that could be calculated from the network. In practically all cases it will be desirable to control for the average degree, and testing hypotheses about β_1 does not seem very meaningful in general.

Many other statistics of the personal network of actor i may be used as the $s_{ki}(x)$ in expression (14) for the objective function. Such statistics are called *effects*. Since the actor has control only over the outgoing tie variables, what is important here is how the effects depend on the outgoing tie variables x_{ij}; effects depending only on incoming tie variables have no consequence on the conditional probability (12). An ample discussion of many statistics which could be included to reflect various theoretically interesting network tendencies and which can be helpful to give a good representation of the dependencies between tie variables is given by Snijders et al. (2010). The following is a (very incomplete) outline.

1. Two fundamental statistics are
   (*a*) the outdegree $\sum_j x_{ij}$, of which the parameter – such as β_1 in example (15) – can be used to fit the level and tendency of the average degree; most other statistics will be correlated with the average degree, which implies that the precise value of this parameter will depend strongly also on the other parameters; and
   (*b*) the reciprocated degree defined as $\sum_j x_{ij}x_{ji}$, the number of reciprocal ties in

which actor i is involved, and also included in (15); in almost all directed social networks reciprocity is a basic tendency, and including this effect will allow a good representation of the tendency toward reciprocation.

2. The local structure of networks is determined by triads, i.e., subgraphs on three nodes (Holland and Leinhardt, 1975). The main dependencies between ties in triads are captured by
   (*a*) transitivity: the tendency that 'friends of friends become, or stay, friends', expressed by the number of transitive triplets in the personal network, $\sum_{j,h} x_{ij}\, x_{jh}\, x_{ih}$, included as the third term in (15); and
   (*b*) three-cycles: the tendency to form closed cycles $i \to j \to h \to i$, measured by $\sum_{j,h} x_{ij}\, x_{jh}\, x_{hi}$. This can represent generalized exchange (Bearman, 1997); however, it is more frequent to observe that this effect has a negative sign, meaning that three-cycles tend to be avoided (Davis, 1970), a sign of local hierarchy.

3. In- and out-degrees are fundamental aspects of individual network position, and creation or termination of ties can be more or less likely depending on the degrees of the actors involved. This is expressed by *degree-related effects*. The basic degree effects are
   (*a*) in-degree popularity, indicating the extent to which those with currently high in-degrees are more popular as receivers of new ties – which is just the Matthew effect mentioned above and the fourth term in (15);
   (*b*) out-degree activity, indicating whether those with currently high out-degrees have a greater tendency to create rather than terminate ties; and analogously
   (*c*) out-degree popularity and
   (*d*) in-degree activity.
   Also higher-order degree effects such as *degree-based assortativity* may be included, which express a stronger or weaker tendency to form and maintain ties depending on the combination of the degrees of both.

4. In addition to these effects based on the network structure itself, it is important to include statistics depending on attributes of the actors – their demographic characteristics, indicators of resources, etc. A given actor variable can be included

as an *ego effect*, reflecting the effect of this variable on the propensity to send ties, and as an *alter effect*, reflecting the effect on the propensity to receive ties. In addition, the combination of sender and receiver usually is important, such as their similarity on salient attributes, reflecting tendencies toward homophily (McPherson, Smith-Lovin, and Cook, 2001).

5. It is also possible to include attributes of pairs of actors – which may be their relatedness in a different network. Such *dyadic covariates* can express, e.g., meeting opportunities, costs and benefits of the dyadic tie, etc.

STATISTICAL INFERENCE FOR ACTOR-BASED MODELS

Varying the parameters α and β can yield very different network dynamics, and for a given longitudinal network data set the question is, how to determine these parameter values to achieve a good fit between model and data. This is the usual question of statistical inference. A technical difficulty here is that no easily computable measure exists for the fit between the model and the data, like the sum of squares in the analysis of variance, and the properties of the model can be assessed in practice only by computer simulation. Indeed the actor-based model can be seen as an agent-based computational model (cf. Macy and Willer, 2002) that is meant to mimic the way in which the network evolves.

