Statistical Methods for Social Network Dynamics

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March 4, 2020

1. Longitudinal modeling of social networks

Social networks: structures of relations between social actors.
Examples:

- friendship between school children
- friendship between colleagues
- advice between colleagues
- alliances between firms
- alliances and conflicts between countries
- etc.......

These can be represented mathematically by graphs
or more complicated structures.

Why are ties formed?

There are many recent approaches to this question
leading to a large variety of mathematical models
for network dynamics.

The approach taken here is for statistical inference:

a flexible class of stochastic models
that can adapt itself well to a variety of network data
and can give rise to the usual statistical procedures:
estimating, testing, model fit checking.

Some example research questions

- Development of preschool children:
  how do well-known principles of network formation,
  namely reciprocity, popularity, and triadic closure,
  vary in importance throughout the network formation period
  as the structure itself evolves?
  (Schaefer, Light, Fabes, Hanish, & Martin, 2010)
- Weapon carrying of adolescents in US High Schools:
  What are the relative contributions of weapon carrying of peers,
  aggression, and victimization
  to weapon carrying of male and female adolescents?
  (Dijkstra, Gest, Lindenberg, Veenstra, & Cillessen, 2012)

More example research questions

- Peer influence on adolescent smoking:
  Is there influence from friends on smoking and drinking?
  (Steglich, Snijders & Pearson, 2010)
- Peer influence on adolescent smoking:
  How does peer influence on smoking cessation differ in magnitude from peer influence on smoking initiation?
  (Haas & Schaefer, 2014)

More example research questions

- Collaboration between collective actors in a policy domain:
  What drives collaboration among collective actors involved in climate mitigation policy? (Ingold & Fischer, 2014)
- Preferential trade agreements and democratization:
  Is there evidence that democracies are more likely to join trade agreements; and for such trade agreements to foster democracy among their members?
  (Manger & Pickup, 2014)

In all such questions, a network approach gives more leverage than a variable-centered approach that does not represent the endogenous dependence between the actors.

In some questions the main dependent variable is the network, in others the characteristic of the actors.

We use the term 'behaviour' to indicate the actor characteristics: behaviour, performance, attitudes, etc.

In the latter type of study, a *co-evolution model* of network and behaviour is often useful.
This represents not only the internal feedback processes in the network, but also the interdependence between the dynamics of the network and the behaviour.

Data collection designs

Many designs possible for collecting network data; e.g.,

1. Non-longitudinal: all ties on one predetermined node set;
2. Longitudinal: panel data with $M \geq 2$ data collection points, at each point all ties on the predetermined node set;
3. Longitudinal: continuous observation of all ties on one node set;
4. Incomplete continuous longitudinal (inter-firm ties): as above, but without recording termination of ties;
5. Snowballing: node set not predetermined (e.g., small world experiment).

Statistical procedures will depend on data collection design.

In some of such questions, networks are *independent variables*.
This has been the case in many studies
for explaining well-being (etc.);
this later led to studies of network resources,
social capital, solidarity,
in which the network is also a *dependent variable*.

Networks are dependent as well as independent variables:
intermediate structures in macro–micro–macro phenomena.

Networks as dependent variables

Here: focus first on networks as dependent variables.

But the network itself also explains its own dynamics:
e.g., reciprocation and transitive closure
(friends of friends becoming friends)
are examples where the network plays both roles
of dependent and explanatory variable.

Single observations of networks are snapshots,
the results of untraceable history.
Everything depends on everything else.

Therefore, explaining them has limited importance.
Longitudinal modeling offers more promise for understanding.
The future depends on the past.

Co-evolution

After the explanation of the
actor-oriented model for network dynamics,
attention will turn to co-evolution, which further combines variables
in the roles of dependent variable and explanation:

co-evolution of networks and behaviour
('behaviour' stands here also for other individual attributes);

co-evolution of multiple networks.

1. Networks as dependent variables

The Stochastic Actor-oriented Model ('SAOM') is a model for repeated
measurements on social networks:
at least 2 measurements (preferably more).

Data requirements:

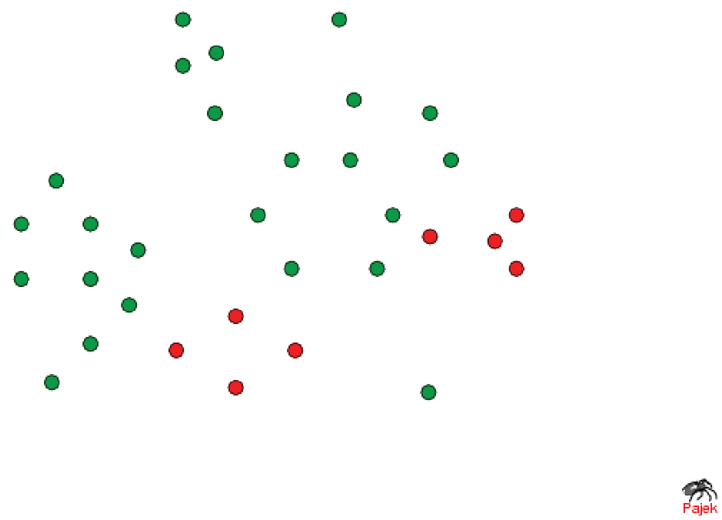
The repeated measurements must be close enough together,
but the total change between first and last observation
must be large enough
in order to give information about rules of network dynamics.

