Analyzing the Joint Dynamics of Several Networks

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Multiple Networks

Social actors are embedded in multiple networks
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friendship, esteem, collaboration, advice, enmity, ...
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collaborative projects, client referral, information sharing, ...
Multiple Networks

Social actors are embedded in multiple networks:

- friendship, esteem, collaboration, advice, enmity, ...
- friendship, bullying, defending, dislike, ...
- collaborative projects, client referral, information sharing, ...

When studying network dynamics, studying between-network dependencies can be illuminating.
A multiple or multivariate social network is a set of $n$ social actors, on which $R$ relations are defined (Wasserman & Faust, 1994; Pattison & Wasserman, 1999).
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The study of multiple networks is quite traditional: e.g., White, Boorman & Breiger (1976); Boorman & White (1976); Pattison (1993); later on, authors including Ibarra, Krackhardt, Padgett, Lazega, Lomi, did empirical research on multiple networks.
... on the variety of how relations can affect relations ...
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(cf. also the algebraic approach; e.g., work by Pattison & Breiger.)
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It’s a multilevel issue (but not nested):

ties, dyads, actors, triads, subgroups, ...
Different relations can impinge on one another in many different ways.

Example: friendship $\Rightarrow$ advice asking; ego is $\otimes$.

*In the first place, within-dyad.*

direct association (within tie)
‘friends become’
Different relations can impinge on one another in many different ways.

Example: *friendship* \(\Rightarrow\) *advice asking*; ego is \(\otimes\).

*In the first place, within-dyad.*

direct association (within tie) ‘friends become advisors’

\[\begin{array}{c}
\otimes \\
\end{array}\]
Different relations can impinge on one another in many different ways.

Example: friendship $\Rightarrow$ advice asking; ego is $\bigotimes$.

*In the first place, within-dyad.*

direct association (within tie)
‘friends become advisors’

mixed reciprocity
‘friendship reciprocated’
Different relations can impinge on one another in many different ways.

Example: friendship $\Rightarrow$ advice asking; ego is $\otimes$.

In the first place, within-dyad.

- Direct association (within tie) ‘friends become advisors’
- Mixed reciprocity ‘friendship reciprocated by asking advice’
A second category operates via actors.

mixed popularity
‘those popular as friends

⨂
A second category operates via actors.

mixed popularity
‘those popular as friends are asked a lot for advice’
A second category operates via actors.

mixed popularity
‘those popular as friends
are asked a lot for advice’

mixed activity
‘those mentioning many friends
’
A second category operates via actors.

mixed popularity
‘those popular as friends are asked a lot for advice’

mixed activity
‘those mentioning many friends also mention many advisors’
Next category: triads.

mixed transitive closure
‘friends of friends
Next category: triads.

mixed transitive closure
‘friends of friends
become advisors’
Next category: triads.

mixed transitive closure
‘friends of friends
become advisors’

agreement
‘those with the same friends
Next category: triads.

mixed transitive closure
‘friends of friends become advisors’

agreement
‘those with the same friends become advisors’
More triads.

other mixed transitive closure
‘advisors of friends

Actor orientation: only the bottom tie is the dependent variable.
More triads.

other mixed transitive closure
‘advisors of friends
become advisors’

Actor orientation: only the bottom tie is the dependent variable.
More triads.

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Actor orientation: only the bottom tie is the dependent variable.

And there are more mixed triads.
This type of cross-network dependencies is discussed for cross-sectional observations in Wasserman & Pattison (1999), with examples in Lazega & Pattison (1999).

For longitudinal observations the dependencies are multiplied, because we must distinguish between the dependent and the explanatory (antecedent – subsequent) relations.

This can also be applied to *signed graphs* in which case balance theory can be applied.
In addition, the actors in the network can be affiliated with various groupings or events:
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this can be represented by two-mode (‘bipartite’) networks, where there are
a set $\mathcal{N}$ of actors (the ‘actor mode’) and a set $\mathcal{M}$ of groupings (the ‘group mode’);
and the tie $i \rightarrow j$ for $i \in \mathcal{N}, j \in \mathcal{M}$ means that $i$ is a member of grouping $j$. 
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For the combination of a one-mode and a two-mode network, other mutual influences between the networks are possible.

*skip bipartite influence models*
dependencies in bipartite networks

*Within-dyad* dependencies are undefined.
dependencies in bipartite networks

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*Actor-level* dependencies are meaningful.
dependencies in bipartite networks

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mixed activity
dependencies in bipartite networks

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mixed activity

\[ \otimes \]

\[ \downarrow \]

\[ \bullet \]

mixed popularity

\[ \otimes \]

\[ \uparrow \]

\[ \bullet \]

⇒ activity

\[ \bullet \]
dependencies in bipartite networks

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⇒ activity
Transitivity for bipartite networks: 4-cycles

An interlude:
for bipartite networks, other structures are important than for one-mode networks.
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We meet each other in various groups.
Transitivity for bipartite networks: 4-cycles

An interlude:
for bipartite networks, other structures are important
than for one-mode networks.

We meet each other
in various groups.

Robins and Alexander (2004):
transitivity in bipartite networks expressed by 4-cycles.
Closed triads are impossible in bipartite networks; but they are possible as mixed patterns.

*One-with-two-mode triads.*

One-mode tie ⇒

two-mode agreement

where my friends are’
Closed triads are impossible in bipartite networks; but they are possible as mixed patterns.

