Actor-Based Models for Dynamics of Two-Sided Relations

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1 Modeling Panel Network Data

Research utilizing the perspective of social networks can shed important light on political processes, as is illustrated, for example, by the special issue *Social Networks and American Politics* of *American Politics Research* (Heaney and McClurg, 2009). This perspective offers some complications for statistical analysis, however. A network approach is so useful because it can represent the interdependence between political actors (see Huckfeldt, 2009) – but statistical modeling is commonly based on independence assumptions. The challenge in statistical modeling of social network data is to represent the dependencies between network ties so that valid inferences can be obtained and misspecification avoided; and, by doing so, to provide methods that allow researchers to test hypotheses about these interdependencies.

This article treats statistical methods for network panel data. It is assumed that the reader has a basic knowledge of networks and the associated terminology; see, e.g., Wasserman and Faust (1994) or Knoke and Yang (2008). For the data structure it is assumed that a fixed set of nodes is being considered – where, however, exceptions are allowed in the sense that some nodes may enter or leave the network – while the change represented by the panel consists of tie changes from one panel wave to the next. The

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purpose of the statistical models treated is to find the rules, or mechanisms, that govern the change in the network ties, in a way that treats the mentioned challenge in an appropriate way. The fact that a fixed set of nodes is considered means that the problem of network delineation, or the 'network boundary problem' (cf. Marsden, 2005) is considered to have been solved before embarking upon the analysis that is the topic of this paper.

This paper has three main parts. First, we present the model for analyzing dynamics of directed network that was introduced by Snijders (2001), with a brief sketch of the associated estimation methods, implemented in the software package *SIENA* ('Simulation Investigation for Empirical Network Analysis'). The actor-based framing used to define this model is helpful in combining the primacy of actors in social science theories with the dyadic nature of network ties. This method is being used in network studies in various of the social sciences and is now also starting to be used in the political sciences; see, e.g., Andrew (2009). Second, this model is extended to non-directed networks, i.e., networks in which the ties are by their nature non-directional, so that a tie from i to j cannot be distinguished from a tie from j to i. Many applications of network analysis to the political sciences are naturally formulated in terms of non-directed networks. For a tie to exist in a non-directed network, some kind of coordination between the two involved actors has to be modeled, and we present new models in which this coordination is added to the actor-based framework of Snijders (2001). The third part is a brief introduction to models for the co-evolution of networks and nodal attributes, which build upon the models for network dynamics but extend the dependent variable: in addition to the network, there is an evolving actor-level variable that can be influenced by the network and that itself can exert influence upon the network. The paper finishes with a discussion which also indicates how these models are positioned in the wider set of statistical network models that have been proposed in the literature.

2 Stochastic Actor-Based Models for Network Dynamics

This section presents a model for network dynamics where as time goes by, ties can be added as well as deleted from the network. The probabilities of these tie changes may depend on variables that are exogenous to the network – where the variables can be monadic (defined at the level of actors) or dyadic (defined at the level of pairs of actors) – but may also depend on the existing configuration of ties in the network as a whole. The latter gives a way of representing dependencies between ties. First the fundamental description of the model is given, followed by possible ingredients for its detailed specification. Finally, procedures for estimation are briefly described. Examples are not given here, but can be found, e.g., in Andrew (2009), Lazega, Mounier, Snijders, and Tubaro (2010), and van de Bunt, Wittek, and de Klepper (2005).

2.1 *Notation*

This section treats directed networks on a given node set $\{1, \ldots, n\}$. Nodes represent social actors. The existence of a tie from node i to node j is indicated by the *tie indicator variable* X_{ij}, having the value 1 or 0 depending on whether there is a tie $i \to j$. For the tie $i \to j$, actor i is called the *sender* and j the *receiver* of the tie. Self-ties are not considered, so that always $X_{ii} = 0$. The matrix with elements X_{ij} is the adjacency matrix of a directed graph, or digraph; the adjacency matrix as well as the digraph will be denoted by X. Outcomes (i.e., particular realizations) of digraphs will be denoted by lower case x.

Replacing an index by a plus sign denotes summation over that index: thus, the number of outgoing ties of actor i, also called the *out-degree* of i, is denoted $X_{i+} = \sum_j X_{ij}$, and the *in-degree*, which is the number of incoming ties, is $X_{+i} = \sum_j X_{ji}$.

For the data structure, it is assumed that there are two or more repeated observations of the network. Observation moments are indicated by $t_1, t_2, \ldots, t_M$ with $M \geq 2$. Besides the network, there may be other variables which can depend on the actors (*monadic or actor covariates*) or on pairs of actors (*dyadic covariates*).

2.2 *Actor-based Models*

One of the difficulties for the use of network analysis in political science is the fact that network data by their nature are dyadic, i.e., refer to pairs of actors, whereas the natural theoretical unit for political science is the actor. This issue is discussed more generally for the social sciences by Emirbayer and Goodwin (1994). For modeling network dynamics, however, a natural combination of network structure and individual agency is possible by basing the model on the postulate that creation and termination of ties are initiated by the actors, as was proposed by Snijders (1996). Here the model is presented for binary directed networks, where it is postulated that changes of ties are under the control of the sending actor. This model is explained more fully in Snijders (2001) and Snijders, van de Bunt and Steglich (2010). In Section 3 a model for non-directed networks is presented.

Like for other statistical models, a number of simplifying assumptions are made.

1. Between observation moments t_1, t_2, etc., time runs on, and changes in the network can and will take place without being directly observed. Thus, while the observation schedule is in discrete time, an underlying process of network evolution is assumed to take place with a continuous time parameter $t \in [t_1, t_M]$.

2. At any given time point $t \in [t_1, t_M]$ when the network changes, not more than one tie variable X_{ij} can change.

3. The probability that a particular variable X_{ij} changes depends on the current state of the network, and not on earlier preceding states.

Assumptions 1 and 3 are expressed mathematically by stating that the network model is a continuous-time Markov process. Assumption 2 simplifies the elements of change to the smallest possible constituent: the creation or termination of a single tie. These assumptions rule out coordination or negotiation between actors. They were proposed as basic simplifying postulates already by Holland and Leinhardt (1977). In future models it will be interesting to allow coordination between actors, but the postulates used here can be regarded as a natural first step to modeling network dynamics.

These three assumptions imply that actors make changes in reaction to each others' changes in between observations. This has strong intuitive validity for many panel observations of networks. For repeated measures of networks created in one step for each new observation, they provide a convenient approximation which has the big advantage that the model is described totally by defining the probability of single tie changes.