Example: Studies Gerhard van de Bunt

Longitudinal study: panel design.

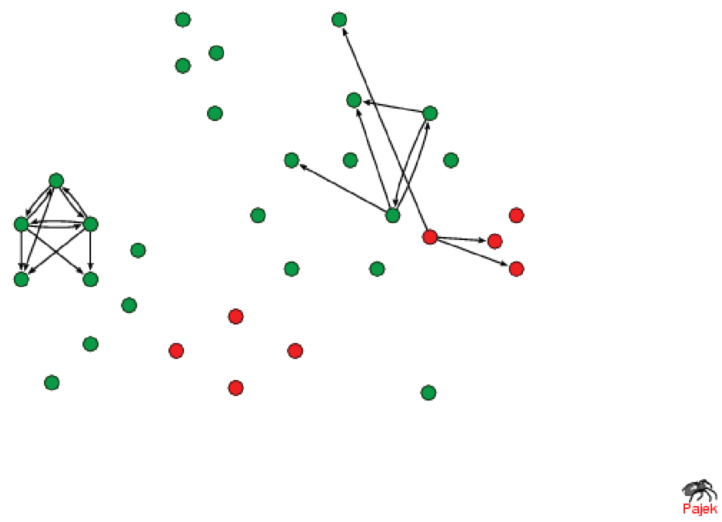
- Study of 32 freshman university students,
  7 waves in 1 year.
  See van de Bunt, van Duijn, & Snijders,
  Computational & Mathematical Organization Theory,
  5 (1999), 167 – 192.

This data set can be pictured by the following graphs
(arrow stands for ‘best friends’).



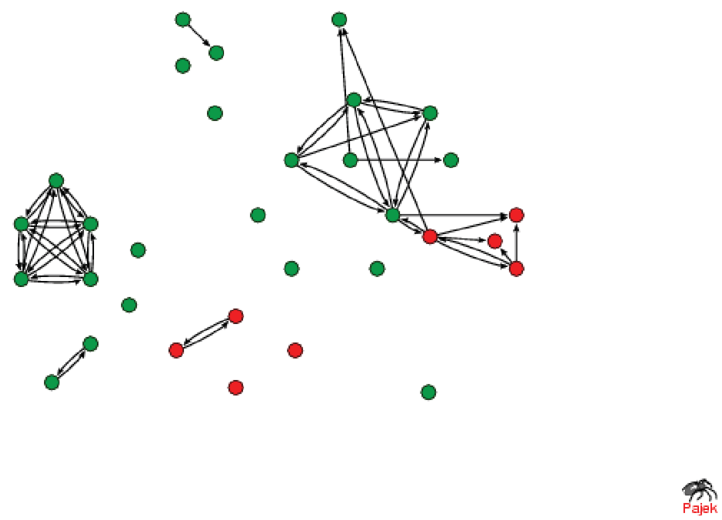
Friendship network time 1.

Average degree 0.0; missing fraction 0.0.



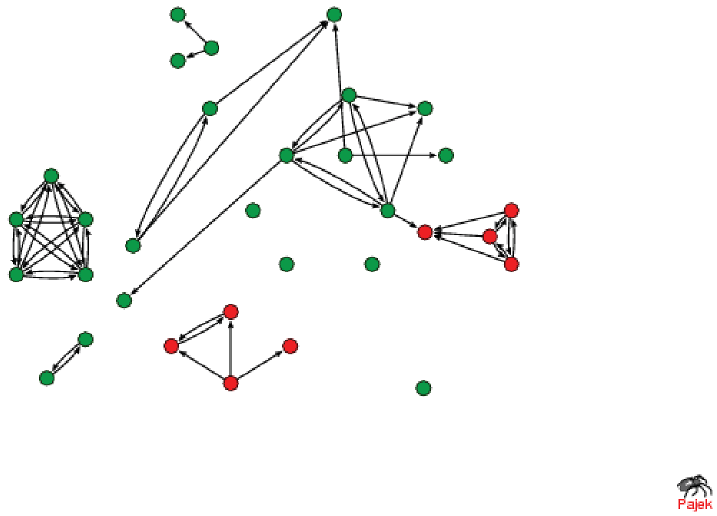
Friendship network time 2.

Average degree 0.7; missing fraction 0.06.