Estimation

For parameter estimation in actor-based models, three methods have been proposed in the literature. In the Method of Moments (Snijders and van Duijn, 1997; Snijders, 2001), a set of statistics of the longitudinal network data set is suitably chosen, one for each estimated parameter, and the parameters are determined so that for these statistics there is a perfect fit between observed values and the expected values in the population of all simulations from this model: the expected values should be equal to the observed values. This can in practice be achieved only approximately, by a stochastic approximation algorithm, with some randomness in the results due to the limited number of simulations actually conducted. Bayesian procedures were proposed by Koskinen and Snijders (2007) and Schweinberger (2007). The Bayesian method postulates a probability distribution of the parameters that represents prior beliefs and/or prior ignorance, and then calculates or approximates the so-called posterior distribution of the parameters. The latter is the conditional distribution of the parameters given the data that were observed, and represents how the prior beliefs have been transformed by the empirical observations. Third, an algorithm to approximate the Maximum Likelihood estimator was developed by Snijders et al. (2010). This algorithm is based on simulating the likely continuous-time process that might have led from one panel wave observation to the next, and then approximating the parameters using an appropriate method of averaging. For data sets that are not too small, and if the model holds to a good approximation, these three methods will yield similar estimation results.

Testing

Connected to the Method of Moments and the Maximum Likelihood method as estimation methods, there are procedures for testing statistical hypotheses about the parameters, following the general principles for constructing statistical tests (see, e.g., Cox and Hinkley, 1974). Often the most straightforward way is to use the parameter estimates and their standard errors. For testing a null hypothesis such as

$$H_0 : \ \beta_k = 0$$

the test statistic then is the ratio of the estimate to the standard error,

$$t \ = \ \frac{\hat{\beta}_k}{\text{s.e.}(\hat{\beta}_k)} \ . \tag{16}$$

This can be tested in a standard normal reference distribution. This may be called a t-test, as it is based on a t-ratio. Multi-parameter tests can be derived in an analogous fashion. For estimates obtained by the method of moments such tests may be called Wald-type tests, for Maximum Likelihood estimates Wald tests.

There is a different way of hypothesis testing which does not require that the tested parameter is estimated. This is the general principle of Rao's efficient score test. For the method of moments a special adaptation is required, which yields the score-type test as developed by Schweinberger (2008). There is a special practical advantage to score or score-type tests for these models, because the Monte Carlo algorithms for parameter estimation may fail to converge in cases when the model is relatively complicated given the amount of information in the data; the score principle then can provide a test even if one does not have a parameter estimate.

Associated with Maximum Likelihood estimation is the likelihood ratio test. An algorithm is presented in Snijders et al. (2010).

The algorithms currently available for Method of Moments are much less time-consuming than those for Maximum Likelihood estimation and testing. However, this is an area of active development, and the computational efficiency of the available algorithms may change.

DYNAMICS OF NETWORKS AND BEHAVIOR

What makes networks important often are the individual behavior and other individual outcomes that are in some way related to the network embeddedness of the actors; see, e.g., Granovetter (1973), Burt (1992), and Lin et al. (2001). Such individual characteristics, however, will also play a role in the explanation of the network dynamics. Thus we encounter the situation where the network and the behavior – a term that we use here as a shorthand for the relevant changeable characteristics of the actors, which also could be attitudes, performance, etc. – both can be considered as dependent variables, changing interdependently. It is assumed here that the behavior variables are ordinal discrete variables, with values 1, 2, etc., up to some maximum value, for instance, several levels of alcohol consumption, several levels of political attitudes on a left-right scale, etc.; a binary variable is a special case. The dependence of the network dynamics on network and behavior jointly will be called the *social selection* process, and the dependence of the behavior dynamics on network and behavior will be called the *social influence* process (An, this volume).

Both social influence and social selection can lead to similarity between tied actors, which is descriptively called *network autocorrelation* (Doreian, 1989; Leenders, 1997). Whether this network autocorrelation is caused mainly by influence or mainly by selection can be an important question. This is demonstrated in Ennett and Bauman (1994) for smoking, and Haynie (2001) and Carrington (this volume) for delinquent behavior.