*One-with-two-mode triads.*

One-mode tie $\rightarrow$

two-mode agreement

'I go to places where my friends are'
Closed triads are impossible in bipartite networks; but they are possible as mixed patterns.

*One-with-two-mode triads.*

One-mode tie $\Rightarrow$

- two-mode agreement

'I go to places where my friends are'

Two-mode agreement $\Rightarrow$

one-mode tie

'Those who go to the same places'
Closed triads are impossible in bipartite networks; but they are possible as mixed patterns.

*One-with-two-mode triads.*

*One-mode tie* \( \Rightarrow \) 
two-mode agreement

’I go to places where my friends are’

*Two-mode agreement* \( \Rightarrow \) 
*one-mode tie*

‘Those who go to the same places become friends’
Closed triads are impossible in bipartite networks; but they are possible as mixed patterns.

One-with-two-mode triads.

One-mode tie \implies two-mode agreement

'I go to places where my friends are'

Two-mode agreement \implies one-mode tie

'Those who go to the same places become friends'
... outline of further presentation ...
specify statistical model:
actor-based model for multiple networks;
... outline of further presentation ...

1 specify statistical model:
actor-based model for multiple networks;

2 sketch procedure for parameter estimation;
... outline of further presentation ...

1. specify statistical model: actor-based model for multiple networks;
2. sketch procedure for parameter estimation;
3. example.
Actor-based models

Actor-based models are defined here as extensions of actor-based models for dynamics of single networks (Snijders 1996, 2001).
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4. The multiple relations together develop stochastically according to a Markov process.
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1. The actors control their outgoing ties.
2. For panel data: employ a continuous-time model to represent unobserved endogenous network evolution.
3. The ties have inertia: they are *states* rather than *events*.
4. The multiple relations together develop stochastically according to a Markov process.
5. At any single moment in time, only one tie variable may change: no coordination.
Changes in each network are modeled as choices by actors in their outgoing ties, with probabilities depending on ‘objective functions’ of the network state that would obtain after this change.

These objective (‘goal’) functions are specified separately for each of the $R$ networks.
Notation

Denote tie variable for \( r^{th} \) relation from \( i \) to \( j \) by

\[
X_{ij}^{(r)} = \begin{cases} 
1 & \text{if } i \rightarrow^r j \\
0 & \text{if not } i \rightarrow^r j,
\end{cases}
\]

where this depends on time \( t \).

By \( X \) we denote the collection of all \( R \) relations: array \( \left( X_{ij}^{(r)} \right) \) for \( r = 1, \ldots, R; \ i = 1, \ldots, n; \ j = 1, \ldots, n \)
The statistical model is a *process model*:

an agent-based simulation model, which simulates the development of the multiple networks from one observation to the next;

statistical modeling consists of fitting such a simulation model to the observed network data, and testing which model components are required to give a good fit.
The model is defined by its smallest possible steps, the ‘microsteps’, which consist of a change in one tie variable: extend one new tie / withdraw one existing tie.
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How rapidly does this happen?
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⇒ How rapidly does this happen?
⇒ What is the probability of this particular tie change, compared to other changes?
Decompose model in:

decompose in:

\[ \frac{\text{average frequency of changes}}{\lambda_i(x)} \]

\[ \text{rate functions} \]

\[ \lambda_i(r) \]

\[ \text{rate at which } i \text{ can change } r \text{-relations; and the probabilities of particular changes, objective functions } f_i(r(x)) \]

\[ \text{changes in } r \text{-relations have higher probabilities accordingly as } f_i(r(x)) \text{ would become higher, } \]

\[ \sim \text{myopic optimization of } f_i(r(x)) + \text{error term.} \]
Decompose model in:

the average *frequency* of changes,

rate functions:

\[ \lambda_i^{(r)}(x) = \text{rate at which } i \text{ can change } r\text{-relations}; \]
Decompose model in:

the average \textit{frequency} of changes,

rate functions:
\[ \lambda_i^{(r)}(x) = \text{rate at which } i \text{ can change } r\text{-relations}; \]

and the \textit{probabilities} of particular changes,

objective functions \( f_i^{(r)} \):

changes in \( r\)-relations have higher probabilities accordingly as \( f_i^{(r)}(x) \) would become higher,

\[ \sim \text{myopic optimization of } f_i^{(r)}(x) + \text{error term.} \]
Model for rate of change often can be simple: rate of change $\lambda_i^{(r)}(x)$ depends only on $r$, some relations change faster than others.
Model for rate of change often can be simple: rate of change $\lambda_i^r(x)$ depends only on $r$, some relations change faster than others.

Rate of change of relation $r$ is $\lambda^r = \sum_i \lambda_i^r$; total rate of change is $\lambda^{(+)} = \sum_r \lambda^r$. 
Outline of model dynamics / simulation algorithm

Model for microstep (smallest possible change):
Outline of model dynamics / simulation algorithm

Model for microstep (smallest possible change):

1. Next event takes place after time interval with exponentially distributed length, average duration $\lambda^{(+)}$.  
   **Step:** Increment $t$ by such a random variable.
Outline of model dynamics / simulation algorithm

Model for microstep (smallest possible change):

1. Next event takes place after time interval with exponentially distributed length, average duration $\lambda_+^{(+)}$.  
   **Step:** Increment $t$ by such a random variable.