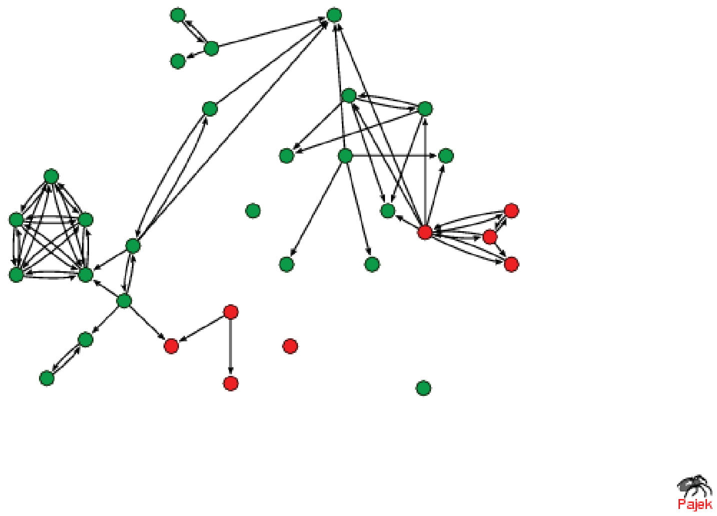
Friendship network time 3.

Average degree 1.7; missing fraction 0.09.



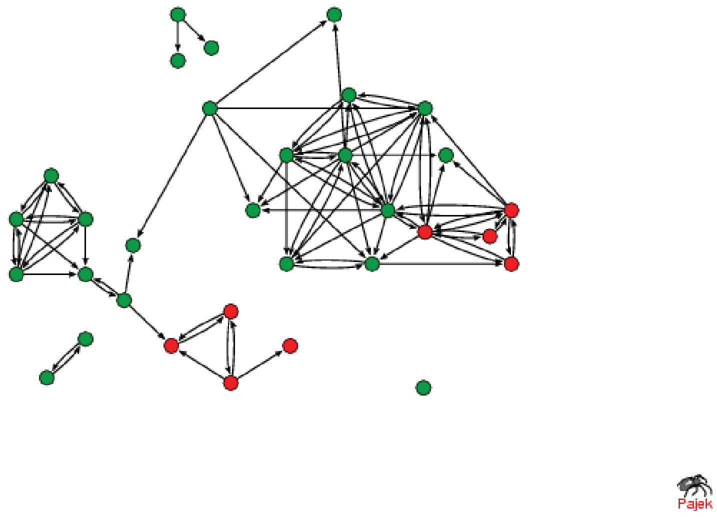
Friendship network time 4.

Average degree 2.1; missing fraction 0.16.



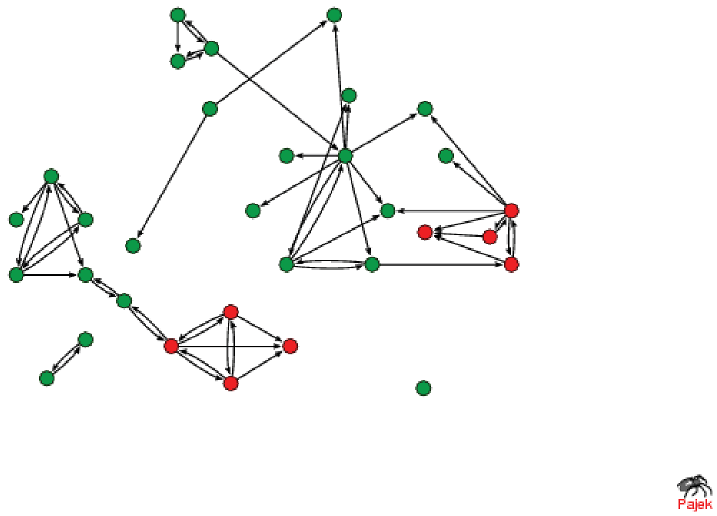
Friendship network time 5.

Average degree 2.5; missing fraction 0.19.



Friendship network time 6.

Average degree 2.9; missing fraction 0.04.



Friendship network time 7.

Average degree 2.3; missing fraction 0.22.

Which conclusions can be drawn from such a data set?

Dynamics of social networks are complicated
because "network effects" are endogenous feedback effects:
e.g., reciprocity, transitivity, popularity, subgroup formation.

For statistical inference, we need models for network dynamics
that are flexible enough to represent
the complicated dependencies in such processes;
while satisfying also the usual statistical requirement
of parsimonious modelling:
complicated enough to be realistic,
not more complicated than empirically necessary and justifiable.

For a correct interpretation of empirical observations
about network dynamics collected in a panel design,
it is crucial to consider a model with *latent change* going on
between the observation moments.

E.g., groups may be regarded as the result of the coalescence
of relational dyads helped by a process of transitivity
("friends of my friends are my friends").
Which groups form may be contingent on unimportant details;
that groups will form is a sociological regularity.

Therefore:
use dynamic models with *continuous time parameter*:
time runs on between observation moments.

Intermezzo

An advantage of using continuous-time models,
even if observations are made at a few discrete time points,
is that a more natural and simple representation may be found,
especially in view of the endogenous dynamics.
(cf. Coleman, 1964).

No problem with irregularly spaced data.

This has been done in a variety of models:
For *discrete data*: cf. Kalbfleisch & Lawless, JASA, 1985;
for *continuous data*:
mixed state space modelling well-known in engineering,
in economics e.g. Bergstrom (1976, 1988),
in social science Tuma & Hannan (1984), Singer (1990s).