Actor-based Models

To answer such questions, it can be helpful to employ process models that represent the interdependent evolution of the tie variables as well as the actors' behavior variables. Here actor-based models are especially natural; such models were specified in Snijders, Steglich and Schweinberger (2007) and in Steglich et al. (2010). They assume that the outgoing ties of an actor, as well as the behavior of the actor, are under this actor's control, subject to various restrictions.

The process model assumes that at random moments, either a network tie or a behavior variable can be changed. The actors have rate functions and objective functions for the network and the behavior separately. That networks and behavior are governed potentially by different processes can be argued, e.g., by regarding network choice and behavior choice as being determined by different decision frames (Lindenberg, 2001). Decomposing the changes in the smallest possible steps here means that at one given ('infinitesimal') moment in time, the possibilities for an actor to change his or her behavioral variable are limited to moving one category up or down on the ordered scale.

We denote the behavior of actor i at time t by $Z_i(t)$, collected in the vector $Z(t)$. It now is assumed that the change probabilities of the network will depend on the current state of the network as well as the behavior; and the change probabilities of the behavior will depend on the current state of the behavior as well as the network. The objective function for actor i for the network is denoted $f_i^X(x, z; \beta)$, and for the behavior $f_i^Z(x, z; \beta)$. Similarly to the objective function for the network, the objective function for behavior is such that changes toward higher values of the objective function are more likely than changes toward lower values. The rate function for actor i for network change is denoted $\lambda_i^X(x, z; \alpha)$, and for behavior change $\lambda_i^Z(x, z; \alpha)$.

Algorithm 3 . *Actor-based dynamics of network and behavior*.

For the network, the same definitions are used as in the algorithm for actor-based network dynamics. For the behavior, for any actor i and a potential increment d, define $z^{(i+d)}$ as the vector of behaviors which is identical to z in all coordinates except that d is added to the i'th coordinate: $z_i^{(i+d)} = z_i + d$.

1. *Define $x = X(t)$, $z = Z(t)$.*
2. *Calculate the ratio*
$$\phi_X = \frac{\sum_{h=1}^{n} \lambda_h^X(x, z; \alpha)}{\sum_{h=1}^{n} \big(\lambda_h^X(x, z; \alpha) + \lambda_h^Z(x, z; \alpha)\big)} \ . \tag{17}$$
With probability ϕ_X, go to item 3 to make a network step; else (with probability $1 - \phi_X$), go to item 5 to make a behavior step.
3. *For $i \in \{1, ..., n\}$, define*
$$\tau_i^X = \frac{\lambda_i^X(x, z; \alpha)}{\sum_{h=1}^{n} \lambda_h^X(x, z; \alpha)} \ . \tag{18}$$
Choose actor i with probability τ_i^X.
4. *For $j \in \{1, ..., n\}$, define*
$$\pi_{ij}^X = \frac{\exp\big(f_i^X(x^{(ij\pm)}, z; \beta)\big)}{\sum_{h=1}^{n} \exp\big(f_i^X(x^{(ih\pm)}, z; \beta)\big)} \ . \tag{19}$$
With probability π_{ij}^X, choose the next network to be $x^{(ij\pm)}$.
Go to step 7.
5. *For $i \in \{1, ..., n\}$, define*
$$\tau_i^Z = \frac{\lambda_i^Z(x, z; \alpha)}{\sum_{h=1}^{n} \lambda_h^Z(x, z; \alpha)} \ . \tag{20}$$
Choose actor i with probability τ_i^Z.

6. *For $d \in \{-1, 0, 1\}$, if $z_i + d$ is in the permitted range of Z, define*

$$\pi_{id}^Z = \frac{\exp\left(f_i^Z(x, z^{(i+d)}; \beta)\right)}{\sum_{k=-1}^{1} \exp\left(f_i^Z(x, z^{(i+k)}; \beta)\right)} . \tag{21}$$

   Values d for which $z_i + d$ would be outside of the permitted range are not included in the denominator.
   With probability π_{id}^Z, choose the next behavior vector to be $z^{(i+d)}$.
   Go to step 7.