The model is actor-based in the sense that tie changes are modeled as the result of choices made by the actor sending the tie. This model is split into two components: timing and choice. The timing component is defined in terms of *opportunities for change*, not in terms of actual change. This is to allow the possibility that an actor is satisfied with the current situation and does not make a tie change, although the opportunity is there.

4. Consider a given current time point t, $t_m \leq t < t_{m+1}$, and denote the current state of the network by $x = X(t)$. Each actor i has a *rate of change*, denoted $\lambda_i(x; \alpha, \rho_m)$, where α and ρ_m are statistical parameters.

5. The waiting time until the next opportunity for change by any actor has the exponential distribution,

$$\begin{aligned} &\mathrm{P}\{\text{Next opportunity for change after } t \text{ is before } t + \Delta t\} \\ &\qquad = 1 - \exp(-\lambda\,\Delta t)\ , \end{aligned} \tag{1}$$

with parameter $\lambda = \lambda_+(x; \alpha, \rho_m)$.

6. The probability that the next opportunity for change is for actor i is given by

$$\mathrm{P}\{\text{Next opportunity for change is by actor } i\} = \frac{\lambda_i(x; \alpha, \rho_m)}{\lambda_+(x; \alpha, \rho_m)}\ . \tag{2}$$

7. Each actor i has an *objective function* $f_i(x; \beta)$ defined on the set of all possible networks x, which may be regarded as the net result of short-term goals and restrictions, determining the probability of the next tie change by this actor; where β is a statistical parameter.

8. To define this probability, the following notation is used. For digraphs x and $i \neq j$, by $x^{(\pm ij)}$ we define the graph which is identical to x in all tie variables except

those for the ordered pair (i, j), and for which the tie variable $i \rightarrow j$ is just the opposite of this tie variable in x, in the sense that $x_{ij}^{(\pm ij)} = 1 - x_{ij}$. In other words, the digraph $x^{(\pm ij)}$ is the same as digraph x except that the tie variable from i to j is toggled. Further, we define $x^{(ii\pm)} = x$ (just as a convenient formal definition). Assume that, at the moment of time $t + \Delta t$ (see point 5) with current network $X(t) = x$, actor i has the opportunity for change. Then the probability that the tie variable changed is X_{ij}, so that the network x changes into $x^{(\pm ij)}$, is given by

$$\frac{\exp\big(f_i(x^{(\pm ij)}; \beta)\big)}{\sum_{h=1}^{n} \exp\big(f_i(x^{(\pm ih)}; \beta)\big)} \ . \tag{3}$$

Expression (3) is the multinomial logit form which is obtained when it is assumed that i makes the choice to toggle the variable X_{ij} that maximizes the objective function of the resulting state plus a random residual,

$$f_i(x^{(\pm ij)}; \beta) + R_j \ ,$$

where the variables R_j are independent and have a standard Gumbel distribution (for a proof, see Maddala, 1983). Thus, this model can be regarded as being obtainable as the result of *myopic stochastic optimization*. Game-theoretical models of network formation often use myopic optimization, e.g., Bala and Goyal (2000).

For extensions of this model without antisymmetry between creating a new tie and terminating an existing tie, see the treatment in Snijders, van de Bunt and Steglich (2010) of the endowment function.

2.3 *Specification of the Actor-based Model*

The specification of the actor-based model amounts to the choice of the rate function $\lambda_i(x; \alpha, \rho_m)$ and the objective function $f_i(x; \beta)$. This choice should be based on theoretical considerations, knowledge of the subject matter, and the hypotheses to be investigated. The focus of modeling normally is on the objective function, as this reflects the choice part of the model.

In many situations, a simple specification of the rate function suffices:

$$\lambda_i(x; \alpha, \rho_m) = \rho_m \ . \tag{4}$$

Note that m is defined above as the index of the observation t_m such that for the current time point t, it holds that $t_m \le t < t_{m+1}$. Including the parameter ρ_m allows to fit exactly the observed number of changes between t_m and t_{m+1}. In model (4), the parameter α is not used and may be omitted from the notation. In some other situations, however, the actors differ in the frequency with which they make changes, e.g., because of the differential resources they devote to optimizing their network position. When this is the case, often this is reflected by the out-degree x_{i+}, and appropriate specifications could be

$$\lambda_i(x;\alpha,\rho_m) = \rho_m \exp(\alpha x_{i+}) \;\text{ or }\; \lambda_i(x;\alpha,\rho_m) = \rho_m \exp(\alpha/(x_{i+}+1)) \;.$$

The exponential function ensures that the rate function is positive. The rate function can also depend on actor covariates. An example where it depends on a covariate v_i and on the out-degree is

$$\lambda_i(x;\alpha,\rho_m) = \rho_m \exp(\alpha_1 v_i + \alpha_2/(x_{i+}+1)) \;.$$

The more important part of the model specification is the objective function. Like in generalized linear modeling, a convenient type of function is the linear combination

$$f_i(x;\beta) \;=\; \sum_{k=1}^{K} \beta_k s_{ki}(x) \;, \tag{5}$$

where the $s_{ki}(x)$ are functions of the network, as seen from the point of view of actor i. These functions are called *effects*. When parameter β_k is positive, tie changes will have a higher probability when they lead to higher values of the effects $s_{ki}(x)$ – and conversely for negative β_k.

Some possible effects are the following. First we discuss some effects depending on the network only, which are important for modeling the dependence between network ties.

1. A basic component is the outdegree, $s_{1i}(x) = \sum_j x_{ij}$. This effect is analogous to a constant term in regression models, and will practically always be included. It fits the level and tendency of the average degree.

2. Reciprocation of choice is a fundamental aspect of almost all directed social networks, because network ties almost always entail some kind of exchange with a tendency toward reciprocity. This is reflected by the the reciprocated degree, $s_{2i}(x) = \sum_j x_{ij}\, x_{ji}$, the number of reciprocal ties in which actor i is involved.

3. The local structure of networks is determined by triads, i.e., subgraphs on three nodes (Holland and Leinhardt, 1975). A first type of triadic dependency is transitivity, in which the pattern $i \to j \to h$ tends to imply the direct tie $i \to h$. This tendency is captured by $s_{3i}(x) = \sum_{j,h} x_{ij}\, x_{jh}\, x_{ih}$, the number of transitive triplets originating from actor i.