Purpose of SAOM

The Stochastic Actor-oriented Model is a statistical model
to investigate network evolution *(dependent var.)* as function of

1. structural effects (reciprocity, transitivity, etc.)
2. explanatory actor variables *(independent vars.)*
3. explanatory dyadic variables *(independent vars.)*

simultaneously.
By controlling adequately for structural effects, it is possible
to test hypothesized effects of variables on network dynamics
(without such control these tests would be incomplete).

The structural effects imply that the presence of ties
is highly dependent on the presence of other ties.

Principles for this approach to analysis of network dynamics:

1. use simulation models as *models for data*
2. comprise a random influence in the simulation model to account for 'unexplained variability'
3. use methods of statistical inference for probability models implemented as simulation models
4. for panel data: employ a continuous-time model to represent unobserved endogenous network evolution
5. condition on the first observation and do not model it: no stationarity assumption.

Stochastic Actor-Oriented Model ('SAOM')

1. *Actors* $i = 1, \ldots, n$ (individuals in the network), pattern X of *ties* between them : one binary network X; $X_{ij} = 0$, or 1 if there is no tie, or a tie, from i to j. Matrix X is *adjacency matrix* of digraph. X_{ij} is a *tie indicator* or *tie variable*.
2. Exogenously determined independent variables: actor-dependent covariates v, dyadic covariates w. These can be constant or changing over time.
3. Continuous time parameter t, observation moments $t_1, \ldots, t_M$.
4. Current state of network $X(t)$ is dynamic constraint for its own change process: Markov process.

'actor-oriented' = 'actor-based'

5. The actors control their outgoing ties.
6. The ties have inertia: they are *states* rather than *events*.
   At any single moment in time,
   only one variable $X_{ij}(t)$ may change.
7. Changes are modeled as
   choices by actors in their outgoing ties,
   with probabilities depending on '*objective function*'
   of the network state that would obtain after this change.

The change probabilities can (but need not)
be interpreted as arising from goal-directed behaviour,
in the weak sense of myopic stochastic optimization.

Assessment of the situation is represented by
objective function, interpreted as
'that which the actors seem to strive after in the short run'.

Next to actor-driven models,
also tie-driven models are possible.

('LERGM', Snijders & Koskinen, Chapter 11 in ERGM book 2013)

At any given moment, with a given current network structure,
the actors act independently, without coordination.
They also act one-at-a-time.

The subsequent changes ('micro-steps') generate
an endogenous dynamic context
which implies a dependence between the actors over time;
e.g., through reciprocation or transitive closure
one tie may lead to another one.

This implies strong dependence between what the actors do,
but it is completely generated by the time order:
the actors are dependent because they constitute
each other's changing environment.

The change process is decomposed into two sub-models,
formulated on the basis of the idea that the actors i control
their outgoing ties $(X_{i1}, \ldots, X_{in})$:

1. waiting times until the next opportunity
   for a change made by actor i:
   rate functions;
2. probabilities of changing (toggling) X_{ij},
   conditional on such an opportunity for change:
   objective functions.

The distinction between rate function and objective function
separates the model for *how many* changes are made
from the model for *which* changes are made.

This decomposition between the timing model and the model for change can be pictured as follows:

At randomly determined moments t,
actors i have opportunity to change one tie variable X_{ij}:
micro step.
(Actors are also permitted to leave things unchanged.)
Frequency of micro steps is determined by *rate functions*.

When a micro step is taken,
the probability distribution of the result of this step depends on the *objective function* :
higher probabilities of moving toward new states that have higher values of the objective function.

Specification: rate function

'how fast is change / opportunity for change ?'

Rate of change of the network by actor i is denoted λ_i :
expected frequency of opportunities for change by actor i.

Simple specification: rate functions are constant within periods.

More generally, rate functions can depend on observation period $(t_{m-1},\ t_m)$, actor covariates, network position (degrees etc.), through an exponential link function.

Formally, for a certain short time interval $(t, t+\epsilon)$,
the probability that this actor randomly gets an opportunity to change one of his/her outgoing ties, is given by $\epsilon\,\lambda_i$.

Specification: objective function

'what is the direction of change?'

The objective function $f_i(\beta, x)$ indicates
preferred 'directions' of change.
β is a statistical parameter, i is the actor (node), x the network.

When actor i gets an opportunity for change,
he has the possibility to change *one* outgoing tie variable X_{ij},
or leave everything unchanged.

By $x^{(\pm ij)}$ is denoted the network obtained
when x_{ij} is changed ('toggled') into $1 - x_{ij}$.
Formally, $x^{(\pm ii)}$ is defined to be equal to x.

Conditional on actor i being allowed to make a change,
the probability that X_{ij} changes into $1 - X_{ij}$ is

$$p_{ij}(\beta, x) = \frac{\exp\left(f_i(\beta, x^{(\pm ij)})\right)}{\sum_{h=1}^{n} \exp\left(f_i(\beta, x^{(\pm ih)})\right)} ,$$

and p_{ii} is the probability of not changing anything.