7. *Increment the time variable t by the amount Δt, being a random variable with the exponential distribution with parameter $\sum_{h=1}^{n} \left(\lambda_h^X(x, z; \alpha) + \lambda_h^Z(x, z; \alpha)\right)$.*

The choice $d = 0$ means that actor i has the opportunity to change her/his behavior, but refrains from doing so. The probability of this will be higher, accordingly as the value of the objective function of the current state, $f_i^Z(x, z; \beta)$ is higher compared to the value of the neighboring states $f_i^Z(x, z^{(i+d)}; \beta)$ for $d = -1, +1$.

Model Specification

For the behavior also, the most convenient expression for the objective function is a linear combination

$$f_i^Z(x, z; \beta) \; = \; \sum_k \beta_k^Z s_{ki}^Z(x, z) \; , \tag{22}$$

where the $s_{ki}^Z(x, z)$ are functions of the behavior and other characteristics of actor i, but may depend also on the personal network, the behavior of those to whom i is tied, etc. In studies of selection and influence, the behavior-dependent selection part is modeled by the specification of the model for network dynamics, e.g., by a term expressing the preference (homophily) for ties to others who are similar on the behavioral variable Z. The network-dependent influence part is modeled by appropriate terms in the objective function for behavior. A basic example of a specification for this function is

$$f_i^Z(x, z; \beta) \; = \; \beta_1^Z z_i \; + \; \beta_2^Z z_i^2 \; + \; \beta_3^Z z_i \left(\frac{\sum_j x_{ij} \, z_j}{\sum_j x_{ij}} \right) \; . \tag{23}$$

The first two terms represent a quadratic preference function for the behavior Z. If preferences are unimodal, then the coefficient of the quadratic term, β_2^Z, is negative. For addictive behaviors, however, this coefficient can be positive. The third term indicates that the 'value' for actor i of behavior z_i depends on the average behavior of those to whom i has an outgoing tie.

EXAMPLES

Because of space constraints, this chapter does not contain an elaborate empirical example. The mentioned methodological articles that further explain the actor-based model for network dynamics can be consulted for some examples. Other published examples of network dynamics (ordered by the age of the population of actors) include Schaefer et al. (2010) about the effects of reciprocity, transitivity, and popularity in friendship dynamics between pre-school children; Selfhout et al. (2010) about the way in which friendship dynamics of adolescents depends on personality characteristics; van Duijn et al. (2003) about the effects of visible and non-visible attributes on dynamics of friendship between university students; and Checkley and Steglich (2007) about how mobility of managers affects inter-firm ties.

Examples of the joint dynamics of networks and behavior have been published only recently, because of the recency of the model. Some of these examples are the following. Burk et al. (2007) present a study on influence and selection processes in the dynamics of friendship and delinquent behavior of adolescents. Steglich et al. (2010) studied the co-evolution of friendship and smoking as well as drinking behavior in a secondary school cohort. Mercken et al. (2009) studied influence and selection processes in smoking initiation among adolescents in a large-scale study with networks in 70 schools in 6 countries. The study by De Klepper et al. (2010) is set in a Naval Academy, and studies the mutual dependence in the evolution of friendships and military discipline.

THE *SIENA* PROGRAM

The actor-based model for network dynamics, as well as the model for dynamics of networks and behavior, are implemented in the *SIENA* ('Simulation Investigation for

Empirical Network Analysis') program. Initially a stand-alone program with a user interface through the program *StOCNET*, since 2009 it is a package within the statistical system R (R Development Core Team, 2009), called RSiena. The R system and its packages are freeware, running on Windows, Mac, as well as Unix/Linux systems. An extensive and frequently updated manual is available (Ripley and Snijders, 2010). This manual gives detailed instructions for installing and working with RSiena.