   Theoretical arguments were formulated already by Simmel (1917), who discussed the consequences of triadic embeddedness on bargaining power of the social actors and on the possibilities of conflicts. Coleman (1988) stressed the importance of triadic closure for social control, where actor i, who has access to j as well as h, has the potential to sanction them in case j behaves opportunistically with respect to h. For networks between individuals, transitivity is found with overwhelming strength (e.g., Davis, 1970). Empirical confirmation for networks of alliances between firms was found, e.g., by Gulati (1998) and Gulati and Gargiulo (1999).

4. Another triadic configuration is the three-cycle, defined by the ties $i \to j \to h \to i$, reflected by the effect $s_{4i}(x) = \sum_{j,h} x_{ij}\, x_{jh}\, x_{hi}$. This can represent generalized exchange (e.g., Molm, Collett, and Schaefer, 2007), or redundancy of exchange flows. If the relation under study has an aspect of deference or hierarchy, then an avoidance of three-cycles is expected. Indeed, a tendency toward transitivity combined with a tendency away from three-cycles can be interpreted as a local (i.e., triadic) representation of a hierarchically ordered structure. Davis (1970) found wide confirmation of the avoidance of three-cycles in empirically observed networks.

In- and out-degrees are fundamental aspects of individual network centrality (Freeman, 1979). They reflect access to other actors and often are linked quite directly to opportunities as well as costs of the network position of the actors. Degrees may be indicators for influence potential, success (de Solla Price, 1976), prestige (Hafner-Burton and Montgomery, 2006), search potential (Scholz, Berardo and Kile, 2008), etc., depending on the context. Accordingly, probabilities of tie creation and dissolution may depend on the degrees of the actors involved. This is expressed by *degree-related effects*, such as the following.

5. In-degree popularity, indicating the extent to which those with currently high in-degrees are more popular as receivers of new ties. This can be expressed by $s_{5i}(x) = \sum_j x_{ij}\, x_{+j}$, the sum of the in-degrees of those to whom i has a tie. When in-degrees are seen as success indicators, this can model Merton's (1968) Matthew effect, which was used by de Solla Price (1976) in his network model of cumulative advantage, rediscovered by Barabási and Albert (1999). This is an example of an effect with emergent (micro-macro) consequences: if individual actors have a preference for being linked to popular (high-indegree) actors, the result is a network with a high dispersion of in-degrees.
   Since degrees may often have diminishing returns, as argued by Hicklin, O'Toole and Meier (2008), an alternative mathematical specification of this effect as defined by $s'_{5i}(x) = \sum_j x_{ij} \sqrt{x_{+j}}$ may also be useful.

6. Out-degree popularity, indicating the extent to which those with currently high out-degrees are more popular as receivers of new ties. This can be expressed by $s_{6i}(x) = \sum_j x_{ij}\, x_{j+}$ (where again the square root transformation might be applied to express diminishing returns of degrees x_{j+}). For this effect the emergent result is less direct: here, if the parameter β_6 is positive, high out-degrees lead to high in-degrees, so that this will result in a positive correlation between out-degrees and in-degrees.

7. In-degree activity, indicating the extent to which those with currently high in-degrees are more active as senders of new ties. This can be expressed by $s_{7i}(x) = \sum_j x_{ij}\, x_{+i}$.

8. Out-degree activity, indicating the extent to which those with currently high out-degrees are more active as senders of new ties. This can be expressed by $s_{8i}(x) = \sum_j x_{ij}\, x_{i+} = \sum_j x_{i+}^2$. In case of a positive parameter β_8, this effect will

lead to a strong dispersion in the out-degrees.

9. In-in degree assortativity: the tendency of actors with high in-degrees to send ties to others with high in-degrees. This can be specified by the effect $s_{9i}(x) = \sum_j x_{ij}\, x_{+i}\, x_{+j}$, and a positive parameter will lead to networks where the in-degrees of tied partners are correlated. Degree-based assortativity was discussed by Morris (1993) and Newman (2002). Such an effect may obtain, e.g., if in-degree reflects social status and actors have the tendency to prefer ties to others with similar status.
   In-in degree assortativity may be regarded as a kind of interaction between in-degree activity and in-degree popularity, and just like for other interaction effects it is advisable to include this assortativity in a model only if also in-degree activity and in-degree popularity are included.

In addition to these effects based on the network structure itself, research questions will naturally lead to effects depending on attributes of the actors – indicators of goals and resources, etc., defined externally to the network. Since network ties involve two actors, a monadic actor variable v_i will lead to potentially several effects for the network dynamics, such as the following. Here the word 'ego' is used for the focal actor, or sender of the tie; while 'alter' is used for the potential candidate for receiving the tie.

10. The *ego effect* $s_{10i}(x) = \sum_j x_{ij}\, v_i = x_{i+} v_i$, reflecting the effect of this variable on the propensity to send ties, and leading to a correlation between v_i and out-degrees.

11. The *alter effect* $s_{11i}(x) = \sum_j x_{ij}\, v_j$, reflecting the effect of this variable on the popularity of the actor for receiving ties, and leading to a correlation between v_i and in-degrees.

12. The *similarity (homophily) effect*, which implies that actors who are similar on salient characteristics have a larger probability to become and stay connected, as reviewed in general terms by McPherson, Smith-Lovin, and Cook (2001). An example is the finding by Huckfeldt (2001) that people tend to select political discussion partners who are perceived to have expertise and who are perceived to

have similar views: this would be reflected by an ego and a similarity effect with respect to (perceived) expertise. This can be implemented by the effect

$$s_{12i}(x) = \sum_j x_{ij} \left(1 - \frac{|v_i - v_j|}{\text{Range}(v)}\right) ,$$

where Range$(v) = \max_i(v_i) - \min_i(v_i)$.

13. The *ego-alter interaction effect*, represented like a product interaction, $s_{13i}(x) = \sum_j x_{ij}\, v_i v_j$, which is a different way to represent how the combination of the values on the covariate of the sender and the receiver of the potential tie may influence the probability of the creation and maintenance of a tie.

Further, it is possible to include attributes of pairs of actors – of which one example is how they are related in a different network. Such *dyadic covariates* can express, e.g., meeting opportunities (e.g., Huckfeldt, 2009), spatial propinquity (e.g., Baybeck and Huckfeldt, 2002), institutional relatedness, competing for the same resources or scarce outcomes, etc.

14. The *dyadic covariate effect* of a covariate w_{ij} is defined as $s_{14i}(x) = \sum_j x_{ij}\, w_{ij}$.

For further possibilities of model specification, see Snijders, van de Bunt, and Steglich (2010) and the *SIENA* manual (Ripley and Snijders, 2010).