Higher values of the objective function indicate
the preferred direction of changes.

One way of obtaining this model specification is to suppose that actors make changes such as to optimize
the objective function $f_i(\beta, x)$
plus a random disturbance that has a Gumbel distribution,
like in random utility models in econometrics:

myopic stochastic optimization,
multinomial logit models.

Actor i chooses the "best" j by maximizing

$$f_i\big(\beta, x^{(\pm ij)}\big) \quad + \quad \underset{\substack{\Uparrow \\ \text{random component}}}{U_i(t, x, j)}\,.$$

(with the formal definition $x^{(\pm ii)} = x$).

Objective functions will be defined as sum of:

1. *evaluation function* expressing satisfaction with network;

And to allow asymmetry creation $\leftrightarrow$ termination of ties:

2. *creation function*
   expressing aspects of network structure
   playing a role only for creating new ties
3. *maintenance = endowment function*
   expressing aspects of network structure
   playing a role only for maintaining existing ties

If creation function = maintenance function,
then these can be jointly replaced by the evaluation function.
This is usual for starting modelling.

Evaluation, creation, and maintenance functions are modeled as linear combinations of theoretically argued components of preferred directions of change. The weights in the linear combination are the statistical parameters.

This is a linear predictor like in generalized linear modeling (generalization of regression analysis).

Formally, the SAOM is a generalized linear statistical model with missing data (the microsteps are not observed).

The focus of modeling is first on the evaluation function; then on the rate and creation – maintenance functions; often, the latter are not even considered.

The objective function does not reflect the eventual 'utility' of the situation to the actor, but short-time goals following from preferences, constraints, opportunities.

The evaluation, creation, and maintenance functions express how the dynamics of the network process depends on its current state.

Stochastic process formulation

This specification implies that X follows a *continuous-time Markov chain* with intensity matrix

$$q_{ij}(x) = \lim_{dt \downarrow 0} \frac{\mathrm{P}\{X(t+dt) = x^{(\pm ij)} \mid X(t) = x\}}{dt} \qquad (i \neq j)$$

given by

$$q_{ij}(x) = \lambda_i(\alpha, \rho, x)\, p_{ij}(\beta, x)\,.$$

Computer simulation algorithm for arbitrary rate function $\lambda_i(\alpha, \rho, x)$

1. Set $t = 0$ and $x = X(0)$.
2. Generate S according to the exponential distribution with mean $1/\lambda_+(\alpha, \rho, x)$ where

$$\lambda_+(\alpha, \rho, x) = \sum_i \lambda_i(\alpha, \rho, x)\,.$$

3. Select $i \in \{1, ..., n\}$ using probabilities

$$\frac{\lambda_i(\alpha, \rho, x)}{\lambda_+(\alpha, \rho, x)}\,.$$

4. Select $j \in \{1, ..., n\}$, $j \neq i$ using probabilities $p_{ij}(\beta, x)$.
5. Set $t = t + S$ and $x = x^{(\pm ij)}$.
6. Go to step 2
   (unless stopping criterion is satisfied).

Note that the change probabilities depend always on
the current network state, not on the last observed state!

Model specification :

Simple specification: only evaluation function;
no separate creation or maintenance function,
periodwise constant rate function.

Evaluation function f_i reflects network effects
(endogenous) and covariate effects (exogenous).
Covariates can be actor-dependentor dyad-dependent.

Convenient definition of evaluation function is a weighted sum

$$f_i(\beta, x) = \sum_{k=1}^{L} \beta_k \, s_{ik}(x) \, ,$$

where the weights β_k are statistical parameters indicating
strength of effect $s_{ik}(x)$ (‘linear predictor’).

Effects

Effects are functions of the network and covariates.

These can be anything; in practice, effects are *local*, i.e., functions of the network neighborhood of the focal actor — this could also be the neighborhood at distance 2.

The RSiena software contains a large collection of effects, all listed in the manual.
This collection is increased as demanded by research needs.

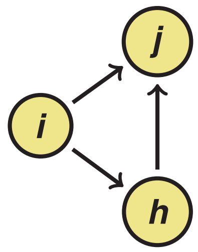
The following slides mention just a few effects.

Some network effects for actor i:
(others to whom actor i is tied are called here i's 'friends')

1. *out-degree effect*, controlling the density / average degree, $s_{i1}(x) = x_{i+} = \sum_j x_{ij}$
2. *reciprocity effect*, number of reciprocated ties $s_{i2}(x) = \sum_j x_{ij}\, x_{ji}$

Various potential effects representing network closure:

3. *transitive triplets effect* ('transTrip'),
   number of transitive patterns in i's ties
   $(i \to j,\ i \to h,\ h \to j)$
   $s_{i3}(x) = \sum_{j,h} x_{ij}\, x_{ih}\, x_{hj}$
   (For each tie $i \to j$, the number
   of intermediate nodes h is added.)