2. The probability that this is an event where actor $i$ may change an $r$-tie is 

   $$\frac{\lambda_i^{(r)}}{\lambda_+^{(+)}}.$$

   **Step:** Choose $r$, $i$ with this probability.
Outline of algorithm – continued

3 For this \( r \) and \( i \), actor \( i \) may change one outgoing \( r \)-tie, or leave all outgoing tie variables \( X_{ij}^{(r)} \) unchanged. The probability of changing toward any new situation \( x \) (\( x \) differs only in one tie variable from current situation!) is proportional to

\[
\exp \left( f_i^{(r)}(x) \right).
\]
Step: Given that actor $i$ may change a tie in relation $r$, the event that tie variable $X_{ij}^{(r)}$ is toggled
Step: Given that actor $i$ may change a tie in relation $r$, the event that tie variable $X_{ij}^{(r)}$ is toggled ($X_{ij}^{(r)} \Rightarrow 1 - X_{ij}^{(r)}$)
Step: Given that actor $i$ may change a tie in relation $r$, the event that tie variable $X_{ij}^{(r)}$ is toggled ($X_{ij}^{(r)} \Rightarrow 1 - X_{ij}^{(r)}$) has probability

\[
\frac{\exp \left( f_i^{(r)} \left( x \text{ changed in } x_{ij}^{(r)} \right) \right)}{\sum_h \exp \left( f_i^{(r)} \left( x \text{ changed in } x_{ih}^{(r)} \right) \right)}.
\]
Model specification

The objective function can be conveniently modeled as a weighted sum (cf. generalized linear modeling),

\[ f_i^{(r)}(\beta, x) = \sum_{k=1}^{L} \beta_k^{(r)} s_{ik}^{(r)}(x), \]

where \( s_{ik}^{(r)}(x) \) are ‘effects’ and \( \beta_k^{(r)} \) their weights, which jointly drive the dynamics for relation \( r \),
given the current state of this and all other relations.
These effects will represent the ‘internal’ dynamics of the network, as dependent on its own current state and on exogenous variables (‘covariates’);
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and, for multiple dependent networks, also the cross-network dependencies.

Testable hypotheses and ‘control mechanisms’ are represented by the choice of the effects $s_{ik}^{(r)}(x)$. 
Within-network dependencies and covariate effects have been discussed extensively elsewhere.

A few examples of cross-network dependencies are presented, with formulae for $s_{ik}^{(\text{red})}(x)$.

Since this a component of the objective function for $X^{(\text{red})}$, this network is the dependent relation – all others have an explanatory role.
direct association

\[ \sum_{j=1}^{n} x^{(\text{blue})}(i, j) x^{(\text{red})}(i, j) \]
direct association

$$\sum_{j=1}^{n} x^{(\text{blue})}(i, j) x^{(\text{red})}(i, j)$$

mixed reciprocity

$$\sum_{j=1}^{n} x^{(\text{blue})}(j, i) x^{(\text{red})}(i, j)$$
mixed transitive closure

\[ \sum_{j,h=1}^{n} x^{(\text{blue})}(i, h) x^{(\text{blue})}(h, j) x^{(\text{red})}(i, j) \]

Other formulae also are defined by mixed expressions incorporating one network in the ‘dependent’ and the others in ‘explanatory’ roles.
Estimation

Assume that \( \left( X_{ij}^{(r)} \right) \) is observed for time points \( t_1, \ldots, t_M \): panel data (repeated measures) on multiple networks.

The estimation conditions on \( X(t_1) \):
model tendencies of change, not initial state.
Estimation

Assume that \( (X_{ij}^{(r)}) \) is observed for time points \( t_1, \ldots, t_M \): panel data (repeated measures) on multiple networks.

The estimation conditions on \( X(t_1) \):
model tendencies of change, not initial state.

Estimation methods:
ML / Bayesian / Method of Moments.

All are computationally intensive MCMC methods.
Method of Moments is computationally faster and quite efficient.

*go to Example 1*
The Method of Moments operates by equating observed statistics to their expected values given the parameter values. For each parameter there must be a statistic that is sensitive to this parameter.

Consider the case of $M = 2$ observations; the estimation conditions on $X(t_1)$. 
If, for a given dependent network $X^{(r)}$, with objective function

$$f_i^{(r)}(\beta, x) = \sum_{k=1}^{L} \beta_k^{(r)} s_{ik}^{(r)}(x),$$

we consider an effect $s_{ik}^{(r)}(x)$ that depends only on this network $x^{(r)}$ itself, then good results are obtained by using the statistic

$$S_k := \sum_i s_{ik}^{(r)}(X(t_2)),$$

and requiring, as part of the moment equation, that

$$E_{\beta}\{S_k \mid X(t_1)\} = s_k^{\text{observed}}.$$

This can be implemented by an MCMC approximation using the Robbins-Monro method of stochastic approximation.
Now consider a statistic that expresses cross-network dependencies.
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Which statistic is sensitive for the parameters expressing cross-network dependencies?
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Which statistic is sensitive for the parameters expressing cross-network dependencies?

The example will be given for the parameter that is the weight of direct association,

$$\sum_{j=1}^{n} x^{(\text{blue})}(i, j) x^{(\text{red})}(i, j)$$

for the case of $M = 2$ repeated observations.
Consider direct association:

\[ i \rightarrow j \quad \text{leading to} \quad j \rightarrow i \]

The statistic for fitting the corresponding parameter is

\[
\sum_{i=1}^{n} \sum_{j=1}^{n} x^{(\text{blue})}(i, j)(t_1) x^{(\text{red})}(i, j)(t_2)
\]

note the use of \( t_1 \) and \( t_2 \): use explanatory network at previous observation, dependent network at the next.
Example 1

Research with Rafael Wittek and Gerhard van de Bunt. Kidney dialysis department in general hospital; four waves, data collected by Gerhard van de Bunt. 49 employees; 4 waves, separated by a few months.