It may be noted that this formulation seems to entail that the objective functions are constant over time and constant across actors. This is not to be taken absolutely, however, as the monadic and dyadic covariates will differentiate between actors and can be defined as time-changing variables. It is more precise, therefore, to say that differences of objective functions between actors and over time are assumed to be captured by available covariates. Since the covariates remain implicit in the notation, they are hidden in the formulae.

2.4 *Parameter Estimation*

If a continuous time record is available from the network evolution process as described above, so that for each tie the exact starting and ending times within the observation period are known, and these starting and ending times are all distinct as assumed by the model even if only by the slightest amounts, then the model can be framed as a generalized linear model and maximum likelihood estimation is possible, in principle, in a straightforward way. This paper, however, focuses on panel data, for which this precise timing available is not available. Estimation in this case is possible by a variety of simulation-based methods.

A method of moments estimator was proposed by Snijders (2001). The method of moments operates in principle by selecting a vector of statistics, one for each parameter coordinate to be estimated, and determining the parameter estimate as the parameter value for which the expected value of this vector of statistics equals the observed value. For the case of this Markov process model for a panel data set, this method is implemented as follows. The observed networks are denoted by $x(t_m)$, while the random networks of which these are realizations are denoted by $X(t_m)$.

Corresponding to the parameter ρ_m, a multiplicative parameter used in (4) and the subsequently given examples for the rate function, the statistic used for estimating this parameter is

$$c_m\big(x(t_m), x(t_{m+1})\big) = \sum_{i,j} \mid x_{ij}(t_{m+1}) - x_{ij}(t_m) \mid \ . \tag{6a}$$

Corresponding to the parameter β_k occurring in the objective function (5), the statistic used is

$$s_k\big(x(t_m)\big) = \sum_i s_{ki}\big(x(t_m)\big) \ . \tag{6b}$$

If ρ_m increases, then $c_m\big(X(t_m), X(t_{m+1})\big)$ is expected to increase. If β_k increases, then $s_k\big(X(t_{m+1})\big)$ is expected to increase for $m = 1, \ldots, M-1$. Therefore the equations defining the moment estimator for parameter θ, defined as a shorthand for $\theta = (\rho_1, \ldots, \rho_{M-1}, \beta_1, \ldots, \beta_K)$, are

$$\mathrm{E}_{\hat\theta}\big\{c_m\big(x(t_m), X(t_{m+1})\big) \mid X(t_m) = x(t_m)\big\} = c_m\big(x(t_m), x(t_{m+1})\big) \qquad (m = 1, \ldots, M-1), \tag{7a}$$

$$\sum_{m=1}^{M-1} \mathrm{E}_{\hat\theta}\big\{s_k\big(X(t_{m+1})\big) \mid X(t_m) = x(t_m)\big\} = \sum_{m=1}^{M-1} s_k\big(x(t_{m+1})\big) \qquad (k = 1, \ldots, K). \tag{7b}$$

The conditionally expected values in (7) cannot be calculated analytically, but can be approximated by Monte Carlo simulation, as the assumptions (1.-8.) described above can be used directly to provide a simulation algorithm for simulating the network $X(t_{m+1})$ given the starting value $X(t_m) = x(t_m)$. An algorithm to approximate the value of $\hat{\theta}$ defined by (7) is described in Snijders (2001).

The Method of Moments estimator has proven to be quite reliable and efficient. More recently, algorithms for likelihood-based estimators have been developed: a Bayes estimator by Koskinen and Snijders (2007) and a Maximum Likelihood estimator by Snijders, Koskinen and Schweinberger (2010).

3 Models for Dynamics of Non-directed Networks

In this section it is assumed that the network is non-directed, i.e., ties have no directionality: $X_{ij} = X_{ji}$ holds by necessity, and the tie variables X_{ij} and X_{ji} are treated as being one and the same variable. This is the case in many types of tie, such as mutual collaboration or agreement. Ties now are indicated by $i \leftrightarrow j$.

3.1 *Two-sided Choices*

For modeling non-directed networks, it is necessary to make assumptions about the negotiation or coordination between the two actors involved in the creation or termination of a tie. In game-theoretic models of networks, it is usually assumed that for a tie to exist, the consent of both actors is involved. This is the basis of Jackson and Wolinsky's (1996) definition of pairwise stability: a network is pairwise stable if no pair of actors can both gain from creation of a new tie between them, and if no single actor can gain from termination of one of the ties in which this actor is involved. We present several models, all based on a two-step process of opportunity and choice, and making different assumptions concerning the combination of choices between the two actors involved in a tie.

For the opportunity, or timing, process, two options are presented.

1. *One-sided initiative*: One actor i is selected and gets the opportunity to make a change.

   This is according to the assumptions 1-6 mentioned above for the directional case.

2. *Two-sided opportunity*: An ordered pair of actors (i, j) (with $i \neq j$) is selected and gets the opportunity to make a new decision about the existence of a tie between them.

   In this case assumptions (1.-3.) are maintained, but (4.-6.) are replaced (in abbreviated description) as follows.

   4.2. Each ordered pair of actors (i, j) has a *rate of change*, denoted $\lambda_{ij}(x; \alpha, \rho_m)$, where α and ρ_m are statistical parameters.

   5.2. The waiting time until the next opportunity for change by any pair of actors has the exponential distribution with parameter $\lambda_{\text{tot}} = \sum_{i \neq j} \lambda_{ij}(x; \alpha, \rho_m)$.

   6.2. The probability that the next opportunity for change is for pair (i, j) is given by

$$\text{P}\{\text{Next opportunity for change is for pair } (i, j)\} = \frac{\lambda_{ij}(x; \alpha, \rho_m)}{\lambda_{\text{tot}}(x; \alpha, \rho_m)} \ . \quad (8)$$

The choice process is modeled as one of three options D(ictatorial), M(utual) and C(ompensatory). We now define, for graphs x and $i \neq j$, by $x^{(+ij)}$ the graph which is identical to x in all tie variables except possibly for the tie between i and j, and to which the tie $i \leftrightarrow j$ is added if it was not already there: $x_{ij}^{(+ij)} = 1$. Thus if $x_{ij} = 0$ then $x^{(+ij)} = x^{(\pm ij)}$; if $x_{ij} = 1$ then $x^{(+ij)} = x$.

In all cases assumption (7.) as defined for the directed case is retained, and assumption (8.) is replaced as indicated below.