transitive triplet

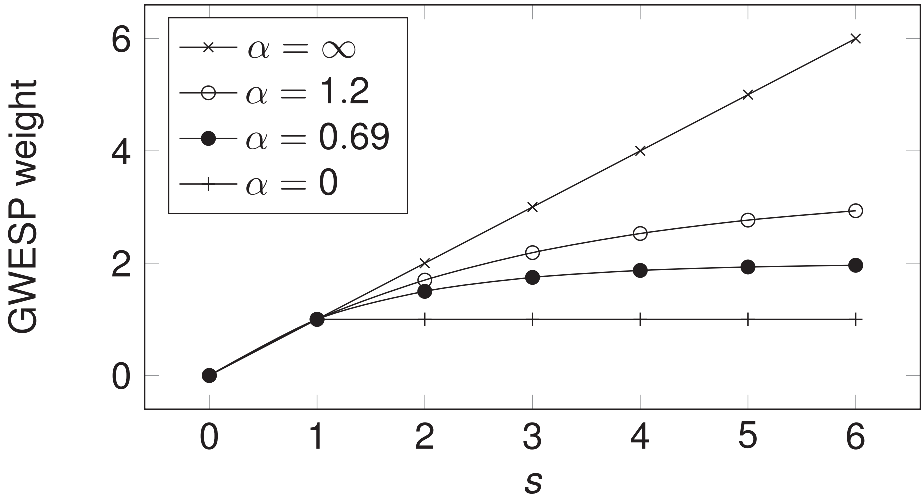
4. *transitive ties effect* ('transTies'),
   number of actors j to whom i is tied indirectly
   (through at least one intermediary: $x_{ih} = x_{hj} = 1$)
   and also directly $x_{ij} = 1$),
   $s_{i4}(x) = \#\{j \mid x_{ij} = 1,\ \max_h(x_{ih}\, x_{hj}) > 0\}$
   (For each tie $i \to j$,
   1 is added if there is at least one intermediate node h.)

5. *geometrically weighted edgewise shared partners*
   ('GWESP'; cf. ERGM)
   is intermediate between transTrip and transTies.

$$\text{GWESP}(i, \alpha) = \sum_j x_{ij}\, e^{\alpha} \left\{ 1 - \left(1 - e^{-\alpha}\right)^{\sum_h x_{ih} x_{hj}} \right\} .$$

for $\alpha \geq 0$ (effect parameter = 100 $\times$ α).
Effect parameters in RSiena are fixed parameters in an effect,
allowing the user
to choose between different versions of the effect.
Default here: $\alpha = \ln(2) \approx 0.69$, effect parameter = 69.

GWESP is intermediate between transitive triplets ($\alpha = \infty$) and transitive ties ($\alpha = 0$).



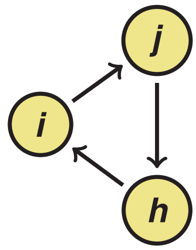
Weight of tie $i \to j$ for $s = \sum_h x_{ih} x_{hj}$ two-paths.

Differences between network closure effects:

- transitive triplets effect: i more attracted to j if there are *more* indirect ties $i \to h \to j$;
- transitive ties effect: i more attracted to j if there is *at least one* such indirect connection ;
- gwesp effect: in between these two;
- balance or Jaccard similarity effects (see manual): i prefers others j who make same choices as i.

Non-formalized theories usually do not distinguish between these different closure effects.
It is possible to 'let the data speak for themselves' and see what is the best formal representation of closure effects.

6. *three-cycle effect*,
   number of three-cycles in i's ties
   $(i \rightarrow j,\ j \rightarrow h,\ h \rightarrow i)$
   $s_{i6}(x) = \sum_{j,h}\ x_{ij}\, x_{jh}\, x_{hi}$



three-cycle

This represents a kind of generalized reciprocity,
and absence of hierarchy.

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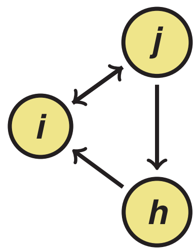
7. *reciprocity* $\times$ *transitive triplets effect*,
   number of triplets in i's ties
   combining reciprocity and transitivity
   as follows
   $(i \leftrightarrow j,\ j \rightarrow h,\ h \rightarrow i)$
   $s_{i7}(x) = \sum_{j,h}\ x_{ij}\, x_{ji}\, x_{jh}\, x_{hi}$



reciprocity $\times$
trans. triplet

Simultaneous occurrence of
reciprocity and network closure
(see Per Block, *Social Networks*, 2015.)

Degree-related effects

Degrees (distinguished in in-degrees and out-degrees)
are important characteristics of actor's network positions.

Direct *degree effects* are about
how indegrees and outdegrees affect themselves and each other.

Degree assortativity effects are about the association
between the in/out-degrees of the nodes at either side of a tie.