1. *Communication during work:*
   At least once a week.

2. *Trust:*
   Confiding for personal matters (work-related or private): strong or very strong trust.

3. *Advice:*
   Ask a colleague for advice or help at least once a week.

*go to Example 2*
Descriptives

Average degrees:

- Trust: 12 – 15.
- Advice: 3.7 – 5.4.

About 20 % missing data.
Descriptives

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- Trust: 12 – 15.
- Advice: 3.7 – 5.4.

About 20 % missing data.

Hypotheses, at this moment, are illustrations.
Hypotheses, dep. var. Communication

C1 *Direct association* Advice $\Rightarrow$ *Communication*: Advice requires communication; therefore an advice tie will tend to confirm existing communication ties.

C2 *Mixed reciprocity* Advice $\Rightarrow$ *Communication*: Likewise, an incoming advice tie will confirm communication.
Hypotheses, dep. var. Communication

C1 *Direct association* Advice $\Rightarrow$ Communication:
Advice requires communication; therefore an advice tie will tend to confirm existing communication ties.

C2 *Mixed reciprocity* Advice $\Rightarrow$ Communication:
Likewise, an incoming advice tie will confirm communication.

C3 ? *Direct association* Trust $\Rightarrow$ Communication:
this is debatable, as trust makes communication easier but also may make it less necessary.
Hypothesis 1, dep. var. Trust

\( T1 \) Direct and Reciprocated association

Communication \( \Rightarrow \) Trust: trust can be a byproduct of communication, mutual communication is required to establish trust.

\[
\begin{align*}
(C) & \Rightarrow T \\
\text{(reciprocated } C) & \Rightarrow T
\end{align*}
\]
Hypothesis 1, dep. var. Trust

**T1** *Direct and Reciprocated association*

*Communication ⇒ Trust:*

trust can be a byproduct of communication, mutual communication is required to establish trust.

\[(C) ⇒ T\]

\[(\text{reciprocated } C) ⇒ T\]
Hypothesis 2, dep. var. Trust

\( T_2 \text{ Agreement concerning Communication} \Rightarrow \text{Trust} \)

{others with whom both communicate could provide social control for the event that trust would be breached.}

(\text{agreement C}) \Rightarrow T
Hypothesis 2, dep. var. Trust

\[ T_2 \quad \text{Agreement concerning Communication} \Rightarrow \text{Trust}: \]
others with whom both communicate
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for the event that trust would be breached.

(\text{agreement } C) \Rightarrow T
Hypotheses, dep. var. Advice

Basic notion: advice receiving can be risky with respect to status and obligation to follow advice.

A1 *Direct association* Trust $\Rightarrow$ Advice:
trust is important for managing critical dependencies.
Hypotheses, dep. var. Advice

Basic notion: advice receiving can be risky with respect to status and obligation to follow advice.

A1 Direct association Trust $\Rightarrow$ Advice:
trust is important for managing critical dependencies.

A2 Direct and reciprocal association Communication $\Rightarrow$ Advice:
advice can be a byproduct of direct and reciprocal communication.
Hypotheses, dep. var. Advice

Basic notion: advice receiving can be risky with respect to status and obligation to follow advice.

A1 Direct association Trust ⇒ Advice: trust is important for managing critical dependencies.

A2 Direct and reciprocal association Communication ⇒ Advice: advice can be a byproduct of direct and reciprocal communication.

A3 Agreement concerning Trustees ⇒ Advice: similar trustees are safeguard against potential problems arising in advice relation.
The hypotheses were tested in the following way:

⇒ Specify for each relation a baseline model of how the relation is influenced by itself: effects of outdegree, reciprocity, transitive triplets, 3-cycles, in- and out-degrees;
⇒ include all direct and reciprocal associations between relations;
⇒ include also any additional tested effects.

First, as descriptives, within-relation analyses are presented.
Results: Communication only

<table>
<thead>
<tr>
<th>Effect</th>
<th>par.</th>
<th>(s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Out-degree</td>
<td>−3.513</td>
<td>(0.507)</td>
</tr>
<tr>
<td>Reciprocity</td>
<td>2.390</td>
<td>(0.585)</td>
</tr>
<tr>
<td>Transitive triplets</td>
<td>0.378</td>
<td>(0.131)</td>
</tr>
<tr>
<td>3-cycles</td>
<td>−0.730</td>
<td>(0.376)</td>
</tr>
<tr>
<td>In-degree popularity (√)</td>
<td>0.218</td>
<td>(0.178)</td>
</tr>
<tr>
<td>Out-degree popularity (√)</td>
<td>−0.624</td>
<td>(0.353)</td>
</tr>
<tr>
<td>Out-degree activity (√)</td>
<td>0.542</td>
<td>(0.104)</td>
</tr>
</tbody>
</table>

Reciprocity; transitivity, negative 3-cycles: local hierarchy.