D. *Dictatorial*: One actor can impose a decision about a tie on the other.

   Like in the directed case, actor i selects the (myopically) best toggle of a single tie variable X_{ij} given the objective function $f_i(x; \beta)$ plus a random disturbance, and actor j just has to accept. Combined with the two opportunity options, this yields the following cases.

8.D.1. The probability that the tie variable changed is X_{ij}, so that the network x changes into $x^{(\pm ij)}$, is given by

$$p_{ij}(x,\beta) = \frac{\exp\big(f_i(x^{(\pm ij)};\beta)\big)}{\sum_{h=1}^{n}\exp\big(f_i(x^{(\pm ih)};\beta)\big)} \ . \tag{9}$$

8.D.2. The probability that network x changes into $x^{(\pm ij)}$, is given by

$$p_{ij}(x,\beta) = \frac{\exp\big(f_i(x^{(\pm ij)};\beta)\big)}{\exp\big(f_i(x;\beta)\big) + \exp\big(f_i(x^{(\pm ij)};\beta)\big)} \ . \tag{10}$$

M. *Mutual*:

Both actors must agree for a tie between them to exist, in line with Jackson and Wolinsky (1996).

8.M.1. In the case of one-sided initiative, actor i selects the best possible choice, with probabilities (9). If currently $x_{ij} = 0$ so that this means creation of a new tie $i \leftrightarrow j$, this is proposed to actor j, who then accepts according to a binary choice based on objective function $f_j(x;\beta)$, with acceptance probability

$$\mathrm{P}\{j \text{ accepts tie proposal}\} = \frac{\exp\big(f_j(x^{(+ij)};\beta)\big)}{\exp\big(f_j(x;\beta)\big) + \exp\big(f_j(x^{(+ij)};\beta)\big)} \ .$$

If the choice by i means termination of an existing tie, the proposal is always put into effect. Jointly these rules lead to the following probability that the current network x changes into $x^{(\pm ij)}$:

$$p_{ij}(x,\beta) = \frac{\exp\big(f_i(x^{(\pm ij)};\beta)\big)}{\sum_{h=1}^{n}\exp\big(f_i(x^{(\pm ih)};\beta)\big)} \left(\frac{\exp\big(f_j(x^{(+ij)};\beta)\big)}{\exp\big(f_j(x;\beta)\big) + \exp\big(f_j(x^{(+ij)};\beta)\big)}\right)^{1-x_{ij}} \ . \tag{11}$$

(Note that the second factor comes into play only if $x_{ij} = 0$, which implies $x^{(+ij)} = x^{(\pm ij)}$.)

8.M.2. In the case of two-sided opportunity, actors i and j both reconsider the value of the tie variable X_{ij}. Actor i proposes a change (toggle) with probability

(10) and actor j similarly. If currently there is no tie, $x_{ij} = 0$, then the tie is created if this is proposed by both actors, which has probability

$$p_{ij}(x, \beta) = \tag{12a}$$
$$\frac{\exp\big(f_i(x^{(+ij)}; \beta)\big)}{\Big(\exp\big(f_i(x; \beta)\big) + \exp\big(f_i(x^{(+ij)}; \beta)\big)\Big)} \frac{\exp\big(f_j(x^{(+ij)}; \beta)\big)}{\Big(\exp\big(f_j(x; \beta)\big) + \exp\big(f_j(x^{(+ij)}; \beta)\big)\Big)} .$$

If currently there is a tie, $x_{ij} = 1$, then the tie is terminated if one or both actors wish to do this, which has probability

$$p_{ij}(x, \beta) = \tag{12b}$$
$$1 - \frac{\exp\big(f_i(x; \beta)\big)}{\Big(\exp\big(f_i(x; \beta)\big) + \exp\big(f_i(x^{(\pm ij)}; \beta)\big)\Big)} \frac{\exp\big(f_j(x; \beta)\big)}{\Big(\exp\big(f_j(x; \beta)\big) + \exp\big(f_j(x^{(\pm ij)}; \beta)\big)\Big)} .$$

C. *Compensatory*: The two actors decide on the basis of their combined interests. The combination with one-sided initiative is rather artificial here, and we only elaborate this option for the two-sided initiative.

8.C.2. The binary decision about the existence of the tie $i \leftrightarrow j$ is based on the objective function $f_i(x; \beta) + f_j(x; \beta)$. The probability that network x changes into $x^{(\pm ij)}$, now is given by

$$p_{ij}(x, \beta) = \frac{\exp\big(f_i(x^{(\pm ij)}; \beta) + f_j(x^{(\pm ij)}; \beta)\big)}{\exp\big(f_i(x; \beta) + f_j(x; \beta)\big) + \exp\big(f_i(x^{(\pm ij)}; \beta) + f_j(x^{(\pm ij)}; \beta)\big)} . \tag{13}$$

The two model components, rate function and objective function, can be put together by considering the so-called transition rates. These give the basic definitions of the continuous-time Markov processes that result from the assumptions formulated above (cf. Taylor and Karlin, 1998, or other textbooks on continuous-time Markov processes). Given that the only permitted transitions between networks are toggles of a single tie variable, the transition rates can be defined as

$$q_{ij}(x) = \lim_{\Delta t \downarrow 0} \frac{\mathrm{P}\{X(t + \Delta t) = x^{(\pm ij)} \mid X(t) = x\}}{\Delta t} \tag{14}$$

for $i \neq j$. Note that this definition implies that the probabilities of toggling a particular tie variable X_{ij} in a short time interval are approximated by

$$\mathrm{P}\{X(t + \Delta t) = x^{(\pm ij)} \mid X(t) = x\} \approx q_{ij}(x)\, \Delta t \, .$$

The transition rates can be computed from the assumptions using the basic rules of probability.