A first requirement is to install R, the package RSiena, and a few auxiliary packages, as described in the RSiena manual. If desired, RSiena can be operated apparently without any knowledge of R, by means of a graphical user interface; after the installation, it is then not necessary to operate R. Once the installation is done, RSiena can be run in three ways:

1. Run the graphical user interface `siena.exe` outside of R. This will call R without the user needing to be aware of this, and after the installation only the RSiena manual is used, and no further knowledge of R is necessary.
2. Run R, load the package RSiena and the auxiliary packages, and run the graphical user interface from within R by the command `siena01Gui()`. This offers the basic functionality of RSiena, with the possibility to integrate the use of RSiena with the use of any other R packages. It has the advantage that no knowledge of the commands of RSiena is required.
3. Run R, load the package RSiena and the auxiliary packages, and run RSiena by using its R commands. This is the best option for users fully conversant with RSiena.

As basic literature, the best combination is to use Snijders et al. (2010) as a tutorial for the methodology, and Ripley and Snijders (2010) (or more recent versions) for the requirements on data formats and the operation of the software.

OUTLOOK AND DISCUSSION

Statistical methods for social network analysis that represent network dependencies in a satisfactory way have been available only since recent years. The methods presented here for analyzing network evolution, and for the co-evolution of networks and behavior, allow researchers to test competing as well as complementary theories about dynamics relating to networks. More reflection now is needed from a theoretical as well as methodological viewpoint to combine the statistical approach with the network approach. The network approach is rich in structural and positional analysis. The statistical approach, by contrast, has a tradition of parsimony, which often limits model specification for hypothesis testing to the choice of tested variables together with a few control variables. Much research in the statistical approach is purely individualistic, ignoring the importance of distinguishing multiple types of unit of analysis and where hypotheses are uniquely formulated without further ado in the scheme of 'X leads to Y, when controlling for A'. Convincing gatekeepers such as reviewers and editors of journals of the importance of a network approach, where theories and statistical models are more complex, can be difficult.

Two major limitations of the purely individualistic approach can be mentioned here. In the first place, most network research is observational rather than experimental, which means that methods of analysis must incorporate adequate control for competing hypotheses or theories, and a good specification of statistical dependencies between observed variables is essential to obtain reliable conclusions. In network phenomena, endogenous (also called self-referential, emergent, self-organizing, feedback) processes are essential, and these lead to dependencies between variables rather than effects of some measured variable X on a dependent variable Y. The failure to specify such dependencies appropriately will lead to hypothesis tests with inadequate control for competing theories.

Second, network dependencies can be a treasure grove of interesting theories and hypotheses, and the infusion of network approaches into theoretical thinking and statistical hypothesis testing, along theoretical lines such as Hedström's (2005) analytical sociology, can lead to better explanations of empirical phenomena and to improved interventions in domains such as public health. A similar kind of progress has started earlier in contextual analysis by multilevel modeling, where the analytical use of several types of unit of analysis is now generally accepted to be fruitful and even necessary, although not yet generally practised; examples are Sampson et al. (2002) and O'Campo (2003).

Such theoretical-methodological advances will be easier when further progress in statistical modeling for network dynamics will have been made along three lines: models for richer data structures, less restrictive models, and richer statistical procedures. With respect to data structures, when remaining within the confines of network panel designs, one can think of extending this type of modeling to data types such as valued networks, multivariate networks, and non-directed networks. Developments should not be limited to panel designs, however. In studies of networks between organizations, sometimes the observation moments are spaced so tightly that it is reasonable to make the approximation that the preceding observation of the network state is used to directly predict the next observation in a network autoregressive model, as done by Leenders (1997) and Gulati and and Gargiulo (1999); sometimes the observations even provide a continuous record of tie creation, although not always of tie termination, such as in Hagedoorn (2002). Second, with respect to models, it will be worthwhile to develop models that are non-Markovian, e.g., models with latent variables or more general hidden Markov models (Cappé et al., 2005). The models presented here assume implicitly that actors have full knowledge of the network, and to model larger networks in a plausible way it will be helpful to develop models that do not assume complete information. Third, statistical procedures have to be developed further. Algorithms should be improved and their mathematical properties investigated. In addition, procedures for assessing goodness of fit should be developed and the robustness of parameter estimators and tests for misspecification should be studied. Together with the software implementation, this implies a considerable amount of methodological work.

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