## Results: Trust only

<table>
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<tr>
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<th>(s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Out-degree</td>
<td>−0.751</td>
<td>0.502</td>
</tr>
<tr>
<td>Reciprocity</td>
<td>0.974</td>
<td>0.108</td>
</tr>
<tr>
<td>Transitive triplets</td>
<td>0.158</td>
<td>0.018</td>
</tr>
<tr>
<td>3-cycles</td>
<td>−0.092</td>
<td>0.031</td>
</tr>
<tr>
<td>In-degree popularity (√)</td>
<td>0.136</td>
<td>0.063</td>
</tr>
<tr>
<td>Out-degree popularity (√)</td>
<td>−0.201</td>
<td>0.115</td>
</tr>
<tr>
<td>Out-degree activity (√)</td>
<td>−0.141</td>
<td>0.062</td>
</tr>
</tbody>
</table>

Reciprocity, local hierarchy.

In-degree popularity: self-reinforcing in-degree differentials.
## Results: Advice only

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</tr>
</thead>
<tbody>
<tr>
<td>Out-degree</td>
<td>-2.968</td>
<td>(1.169)</td>
</tr>
<tr>
<td>Reciprocity</td>
<td>2.084</td>
<td>(0.930)</td>
</tr>
<tr>
<td>Transitive triplets</td>
<td>0.415</td>
<td>(0.291)</td>
</tr>
<tr>
<td>3-cycles</td>
<td>0.543</td>
<td>(0.554)</td>
</tr>
<tr>
<td>In-degree popularity (√)</td>
<td>0.581</td>
<td>(0.147)</td>
</tr>
<tr>
<td>Out-degree popularity (√)</td>
<td>-1.527</td>
<td>(1.207)</td>
</tr>
<tr>
<td>Out-degree activity (√)</td>
<td>0.468</td>
<td>(0.143)</td>
</tr>
</tbody>
</table>

Global hierarchy as evidenced by degree-related effects; reciprocity; no significant triadic effects.
## Results: Communication

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<th>(s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Out-degree</td>
<td>-1.625</td>
<td>(0.513)</td>
</tr>
<tr>
<td>Reciprocity</td>
<td>0.457</td>
<td>(0.124)</td>
</tr>
<tr>
<td>Transitive triplets</td>
<td>0.113</td>
<td>(0.015)</td>
</tr>
<tr>
<td>3-cycles</td>
<td>-0.058</td>
<td>(0.029)</td>
</tr>
<tr>
<td>In-degree popularity (✓)</td>
<td>0.296</td>
<td>(0.078)</td>
</tr>
<tr>
<td>Out-degree popularity (✓)</td>
<td>-0.152</td>
<td>(0.141)</td>
</tr>
<tr>
<td>Out-degree activity (✓)</td>
<td>-0.065</td>
<td>(0.054)</td>
</tr>
<tr>
<td>Trust ⇒ Communication (C3)</td>
<td>1.164</td>
<td>(0.190)</td>
</tr>
<tr>
<td>Incoming trust ⇒ Communication</td>
<td>0.419</td>
<td>(0.137)</td>
</tr>
<tr>
<td>Advice ⇒ Communication (C1)</td>
<td>0.938</td>
<td>(0.219)</td>
</tr>
<tr>
<td>Incoming advice ⇒ Communication (C2)</td>
<td>0.442</td>
<td>(0.210)</td>
</tr>
</tbody>
</table>
Trust and advice lead to communication, both directly and reciprocally.
## Results: Trust

<table>
<thead>
<tr>
<th>Effect</th>
<th>par.</th>
<th>(s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Out-degree</td>
<td>-1.590</td>
<td>(0.769)</td>
</tr>
<tr>
<td>Reciprocity</td>
<td>-0.144</td>
<td>(0.350)</td>
</tr>
<tr>
<td>Transitive triplets</td>
<td>0.184</td>
<td>(0.026)</td>
</tr>
<tr>
<td>3-cycles</td>
<td>-0.084</td>
<td>(0.036)</td>
</tr>
<tr>
<td>In-degree popularity (√)</td>
<td>0.017</td>
<td>(0.072)</td>
</tr>
<tr>
<td>Out-degree popularity (√)</td>
<td>-0.125</td>
<td>(0.162)</td>
</tr>
<tr>
<td>Out-degree activity (√)</td>
<td>-0.103</td>
<td>(0.068)</td>
</tr>
<tr>
<td>Communication ⇒ Trust</td>
<td>1.712</td>
<td>(0.460)</td>
</tr>
<tr>
<td>Incoming communication ⇒ Trust (T1)</td>
<td>1.532</td>
<td>(0.426)</td>
</tr>
<tr>
<td>Communic. agreement ⇒ Trust (T2)</td>
<td>-0.084</td>
<td>(0.027)</td>
</tr>
<tr>
<td>Advice ⇒ Trust</td>
<td>-0.032</td>
<td>(0.222)</td>
</tr>
<tr>
<td>Incoming advice ⇒ Trust</td>
<td>0.000</td>
<td>(0.244)</td>
</tr>
</tbody>
</table>
Communication leads to trust, directly and reciprocally; advice does not appear to lead to trust; communicating to the same third parties leads to less trust. Reciprocity and degree-related effects on trust are taken over by effects of advice and communication.
## Results: Advice