In this derivation account must be taken of the fact that toggling variable X_{ij} is the same as toggling X_{ji}, and the rules described above give different roles for the first and the second actor in the pair (i, j). For the models with one-sided initiative, the transition rate is

$$q_{ij}(x) = \lambda_i(x; \alpha, \rho_m)\, p_{ij}(x, \beta) + \lambda_j(x; \alpha, \rho_m)\, p_{ji}(x, \beta) \; , \tag{15}$$

and for the models with two-sided opportunity

$$q_{ij}(x) = \lambda_{ij}(x; \alpha, \rho_m)\, p_{ij}(x, \beta) + \lambda_{ji}(x; \alpha, \rho_m)\, p_{ji}(x, \beta) \; . \tag{16}$$

3.2 *Comparison in simple cases*

It may be illuminating to treat the simplest – and thereby trivial – case obtained from these models when the objective function is zero, so that all decisions taken by actors are equiprobable choices in some permitted option set; and the rate function is constant, $\lambda_i = \rho$ or $\lambda_{ij} = \rho$, respectively. Although substantively this is an uninteresting case, it gives insight in the differences in behavior of the models. This specification yields the following transition rates:

D.1. $$q_{ij}(x) = 2\,\rho\,\frac{1}{n} = \frac{2\rho}{n}$$

D.2, C.2. $$q_{ij}(x) = 2\,\rho\,\frac{1}{2} = \rho$$

M.1. $$q_{ij}(x) = \begin{cases} 2\,\rho\,\dfrac{1}{2n} = \dfrac{\rho}{n} & (x_{ij} = 0) \\ 2\,\rho\,\dfrac{1}{n} = \dfrac{2\rho}{n} & (x_{ij} = 1) \end{cases}$$

M.2. $$q_{ij}(x) = \begin{cases} 2\,\rho\,\dfrac{1}{4} = \dfrac{\rho}{2} & (x_{ij} = 0) \\ 2\,\rho\,\dfrac{3}{4} = \dfrac{3\rho}{2} & (x_{ij} = 1) \; . \end{cases}$$

From these expressions for the transition rates we can draw three conclusions.

1. The transition rates do not depend on elements of x other than x_{ij} itself. This means that the $n(n-1)/2$ dyad processes $X_{ij}(t)$ are independent across dyads.

2. Elementary properties of continuous-time Markov chains imply that the tie probabilities $\mathrm{P}\{X_{ij}(t) = 1\}$ have a limiting value which is equal to the ratio of the transition rate for the value 0 changing to 1, compared to the rate of 1 changing to 0. Thus, models D.1, D.2, and C.2 lead to randomly changing graphs with a limiting distribution where each tie has probability 1/2; M.1 leads to a randomly changing graph with a limiting tie probability of 1/3; while M.2 yields a randomly changing graph with a limiting tie probability of 1/4.

3. The transition rates in the one-sided initiative models have a factor $1/n$ which does not occur in the two-sided opportunity models. This reflects the fact that, contrary to the two-sided opportunity models, the one-sided initiative models contain a step of choosing between n options. In models with more general objective functions there will not be a direct proportionality factor of $1/n$, but still the parameter estimates for different models on the same data set will normally be such that the rate function $\lambda_i(x; \alpha, \rho_m)$ for the one-sided initiative model will be much larger than the rate function $\lambda_{ij}(x; \alpha, \rho_m)$ for the two-sided opportunity model. Another way of understanding this is that the former are rates at which a given actor may change any of her/his ties, whereas the latter are rates at which a given dyadic tie variable may change, which happens less frequently.

The second conclusion for this trivial example shows that the requirement in the mutual models that both actors agree with the existence of a tie, leads to lower tie probabilities compared to the dictatorial and compensatory models, given the same objective function. This would not necessarily be the case in modifications of the mutual models where the actors look ahead one step for the behavior of their potential partner, and stochastically maximize the expected objective function one step ahead. Here again there are various different possibilities; we leave this as a topic for future research.

Again a convenient and flexible class of objective functions can be represented by the linear combination (5). The same effects can be used as for directed networks, but some are redundant. For example, the reciprocity effect s_{2i} is the same as the degree effect s_{1i}, for monadic covariates the ego effect s_{10i} is the same as the alter effect s_{11i}, etc.

It is possible to go beyond the trivial case elaborated above, and derive exact or approximate identities between the models in still simple, but nontrivial cases. This may help to obtain some insight into how these different models behave when applied with the same objective function to the same data set. The perhaps clearest example is the following. If all effects s_{ki} included in a model (5) are such that the contributions of ties are the same for both actors involved (which is the case, for example, for the degree effect s_{1i} and the similarity effect s_{12i}), then the compensatory dyadic model C.2 is identical to the dictatorial dyadic model D.2, except that the parameters β_k are twice as small for C.2 compared to D.2., because of the addition of the two objective functions in (13). For general models this identity will not hold, but in a first-order approximation it still may be expected that the β_k parameters in model C.2 are about twice as small as those in D.2, and the ρ_m parameters are quite similar. For the objective function specified as $f_i(x;\beta) = \beta_1 x_{i+}$, models D.2 and C.2 yield graphs with randomly changing tie variables where the limiting tie probability can be an arbitrary number between 0 and 1, depending on the value of β_1.

3.3 *Estimation and Examples*

Method of moment estimators can be obtained for these models in exactly the same way as described in the previous section for models for directed networks. This is because the algorithm for these estimators is based directly on simulation of the network evolution, and the assumptions in this section can be used straightforwardly for simulating the evolution of a non-directed network.

An example of model M.1, which seems theoretically the most appealing version, is given by van de Bunt and Groenewegen (2007) in an interorganizational setting.

4 Models for Co-evolution of Networks and Nodal Attributes

A major reason for the fruitfulness of a network-oriented research perspective is the entwinement of networks and individual behavior, performance, attitudes, etc., of political actors. The effect of peers on individual political behavior is a well-studied issue, starting from Lazarsfeld, Berelson, and Gaudet (1948); a recent example is Klofstad (2007). Huckfeldt (2009) argues that, since social interaction leads to influence with respect to political behaviors, the composition of the social context of individuals influences their own attitudes and behaviors, and he draws attention to the endogeneity of the network of interaction partners. Inter-organizational studies have also drawn attention to the importance of networks for organization-level outcomes. Scholz, Berardo and Kile (2008) show that the position of organizations in general contact networks influences their propensity to collaborate and to perceive agreement between stakeholders. Berardo (2009) shows that cooperation between governmental and nongovernmental organizations enhances organizational performance.

Studying the entwinement of networks and actor-level outcomes is made difficult because of the endogeneity of both: the network affects the outcomes while the outcomes affect the network. One way to get a handle on this is to model these dynamic dependencies both ways in studies of the co-evolution of networks and nodal attributes. This is elaborated here. For the nodal attributes in the role of dependent variables we use the term 'behavior' as a catch-word that also can represent other outcomes such as performance, attitudes, etc.