8. *in-degree related popularity effect*, sum friends' in-degrees
   $s_{i8}(x) = \sum_j x_{ij}\, x_{+j} = \sum_j x_{ij} \sum_h x_{hj}$
   related to dispersion of in-degrees
9. *out-degree related popularity effect*,
   sum friends' out-degrees
   $s_{i9}(x) = \sum_j x_{ij}\, x_{j+} = \sum_j x_{ij} \sum_h x_{jh}$
   related to association in-degrees — out-degrees;
10. *Outdegree-related activity effect* ,
   $s_{i10}(x) = \sum_j x_{ij}\, x_{i+} = x_{i+}^2$
   related to dispersion of out-degrees;
11. *Indegree-related activity effect* ,
   $s_{i11}(x) = \sum_j x_{ij}\, x_{+i} = x_{i+}\, x_{+i}$
   related to association in-degrees — out-degrees;

(These effects can also be defined with a $\sqrt{}$ sign.)

12. *Assortativity effects:*
    Preferences of actors dependent on their degrees.
    Depending on their own out- and in-degrees,
    actors can have differential preferences for ties
    to others with also high or low out- and in-degrees.
    Together this yields 4 possibilities:
    - out ego - out alter degrees
    - out ego - in alter degrees
    - in ego - out alter degrees
    - in ego - in alter degrees

    All these are product interactions between the two degrees.
    Here also the degrees could be replaced by their square roots.

How to specify structural part of the model?

1. Always: outdegree effect (like constant term in regression)
2. Almost always: reciprocity
3. Triadic effects: transitivity, reciprocity $\times$ transitivity, 3-cycles, etc.
4. Degree-related effects:
   inPop, outAct; outPop or inAct;
   perhaps $\surd$ versions; perhaps assortativity.

Of course, there are more.
Model selection:
combination of prior and data-based considerations
(Goodness of fit; function *sienaGOF*).

For the effects of an actor variable v_i , a transformation from the actor level to the tie (dyadic) level is necessary.

13. *covariate-related popularity*, ‘alter’
    sum of covariate over all of i ’s friends
    $s_{i13}(x) = \sum_j x_{ij}\, v_j$;
14. *covariate-related activity*, ‘ego’
    i’s out-degree weighted by covariate
    $s_{i14}(x) = v_i\, x_{i+}$;
15. For a binary or other categorical variable: *same covariate*, ‘same’
    number of i’s ties to alters with same covariate
    $s_{i15}(x) = \sum_j x_{ij}\, I\{v_j \,=\, v_i\}$,
    where $I\{v_j \,=\, v_i\} \,=\, 1$ if $v_j = v_i$ and else 0.

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16. For homophily, *covariate-related similarity*,
    sum of measure of covariate similarity
    between i and his friends,
    $s_{i16}(x) = \sum_j x_{ij}\, \mathrm{sim}(v_i, v_j)$
    where $\mathrm{sim}(v_i, v_j)$ is the similarity between v_i and v_j,
    $$\mathrm{sim}(v_i, v_j) \,=\, 1 - \frac{|v_i - v_j|}{R_V} \;,$$
    R_V being the range of V;
17. Another type of combination is the product interaction,
    covariate-related interaction, ‘ego $\times$ alter’
    $s_{i17}(x) = v_i \sum_j x_{ij}\, v_j$;

Later on, I will discuss how to treat the specification of effects of for numerical actor variables.

Evaluation function effect for dyadic covariate w_{ij} :

18. *covariate-related preference*,
 sum of covariate over all of i's friends,
 i.e., values of w_{ij} summed over all others to whom i is tied,
 $s_{i18}(x) = \sum_j x_{ij}\, w_{ij}$.
 If this has a positive effect, then the value of a tie $i \rightarrow j$ becomes higher when w_{ij} becomes higher.

Here no transformation is necessary!

Of course, more complicated effects are possible.

(E.g., for W = 'living in the same house', the 'compound' effect 'being friends with those living in the same house as your friends'.)

Example

Data collected by Gerhard van de Bunt:
group of 32 university freshmen,
24 female and 8 male students.

Three observations used here ($t_1,\ t_2,\ t_3$) :
at 6, 9, and 12 weeks after the start of the university year.
The relation is defined as a 'friendly relation'.

Missing entries $x_{ij}(t_m)$ set to 0
and not used in calculations of statistics.

Densities increase from 0.15 at t_1 via 0.18 to 0.22 at t_3 .

Very simple model: only out-degree and reciprocity effects

	Model 1	
Effect	par.	(s.e.)
Rate $t_1 - t_2$	3.51	(0.54)
Rate $t_2 - t_3$	3.09	(0.49)
Out-degree	-1.10	(0.15)
Reciprocity	1.79	(0.27)

rate parameters:
per actor about 3 opportunities for change between observations;

out-degree parameter negative:
on average, cost of friendship ties higher than their benefits;

reciprocity effect strong and highly significant ($t = 1.79/0.27 = 6.6$)
(test using the ratio parameter estimate / standard error).

Evaluation function is

$$f_i(x) = \sum_j \Big(-1.10\, x_{ij} \;+\; 1.79\, x_{ij}\, x_{ji} \Big).$$

This expresses 'how much actor *i* likes the network'.

