# One-mode – two-mode co-evolution for studying social influence on behaviour

Tom A.B. Snijders



University of Groningen University of Oxford

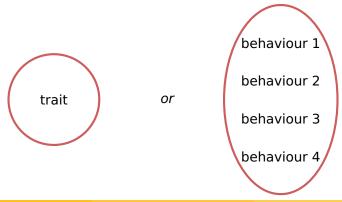
January 2023



### Social influence: at which level?

Question for this presentation:

### Does social influence in a given domain take place at the level of a general trait or at the level of concrete behaviours?



# 1. Social influence: at which level?

Consider a set of items in the given domain, for which the trait could be measured by a sum score, but each item of which is also a meaningful concrete behaviour.

Suppose that for a group, repeated measures are available about the network of interest and these items.

For influence at the level of a trait, the Stochastic Actor-oriented Model for Network Dynamics ('SAOM') can be used for network-behaviour co-evolution.

For influence at the level of the behaviours, the SAOM (RSiena) can be used for one-mode – two-mode network co-evolution.

The second will be elaborated in this presentation with as example delinquency in early secondary schools (data collected by Andrea Knecht)

# 2. Delinquency

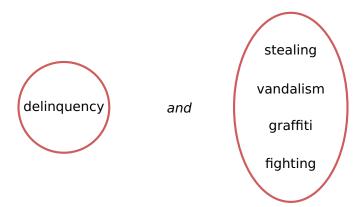
Andrea Knecht (supervision Chris Baerveldt) collected network data in 126 first-grade classrooms in 14 secondary schools.

The Netherlands, 2003-2004, 4 waves

Four questions about delinquency were posed: self-reported frequencies of stealing, vandalism, graffiti, fighting.

Social influence and selection for the trait of delinquency was studied in

Andrea Knecht, Tom A.B. Snijders, Chris Baerveldt, Christian Steglich, and Werner Raub (2010). Friendship and delinquency: Selection and influence processes in early adolescence. *Social Development*, 19, 494—514, using a sum score. Here we shall study the influence and selection simultaneously for the trait and the behaviors:



# 3. One-mode – two-mode co-evolution

This is analyzed with the SAOM for co-evolution of one-mode and two-mode networks Snijders, Lomi & Torlò in *Social Networks, 2013*.

X: one-mode friendship network;
Z: two-mode delinquency network,
where the 2nd mode consists of the four items
stealing, vandalism, graffiti, fighting
dichotomized: 'at least once in past 3 months', for fighting 'at least twice in past 3 months'.

X influences changes in Z, and Z influences changes in X.

The following slides present the main effects for representing the one-mode – two-mode selection and influence process.

# Notation for the effects

 $x_{ii}$ : indicator that *i* mentions *j* as a friend;

- $z_{ih}$ : indicator that *i* mentions delinquent behaviour *h*;
- $s^{\chi}$  represents effects for friendship,  $s^{Z}$  effects for delinguency;

in the pictures:

circles are actors, squares delinguent behaviours;

means there is a friendship tie from *i* to *j*;

means *i* mentions delinguent behaviour *h*.

I shall be using also the "shortNames" as used in RSiena.

# Influence at the level of the sum score

average Z-outdegree of i's X-friends  $\Rightarrow$  Z<sub>i</sub>

Effect for influence at the level of the sum score:

outOutAvIntn, the total number of delinguent behaviours reported by *i* multiplied by the average number of delinguent behaviours (centered) reported by all i's friends,

$$s_{\text{od\_av},i}^{Z}(x,z) = \sum_{h} z_{ih} \frac{\sum_{j} x_{ij} \{\sum_{\ell} (z_{j\ell} - \bar{z})\}}{\sum_{j} x_{ij}}$$

where  $\bar{z}$  is the average observed outdegree for Z in the group.

$$h \nleftrightarrow i \longrightarrow j \rightarrowtail l$$

# Selection at the level of the sum score

### higher $Z_i \Rightarrow i$ will prefer X-friends with higher $Z_j$

Effect for selection at the level of the sum score:

 $outActIntn \times outPopIntn$ , interaction between friendship activity and friendship popularity based on delinquency outdegree,

$$s_{\text{oaxop},i}^{X}(x,z) = \sum_{j} x_{ij} \{ \sum_{h} (z_{ih} - \bar{z}) \} \{ \sum_{\ell} (z_{j\ell} - \bar{z}) \}$$

$$h \longleftarrow i \longrightarrow j \longrightarrow \ell$$

(same picture)

### Influence and selection at the behaviour level

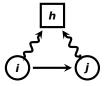
X-friends *i* and *j* will do the same delinquent acts  $z_{ih}$ ,  $z_{jh}$ 

'Regular' influence and selection effects for one-two co-evolution:

to and from, the number of friendships of *i* weighted by the number of delinquent behaviours *i* and *j* have in common,

$$s^V_{{
m triad},i}(x,z) = \sum_{j,h} x_{ij} z_{ih} z_{jh}$$
 ,

for V = X (selection) and V = Z (influence).



# Further cross-network effects

#### Hierarchy requires control for lower-order effects

In the first place, the regular mixed degree effects:

outActIntn, mixed outdegree activity, the product of the number of outgoing friendships and the number of delinguent behaviours of i,

$$s_{\text{oa},i}^{X}(x,z) = \sum_{j} x_{ij} \sum_{h} (z_{ih} - \bar{z}) ,$$
$$s_{\text{oa},i}^{Z}(x,z) = \sum_{j} z_{ij} \sum_{h} (x_{ih} - \bar{x}) ,$$
$$\boxed{h} \checkmark \checkmark (i) \longrightarrow (j)$$

#### **Fffects**

# Further cross-network effects (2)

The product of the number of incoming friendships and the number of delinguent behaviours is effect *outPopIntn* for friendship:

$$s_{ia,i}^{X}(x,z) = \sum_{j} x_{ij} \sum_{h} (z_{ih} - \bar{z}),$$

and *inActIntn* for delinguency (where *i* and *j* are reversed),

$$s_{\text{op},i}^{Z}(x,z) = \sum_{j} x_{jj} \sum_{h} (z_{ih} - \bar{z}) .$$

# Further effects (3)

#### Group mean of delinguency

In addition, there is the effect *avDeg* of the group average of the delinguency outdegree on delinguency:

$$s_{ga,i}^{Z}(x,z) = \sum_{j} z_{ij} \left( \frac{1}{n} \sum_{h} z_{h+} - p \right)$$

where p is a parameter for centering.

This is the effect of the average of the entire group.

It could be regarded as a group norm;

since  $z_{h+}$  can be regarded as an unreliable measurement of 'delinguency' of actor h, this effect, with a negative parameter, will also reflect regression to the mean ( $\Rightarrow$  stability).

# Overview of effects

The one-mode – two-mode selection and influence model contains the following effects. Between square brackets are the analogous effects for network-behaviour co-evolution.

For friendship:

- outActIntn (degrees) [egoX]
- outPopIntn (degrees) [altX]
- outActIntn × outPopIntn (degrees for selection) [egoXaltX]
- from (triadic) (selection on specific behaviours)

For delinquency:

- outActIntn (degrees) [outdeg]
- InActIntn (degrees) [indeg]
- outOutAvIntn (influence from average friends' sum score) [avAlt]
- to (triadic) (influence from specific behaviours)
- avDeg (group average outdegree) [avGroup]

# Results for the example

The model was estimated by a random coefficient multilevel model (function *sienaBayes*), using 3 parallel chains of 70,000 steps. Convergence was good.

Waves 2-3-4 were used, selection based on missing data and enough stability (Jaccard coeff.) led to including 82 classrooms.

Prior distributions were specified according to the advice in the RSiena manual.

# Descriptive statistics 1

Overall means and similarity coefficients of delinquent acts

	mean	Jaccard similarity			
		stealing	vandalism	graffiti	fighting
stealing	0.13	-	0.29	0.22	0.27
vandalism	0.20	0.29	-	0.27	0.33
graffiti	0.17	0.22	0.27	-	0.23
fighting	0.21	0.27	0.33	0.23	-

Delinquency is present but low;

the four delinquent behaviours are weakly positively associated.