4.1 *Dynamics of Networks and Behavior*

The modeling framework used above for an evolving network $X(t)$ now is extended by considering a simultaneously and interdependently evolving vector of H behavior variables $Z(t) = (Z_1(t), \ldots, Z_H(t))$. The value of the h'th variable for the i'th actor is denoted $Z_{ih}(t)$. We assume that all components of the behavior vector $Z(t)$ are *ordinal discrete variables* with values coded as an interval of integers.

For modeling the joint dynamics of the network and behavior $\big(X(t), Z(t)\big)$, we follow the same principles as those used to model the development of $X(t)$ alone: time t is a continuous parameter; changes in network and behavior can take place at arbitrary moments between observations; at any single time point, only one variable can change, either a tie variable X_{ij} or a behavior variable Z_{ih}; and the process $\big(X(t), Z(t)\big)$ evolves as a Markov process, i.e., change probabilities depend on the current state of the process, not on earlier states. The principle of decomposing the dynamics in the smallest possible steps is carried further by requiring that a change of a behavior variable at one single moment can only be one step up or down the ladder of ordered values – i.e., by a value ± 1, as these variables have integer values.

These principles are elaborated by Snijders, Steglich and Schweinberger (2007) and Steglich, Snijders and Pearson (2010) in a model that has the following basic components.

- For the network changes the *network rate function* $\lambda_i^X(x, z; \alpha^X, \rho^X)$ indicates the average frequency with which actor i has the opportunity to make changes in one outgoing network variable.
- For each behavior variable Z_h the *behavior rate function* $\lambda_i^{Zh}(x, z; \alpha^{Zh}, \rho^{Zh})$ indicates the average frequency with which actor i has the opportunity to make changes in this behavior variable.
- For the network the *network objective function* $f_i^X(x, z; \beta^X)$ reflects the net result of short-term goals and restrictions, determining the probability of the next tie change by actor i.
- For each behavior variable Z_h the *behavior objective function* $f_i^{Zh}(x, z; \beta^{Zh})$ reflects the net result of short-term goals and restrictions, determining the probability of the next behavior change by actor i.

The network dynamics proceeds just as defined above for the network-only case. The behavior dynamics is analogous. Here the option set for the decision of change is different, however, in the following way. For notational simplicity, we give the formulae only for the case of $H = 1$ dependent behavior variable. In a process driven by the rate functions $\lambda_i^Z(x, z; \alpha^Z, \rho^Z)$, actor i now and then gets the opportunity to change the value of her behavior Z_i. When this happens, and the current value is denoted z, the actor has three options: increase by 1, stay constant, or decrease by 1. If the current value is at the minimum or maximum of the range, one of these options is excluded. Of the three (or two) allowed new values z', the actor chooses by myopic stochastic optimization: the value z' is selected that has the highest value of the objective function of the new state plus a random residual,

$$f_i^Z(x, z'; \beta^Z) + R_j^Z \ ,$$

where again the variables R_j^Z are independent and have a standard Gumbel distribution. The resulting choice probabilities again have a multinomial logit form, the probability of choosing z' (with permitted values $z-1, z, z+1$) being

$$\frac{\exp\left(f_i^Z(x, z'; \beta^Z)\right)}{\sum_{d=-1}^{1} \exp\left(f_i^Z(x, z+d; \beta^Z)\right)} \ , \tag{17}$$

with obvious modifications in case z is at the boundary of its range.

This model for the co-evolution of networks and behavior permits the expression of both *social selection* (e.g., homophilous selection), where the values of Z_{ih} and Z_{jh} influence the probability of creating, or of maintaining, a tie from i to j; and of *social influence*, or contagion, where for actor i the probability of changes in Z_{ih} depends on the behaviors Z_{jh} of those actors j with whom i is tied.

4.2 *Specification of Behavior Dynamics*

The main extra component of the model specification regards the objective function for behavior. Here also we use notation just for one single behavior variable Z. Again, a linear combination is considered:

$$f_i^Z(x, z; \beta^Z) \ = \ \sum_k \beta_k^Z s_{ki}^Z(x, z) \ , \tag{18}$$

where the effects $s_{ki}^Z(x, z)$ depend on the network and the behavior. A baseline is a quadratic function of the actor's own behavior as the expression of short-term goals and restrictions.

1. This includes the linear term $s^Z_{1i}(x, z) = z_i$, and

2. the quadratic term $s^Z_{2i}(x, z) = z_i^2$.

Several statistics could be specified to represent social influence (contagion), such as the following two.

3. The similarity between the behavior of actor i and the actors to whom i is tied, measured just like the analogous effect s_{12i} for the network dynamics,
$$s^Z_{3i}(x) = \sum_j x_{ij} \left(1 - \frac{|z_i - z_j|}{\text{Range}(z)} \right) .$$

4. The product of the own behavior z_i with the average behavior of the other actors to whom i is tied, $s^Z_{4i}(x, z) = z_i \big(\sum_j x_{ij}\, z_j \big) / \big(\sum_j x_{ij} \big)$ (defined as 0 if this is 0/0). Together with the two terms s^Z_{1i} and s^Z_{2i}, this yields a quadratic function of which (if the coefficient of s^Z_{2i} is negative) the location of the maximum is a linear function of the average behavior in the 'personal network' of i.

The effects s^Z_{3i} and s^Z_{4i} both express the concept of social influence, albeit in different mathematical ways. The choice between them can be based on theoretical grounds, if any theoretical preferences exist – else on empirical grounds.

The behavior dynamics can also depend on network position directly, for example, on the degrees of the actor.

5. It can depend, e.g., on the 'popularity' of actor i as measured by the indegree, i.e., the number of incoming ties, $s^Z_{5i}(x) = z_i\, x_{+i}$, and/or

6. on the 'activity' of actor i as measured by the outdegree, i.e., the number of outgoing ties, $s^Z_{6i}(x) = z_i\, x_{i+}$.

In addition, it will often be important to include effects of other actor-level variables on z_i.

The parameter estimation for this model is treated in Snijders, Steglich, and Schweinberger (2007).