<table>
<thead>
<tr>
<th>Effect</th>
<th>par.</th>
<th>(s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Out-degree</td>
<td>-4.473</td>
<td>(0.822)</td>
</tr>
<tr>
<td>Reciprocity</td>
<td>1.704</td>
<td>(0.784)</td>
</tr>
<tr>
<td>Transitive triplets</td>
<td>0.266</td>
<td>(0.195)</td>
</tr>
<tr>
<td>3-cycles</td>
<td>0.532</td>
<td>(0.417)</td>
</tr>
<tr>
<td>In-degree popularity (✓)</td>
<td>0.625</td>
<td>(0.135)</td>
</tr>
<tr>
<td>Out-degree popularity (✓)</td>
<td>-1.310</td>
<td>(0.774)</td>
</tr>
<tr>
<td>Out-degree activity (✓)</td>
<td>0.559</td>
<td>(0.110)</td>
</tr>
<tr>
<td>Communication ⇒ Advice (A2)</td>
<td>1.006</td>
<td>(0.419)</td>
</tr>
<tr>
<td>Incoming communication ⇒ Advice (A2)</td>
<td>0.064</td>
<td>(0.348)</td>
</tr>
<tr>
<td>Trust ⇒ Advice (A1)</td>
<td>0.511</td>
<td>(0.265)</td>
</tr>
<tr>
<td>Incoming trust ⇒ Advice</td>
<td>0.289</td>
<td>(0.295)</td>
</tr>
<tr>
<td>Trust agreement ⇒ Advice (A3)</td>
<td></td>
<td>p = 0.18</td>
</tr>
</tbody>
</table>
Communication leads to advice;
weak evidence that trust leads to advice;
other hypotheses not supported.
Conclusions

As hypothesized, increased/sustained communication follows on advice and trust;

the hypothesis that reciprocal communication is an antecedent for trust is also borne out;

no significant effect for incoming communication on advice, when controlling for direct effect;

reciprocity in trust explained away by advice and communication.
Conclusions — continued

Communicating with same others has a **negative** effect on trust (trusting the same others also leads to more communication);

trusting the same others has no significant effect on advice;
differential out-degrees on communication explained away by joint dynamics depending on trust and advice
Conclusions — continued

Communicating with same others has a \textit{negative} effect on trust (trusting the same others also leads to more communication);

trusting the same others has no significant effect on advice;
differential out-degrees on communication explained away by joint dynamics depending on trust and advice

For further analysis, the \textit{work organization} needs to be taken into account:

hierarchy, sharing shift work.
Example 2

Research with Vanina Torlo and Alessandro Lomi.

International MBA program in Italy;
75 students; 3 waves.

1. *Friendship*

2. *Advice:*
   To whom do you go for help if you missed a class, etc.
Example 2

Research with Vanina Torlo and Alessandro Lomi.

International MBA program in Italy;
75 students; 3 waves.

1. Friendship
2. Advice:
   To whom do you go for help if you missed a class, etc.
3. Two mode: organizational preference:
   in which organizations are you interested
   as potential employer.
   A total of 100 organizations were mentioned.
## Results: Friendship, univariate

<table>
<thead>
<tr>
<th>Effect</th>
<th>par.</th>
<th>(s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Out-degree</td>
<td>−1.840</td>
<td>(0.233)</td>
</tr>
<tr>
<td>Reciprocity</td>
<td>1.604</td>
<td>(0.097)</td>
</tr>
<tr>
<td>Transitive triplets</td>
<td>0.188</td>
<td>(0.017)</td>
</tr>
<tr>
<td>3-cycles</td>
<td>−0.095</td>
<td>(0.030)</td>
</tr>
<tr>
<td>Indegree popularity (√)</td>
<td>0.218</td>
<td>(0.062)</td>
</tr>
<tr>
<td>Outdegree popularity (√)</td>
<td>−0.383</td>
<td>(0.065)</td>
</tr>
<tr>
<td>Outdegree activity (√)</td>
<td>−0.079</td>
<td>(0.041)</td>
</tr>
<tr>
<td>Same nationality</td>
<td>0.240</td>
<td>(0.080)</td>
</tr>
<tr>
<td>Sex alter</td>
<td>−0.016</td>
<td>(0.070)</td>
</tr>
<tr>
<td>Sex ego</td>
<td>−0.158</td>
<td>(0.070)</td>
</tr>
<tr>
<td>Same sex</td>
<td>0.277</td>
<td>(0.065)</td>
</tr>
<tr>
<td>Performance alter</td>
<td>−0.015</td>
<td>(0.023)</td>
</tr>
<tr>
<td>Performance ego</td>
<td>−0.076</td>
<td>(0.024)</td>
</tr>
<tr>
<td>Performance similarity</td>
<td>0.764</td>
<td>(0.188)</td>
</tr>
</tbody>
</table>
## Results: Advice, univariate

<table>
<thead>
<tr>
<th>Effect</th>
<th>par.</th>
<th>(s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Out-degree</td>
<td>−2.267</td>
<td>(0.321)</td>
</tr>
<tr>
<td>Reciprocity</td>
<td>1.329</td>
<td>(0.131)</td>
</tr>
<tr>
<td>Transitive triplets</td>
<td>0.320</td>
<td>(0.038)</td>
</tr>
<tr>
<td>3-cycles</td>
<td>−0.065</td>
<td>(0.061)</td>
</tr>
<tr>
<td>Indegree popularity (✓)</td>
<td>0.245</td>
<td>(0.057)</td>
</tr>
<tr>
<td>Outdegree popularity (✓)</td>
<td>−0.346</td>
<td>(0.143)</td>
</tr>
<tr>
<td>Outdegree activity (✓)</td>
<td>−0.088</td>
<td>(0.062)</td>
</tr>
<tr>
<td>Same nationality</td>
<td>0.450</td>
<td>(0.124)</td>
</tr>
<tr>
<td>Sex alter</td>
<td>−0.043</td>
<td>(0.092)</td>
</tr>
<tr>
<td>Sex ego</td>
<td>−0.269</td>
<td>(0.096)</td>
</tr>
<tr>
<td>Same sex</td>
<td>0.168</td>
<td>(0.086)</td>
</tr>
<tr>
<td>Performance alter</td>
<td>0.129</td>
<td>(0.036)</td>
</tr>
<tr>
<td>Performance ego</td>
<td>−0.107</td>
<td>(0.034)</td>
</tr>
<tr>
<td>Performance similarity</td>
<td>0.735</td>
<td>(0.245)</td>
</tr>
</tbody>
</table>
## Results: Organizational Preference, univariate