# Descriptive statistics 2

#### Network descriptives: means and between-group standard deviations

	wave 1		wave 2		wave 3	
	mean	(s.d.)	mean	(s.d.)	mean	(s.d.)
Friendship						
outdegree	4.01	(0.66)	4.17	(0.60)	4.03	(0.68)
Jaccard with next wave	0.50	(0.09)	0.52	(0.08)		
proportion missings	0.03	(0.03)	0.07	(0.06)	0.06	(0.04)
Delinquency						
outdegree	0.76	(0.29)	0.91	(0.34)	0.92	(0.35)
Jaccard with next wave	0.39	(0.09)	0.43	(0.10)		
proportion missings	0.01	(0.02)	0.06	(0.05)	0.06	(0.05)

# Parameter estimates: friendship

del = delinquency

Effect	par.	(psd)	betw. sd
outdegree (density)	-2.194	(0.066)	0.362
reciprocity	2.052	(0.065)	0.387
transitive triplets	0.465	(0.016)	0.103
transitive reciprocated triplets	-0.159	(0.017)	
indegree - popularity	-0.072	(0.012)	0.092
outdegree - activity	0.036	(0.008)	0.055
reciprocal degree - activity	-0.184	(0.015)	0.099
same non-Dutch language	0.698	(0.209)	
same sex	0.662	(0.025)	
log classroom size	-0.264	(0.225)	
advice similarity	0.100	(0.085)	0.255
outdegree del popularity	0.008	(0.018)	
outdegree del activity	-0.033	(0.019)	
outdeg. del activity x outdeg. del popularity	0.018	(0.017)	
triadic: same delinquent behaviors	-0.011	(0.059)	

par = posterior mean; psd = posterior standard deviation of the mean;

betw. sd = posterior between-groups stand. deviation.

# Parameter estimates: delinquency

#### del = delinquency

Effect	par.	(psd)	betw. sd
outdegree (density)	-2.420	(0.130)	0.550
indegree - popularity	0.016	(0.018)	0.081
outdegree - activity	0.440	(0.020)	0.100
classroom average outdegree	-0.945	(0.166)	
sex ego	0.202	(0.042)	
advice ego	0.028	(0.022)	
classroom mean advice	-0.127	(0.044)	
indegree friends activity	-0.004	(0.013)	
outdegree friends activity	-0.065	(0.017)	
average friends' del outdegree	-0.057	(0.059)	
triadic: behaviors mentioned by friends	0.241	(0.042)	

par = posterior mean; psd = posterior standard deviation of the mean; betw. sd = posterior between-groups stand. deviation.

# Parameter estimates:

# focus on selection and influence

del = delinquency

Effect	par.	(psd)
friendship		
outdegree del popularity	0.008	(0.018)
outdegree del activity	-0.033	(0.019)
outdeg. del activity x outdeg. del popularity	0.018	(0.017)
triadic: same delinquent behaviors	-0.011	(0.059)
delinquency		
classroom average outdegree	-0.945	(0.166)
indegree friends activity	-0.004	(0.013)
outdegree friends activity	-0.065	(0.017)
average friends' del outdegree	-0.057	(0.059)
triadic: behaviors mentioned by friends	0.241	(0.042)

# Conclusion

This data set on delinquency gives strong evidence for social influence on the level of specific delinquent behaviours, and not on the level of the aggregate measure of delinquency.

Furthermore, there is regression to the mean w.r.t delinquency.

There is no evidence for friendship selection based on delinquency, neither on the aggregate measure nor on doing the same behaviours.

### Discussion

When there is a set of binary items indicating specific behaviours in a given domain, for which the trait could be measured by a sum score, then this method of one-mode – two-mode co-evolution can be used in studies where selection and influence are considered simultaneously at the level of the sum score and at the level of the specific individual behaviours.

The issue of regression to the mean merits further thought.

Johan Koskinen and Tom A.B. Snijders (2023). Multilevel Longitudinal Analysis of Social Networks. Journal of the Royal Statistical Society, Series A, 186:376—400.