5 Discussion

Network-related research questions lead to various issues at the interface between theory and methodology – in political as well as other sciences. One issue is how to make the combination of, on the one hand, theories in which individual actors have primacy and which recognize the embeddedness in the social context (cf. DiPrete and Forrestal, 1994; Udehn, 20002; Huckfeldt, 2009) and, on the other hand, empirical research with data sets including dyadic as well as monadic variables. Another issue is the fact that hypotheses about dyadic relations between social actors usually will imply non-independence between dyadic tie variables, and also between dyadic and monadic variables, which requires new and perhaps unusual statistical methods. This non-independence sometimes can be regarded as a consequence of endogeneity, i.e., resulting from different but interdependent choices: for example, in studies of how actors are influenced by those actors to whom they are tied it is important to recognize that the network may be endogenous, and in studies of homophilous choice of interaction partners the behavior that is the dimension for homophily may be endogenous. Dropping the assumption of independence implies that the dependence between variables has to be specified in a plausible way in order for the statistical analysis to be reliable. However, our theories mostly give only a very incomplete handle on this specification; few statistical models representing dependencies between dyadic variables have been proposed, and the available models are currently in various stages of development; and as yet we know little about the sensitivity of conclusions for the misspecification of such statistical models.

The methods discussed and proposed in this article are meant to provide an inroad to tackling these two issues for panel data on ‘complete networks’. The latter specification means that the network consists of the pattern of ties between all actors in a well-delineated group, and ties of these actors with others outside the group may be ignored: the network boundary problem (Marsden, 2005) is assumed to have been solved in an earlier phase of the research. Within these models the wider context outside of this group, to which every group member is exposed, is therefore kept constant, and its influence is not considered. This implies, in terms of Huckfeldt's (2009: 928) statement that '(p)olitical communication networks are created as the complex product of this intersection between human choices and environmentally imposed options', that the methods treated here focus on the 'human choice' component but not on effects of the composition of the network.

The models presented here are for data of dynamics of binary networks, and for the interdependent dynamics of networks and individual outcome variables ('behavior') that can be represented by ordinal discrete variables. They are implemented in the program *SIENA*, 'Simulation Investigation for Empirical Network Analysis', which is available as an R package (Ripley and Snijders, 2010). These models were first proposed, respectively, in Snijders (2001) and Snijders, Steglich and Schweinberger (2007), and have been applied across the social sciences. The model for dynamics of non-directed networks is new, although an application to choice by firms of collaboration partners already appeared in van de Bunt and Groenewegen (2007). Work is in progress to extend these models to more general types of data, such as valued networks and multivariate networks.

The definition of this model in terms of choices by individual actors means that changing dyadic and monadic variables can be analyzed in a coherent framework according to theories where the analytical primacy is with structurally embedded individual actors, in line with Udehn's (2002) remarks on structural individualism. The fact that choices are assumed to be myopic does not exclude strategic considerations, but means that these have to be represented by the short-term goals through which actors attempt to reach their long-term objectives. Theoretical arguments given in the literature for the occurrence of structural effects such as reciprocity and transitivity are mostly based on their importance as intermediate goals serving the purpose of ulterior objectives; this is the case, e.g., for Coleman's (1988) argument that transitivity (triadic closure) gives opportunities for social control and sanctioning, as well as for Burt's (2002) theory about the importance of structural holes as a means for obtaining positional advantage. Nevertheless it would be interesting to extend the myopic models presented here to models that incorporate some kind of foresight or explicit strategic interdependence.

The definition of the model for repeated observations in discrete time as a probability model for unobserved small changes occurring in continuous time, is a principle that goes back at least to Coleman (1964), and that was proposed for network dynamics by Holland and Leinhardt (1977). This idea is generally useful to obtain relatively simple representations for time series incorporating feedback between multiple variables and for unequally spaced time series. An overview is given by Singer (2008).

A variety of models have been proposed, and used in the political science literature, for treating network data. To identify the position occupied by the models treated here in the wider array of statistical models for network data, four broad categories of methods may be discerned for dealing with network dependencies. With 'models for network data' we refer to statistical models where the dependent variable is dyadic and can be represented by X_{ij}, where i and j range in a common set of actors. In discussing these four approaches we amalgamate models for cross-sectional and for longitudinal network data.

A first approach is to compute variables representing network structure and use these among the independent variables in otherwise traditional statistical models, assuming independent residuals. An example is the paper by Hafner-Burton and Montgomery (2006) who used pooled time-series analysis for binary dyadic outcomes. This is flexible because the network variables can be calculated in any way that follows from the research questions; but this also is risky because the residuals are likely to be dependent, e.g., residuals for the X_{ij} with the same sender i or the same receiver j may be dependent, and there may be more complicated types such as reciprocal and triadic dependencies. This will often lead to misspecification of models postulating independent residuals, and hence to the possibility of incorrect type-I error rates of hypothesis tests. This approach can be criticized also on conceptual grounds, because the use of network-based variables presupposes dependencies between dyads, which however are ignored in the specification of the distribution for the residuals.

A second approach is to take account of network dependencies without explicitly modeling them. The most well-known method here is the quadratic assignment procedure (Krackhardt, 1988; Dekker, Krackhardt, and Snijders, 2007) which is a permutation test respecting the network structure. This procedure was applied by Shrestha and Feiock (2009) in a study of multivariate service networks. In many situations this approach, when applied properly, leads to tests with approximately the correct type-I error rate for hypotheses about relations between dyadic variables observed in a network structure. Since the network structure is not modeled explicitly, this approach does not allow testing hypotheses about network dependencies such as reciprocity or transitivity.

The other two approaches represent network dependencies explicitly. One possibility to do this is by latent variable models. Hoff and Ward (2004) modeled international relations using models with random effects for countries. Hoff, Raftery and Handcock (2002) proposed a model for network data where the nodes are represented by latent positions in Euclidean space, assuming that nearby actors have a larger probability of being tied, so that the latent positions can represent transitivity and clustering. This model was adapted to rank data by Gormley and Murphy (2007) and applied to Irish election data. The fourth possibility, finally, is to model dependencies as such. For cross-sectional network studies, this can be done by the Exponential Random Graph Model or p^* model, see Wasserman and Pattison (1996) and Snijders, Pattison, Robins, and Handcock (2006). This model was applied by Thurner and Binder (2001) in a study of transgovernmental networks between the EU member states. For longitudinal network studies, the actor-based models presented are examples of this approach. These models can represent network dependencies in many differentiations; for example, they allow to test hypotheses about triadic closure and three-cycles. Whether the third or fourth model gives a better representation of network dependencies is an empirical question which has not yet been investigated, so it seems. The quality of this representation will also depend strongly on the model specification within either of these model types, and on the data set. For some research questions the representation with random effects and latent spatial positions will be more natural, for others the representation by differentiated types of network dependencies.

The first of these four approaches may be considered more and more to be inadequate, given the risks of misspecification and the availability of specific network-oriented methods. The other three approaches all have their own pros and cons and domains of applicability, and all may be expected to be fruitful for research in political science.

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