<table>
<thead>
<tr>
<th>Effect</th>
<th>par.</th>
<th>(s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Out-degree</td>
<td>-2.595</td>
<td>(0.118)</td>
</tr>
<tr>
<td>Four-cycles</td>
<td>0.090</td>
<td>(0.019)</td>
</tr>
<tr>
<td>Indegree popularity</td>
<td>0.086</td>
<td>(0.014)</td>
</tr>
<tr>
<td>Outdegree activity</td>
<td>0.085</td>
<td>(0.019)</td>
</tr>
<tr>
<td>Outd.-ind. assortativity</td>
<td>-0.012</td>
<td>(0.003)</td>
</tr>
</tbody>
</table>
now the multivariate results ... :
# Results: Friendship (1/2)

<table>
<thead>
<tr>
<th>Effect</th>
<th>par.</th>
<th>(s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Out-degree</td>
<td>-2.980</td>
<td>(0.284)</td>
</tr>
<tr>
<td>Reciprocity</td>
<td>1.280</td>
<td>(0.119)</td>
</tr>
<tr>
<td>Transitive triplets</td>
<td>0.153</td>
<td>(0.018)</td>
</tr>
<tr>
<td>3-cycles</td>
<td>-0.061</td>
<td>(0.032)</td>
</tr>
<tr>
<td>Indegree popularity (✓)</td>
<td>0.386</td>
<td>(0.064)</td>
</tr>
<tr>
<td>Outdegree popularity (✓)</td>
<td>-0.354</td>
<td>(0.073)</td>
</tr>
<tr>
<td>Outdegree activity (✓)</td>
<td>0.023</td>
<td>(0.045)</td>
</tr>
<tr>
<td>Same nationality</td>
<td>0.203</td>
<td>(0.084)</td>
</tr>
<tr>
<td>Sex alter</td>
<td>-0.033</td>
<td>(0.070)</td>
</tr>
<tr>
<td>Sex ego</td>
<td>-0.147</td>
<td>(0.074)</td>
</tr>
<tr>
<td>Same sex</td>
<td>0.237</td>
<td>(0.072)</td>
</tr>
<tr>
<td>Performance alter</td>
<td>-0.022</td>
<td>(0.028)</td>
</tr>
<tr>
<td>Performance ego</td>
<td>-0.098</td>
<td>(0.026)</td>
</tr>
<tr>
<td>Performance similarity</td>
<td>0.789</td>
<td>(0.189)</td>
</tr>
</tbody>
</table>
### Results: Friendship (2/2)

<table>
<thead>
<tr>
<th>Effect</th>
<th>par.</th>
<th>(s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advice ⇒ Friendship</td>
<td>1.672</td>
<td>(0.227)</td>
</tr>
<tr>
<td>Reciprocal advice ⇒ Friendship</td>
<td>0.730</td>
<td>(0.193)</td>
</tr>
<tr>
<td>Advice ind. ⇒ Friendship popularity</td>
<td>–0.151</td>
<td>(0.048)</td>
</tr>
<tr>
<td>Advice outd. ⇒ Friendship activity</td>
<td>–0.214</td>
<td>(0.076)</td>
</tr>
<tr>
<td>Empl. choice outd. ⇒ Friendship activity</td>
<td>0.235</td>
<td>(0.101)</td>
</tr>
</tbody>
</table>
## Results: Advice (1/2)

<table>
<thead>
<tr>
<th>Effect</th>
<th>par.</th>
<th>(s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Out-degree</td>
<td>-4.135</td>
<td>(0.424)</td>
</tr>
<tr>
<td>Reciprocity</td>
<td>0.517</td>
<td>(0.158)</td>
</tr>
<tr>
<td>Transitive triplets</td>
<td>0.243</td>
<td>(0.038)</td>
</tr>
<tr>
<td>Three-cycles</td>
<td>-0.087</td>
<td>(0.062)</td>
</tr>
<tr>
<td>Indegree popularity</td>
<td>0.330</td>
<td>(0.053)</td>
</tr>
<tr>
<td>Outdegree popularity</td>
<td>0.062</td>
<td>(0.152)</td>
</tr>
<tr>
<td>Outdegree activity</td>
<td>0.013</td>
<td>(0.075)</td>
</tr>
<tr>
<td>Same nationality</td>
<td>0.327</td>
<td>(0.125)</td>
</tr>
<tr>
<td>Sex (M) alter</td>
<td>0.038</td>
<td>(0.098)</td>
</tr>
<tr>
<td>Sex (M) ego</td>
<td>-0.182</td>
<td>(0.099)</td>
</tr>
<tr>
<td>Same Sex</td>
<td>0.052</td>
<td>(0.094)</td>
</tr>
<tr>
<td>Performance alter</td>
<td>0.151</td>
<td>(0.040)</td>
</tr>
<tr>
<td>Performance ego</td>
<td>-0.059</td>
<td>(0.038)</td>
</tr>
<tr>
<td>Performance similarity</td>
<td>0.465</td>
<td>(0.261)</td>
</tr>
</tbody>
</table>
## Results: Advice (2/2)

<table>
<thead>
<tr>
<th>Effect</th>
<th>par.</th>
<th>(s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friendship ⇒ Advice</td>
<td>1.792</td>
<td>(0.220)</td>
</tr>
<tr>
<td>Reciprocal friendship ⇒ Advice</td>
<td>0.356</td>
<td>(0.180)</td>
</tr>
<tr>
<td>Friendship ind. ⇒ Advice popularity</td>
<td>−0.273</td>
<td>(0.064)</td>
</tr>
<tr>
<td>Friendship outd. ⇒ Advice activity</td>
<td>−0.300</td>
<td>(0.055)</td>
</tr>
<tr>
<td>Empl. choice outd. ⇒ Advice activity</td>
<td>0.202</td>
<td>(0.124)</td>
</tr>
<tr>
<td>Employment pref. agreement ⇒ Advice</td>
<td>0.151</td>
<td>(0.078)</td>
</tr>
</tbody>
</table>
## Results: Organizational Preference

<table>
<thead>
<tr>
<th>Effect</th>
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<th>(s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Out-degree</td>
<td>-2.525</td>
<td>(0.141)</td>
</tr>
<tr>
<td>Four-cycles</td>
<td>0.085</td>
<td>(0.031)</td>
</tr>
<tr>
<td>Indegree popularity</td>
<td>0.071</td>
<td>(0.018)</td>
</tr>
<tr>
<td>Outdegree activity</td>
<td>0.074</td>
<td>(0.021)</td>
</tr>
<tr>
<td>Outd.-ind. assortativity</td>
<td>-0.010</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Friendship outd. ⇒ Org. pref. activity</td>
<td>0.010</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Advice outd. ⇒ Org. pref. activity</td>
<td>-0.014</td>
<td>(0.021)</td>
</tr>
<tr>
<td>Friendship ⇒ Org. pref. agreement</td>
<td>-0.065</td>
<td>(0.074)</td>
</tr>
<tr>
<td>Advice ⇒ Org. pref. agreement</td>
<td>0.274</td>
<td>(0.153)</td>
</tr>
</tbody>
</table>
Conclusions (1)

Positive dyad-level effects, 
direct effects stronger than reciprocal effects.

Negative actor-level effects friendship ⇔ advice: 
Specialization between friendship / advice, 
w.r.t. incoming ties as well as outgoing ties.
Conclusions (1)

Positive dyad-level effects, direct effects stronger than reciprocal effects.

Negative actor-level effects friendship $\Leftrightarrow$ advice: Specialization between friendship / advice, w.r.t. incoming ties as well as outgoing ties.

Internal dynamics of organizational preference is not changed strongly by co-dependence on friendship and advice.
Conclusions (2)

Cross-dependencies between friendship and advice do change the representation of the internal dynamics: tendency toward reciprocation is partly mediated by the other relation, especially for advice.

Also homophily in advice is partially mediated by friendship.
Conclusions (2)

Cross-dependencies between friendship and advice do change the representation of the internal dynamics: tendency toward reciprocation is partly mediated by the other relation, especially for advice.

Also homophily in advice is partially mediated by friendship.

Positive interrelations employment choice outdegree and friendship outdegree.

Having the same empl. prefs. leads to advice ties, and vice versa.
Discussion

⇒ See Snijders, Lomi & Torlò in *Social Networks*, 2013.
Discussion

⇒ See Snijders, Lomi & Torlò in *Social Networks*, 2013.

⇒ Testing cross-network dependencies in dynamics of multiple networks gives interesting new possibilities for hypothesis testing.
Discussion

⇒ See Snijders, Lomi & Torlò in *Social Networks*, 2013.

⇒ Testing cross-network dependencies in dynamics of multiple networks gives interesting new possibilities for hypothesis testing.

⇒ Elaborated along the lines of actor-based modeling.
Discussion

⇒ See Snijders, Lomi & Torlò in *Social Networks*, 2013.
⇒ Testing cross-network dependencies in dynamics of multiple networks gives interesting new possibilities for hypothesis testing.
⇒ Elaborated along the lines of actor-based modeling.
⇒ Compared to modeling dynamics of single networks, this approach attenuates the Markov assumption by extending the state space to a multiple network.
New perspectives possible by combining one-mode and two-mode networks.
⇒ New perspectives possible by combining one-mode and two-mode networks.
⇒ The method is available in RSiena. This works for a small number (e.g., 2–6) of networks, and a limited number of actors (up to a few hundred).
⇒ New perspectives possible by combining one-mode and two-mode networks.

⇒ The method is available in RSiena. This works for a small number (e.g., 2–6) of networks, and a limited number of actors (up to a few hundred).

⇒ If there are implication relations between the networks, e.g., two networks might be mutually exclusive, or one might be a sub-network of the other, then this constraint is observed, noted in the print01Report, and respected in the simulations. This gives possibilities for networks with valued ties by using different dichotomies.