Adding a reciprocated tie (i.e., for which $x_{ji} = 1$) gives

$$-1.10 + 1.79 \;=\; 0.69.$$

Adding a non-reciprocated tie (i.e., for which $x_{ji} = 0$) gives

$$-1.10,$$

i.e., this has negative 'benefits'.
Gumbel distributed disturbances are added:
these have standard deviation $\sqrt{\pi^2/6} \;=\; 1.28$.

Conclusion: reciprocated ties are valued positively,
unreciprocated ties negatively;
actors will be reluctant to form unreciprocated ties;
by 'chance' (the random term),
such ties will be formed nevertheless
and these are the stuff on the basis of which
reciprocation by others can start.

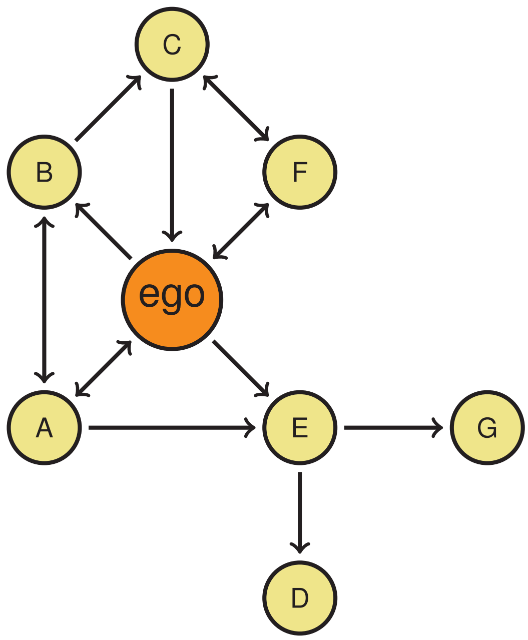
(Incoming unreciprocated ties, $x_{ji} = 1, x_{ij} = 0$ do not play a role
because for the objective function
only those parts of the network are relevant
that are under control of the actor,
so terms not depending on the outgoing relations of the actor
are irrelevant.)

For an interpretation, consider the simple model
with only the transitive ties network closure effect. The estimates are:

Structural model with one network closure effect

	Model 3	
Effect	par.	(s.e.)
Rate $t_1 - t_2$	3.86	(0.60)
Rate $t_2 - t_3$	3.04	(0.48)
Out-degree	-2.13	(0.36)
Reciprocity	1.57	(0.28)
Transitive ties	1.29	(0.40)

Example: Personal network of ego.



for ego:

out-degree $x_{i+} = 4$
#{recipr. ties} = 2,
#{trans. ties } = 3.

The evaluation function is

$$f_i(x) = \sum_j \Big(-2.13\, x_{ij} \;+\; 1.57\, x_{ij}\, x_{ji} \;+\; 1.29\, x_{ij} \max_h \big(x_{ih}\, x_{hj}\big)\Big)$$

$\Big($ note: $\sum_j x_{ij}\ \max_h \big(x_{ih}\, x_{hj}\big)$ is #{trans. ties } $\Big)$

so its current value for this actor is

$$f_i(x) = -2.13 \times 4 \;+\; 1.57 \times 2 \;+\; 1.29 \times 3 \;=\; -1.51 .$$

Options when ‘ego’ has opportunity for change:

	out-degr.	recipr.	trans. ties	gain	prob.
current	4	2	3	0.00	0.071
new tie to C	5	3	5	+2.02	0.532
new tie to D	5	2	4	–0.84	0.031
new tie to G	5	2	4	–0.84	0.031
drop tie to A	3	1	0	–3.30	0.003
drop tie to B	3	2	1	–0.45	0.045
drop tie to E	3	2	2	+0.84	0.164
drop tie to F	3	1	3	+0.56	0.124

The actor adds random influences to the gain (with s.d. 1.28), and chooses the change with the highest total ‘value’.

Model with more structural effects

Effect	par.	(s.e.)
Rate 1	3.90	(0.62)
Rate 2	3.21	(0.52)
Out-degree	–1.46	(0.39)
Reciprocity	2.55	(0.52)
Transitive ties	0.51	(0.40)
Transitive triplets	0.62	(0.14)
Transitive reciprocated triplets	–0.65	(0.23)
Indegree - popularity	–0.18	(0.07)

convergence t ratios all < 0.08.

Overall maximum convergence ratio 0.13.

Conclusions:

Reciprocity, transitivity;
negative interaction
transitivity – reciprocity;
negative popularity effect;
transitive ties not needed.

Add effects of gender & program, smoking similarity

Effect	par.	(s.e.)
Rate 1	4.02	(0.64)
Rate 2	3.25	(0.52)
outdegree (density)	–1.52	(0.41)
reciprocity	2.35	(0.46)
transitive triplets	0.61	(0.13)
transitive recipr. triplets	–0.58	(0.21)
indegree - popularity	–0.16	(0.07)
sex alter	0.72	(0.27)
sex ego	–0.04	(0.26)
same sex	0.42	(0.23)
program similarity	0.69	(0.26)
smoke similarity	0.29	(0.19)

convergence t ratios all < 0.1.

Overall maximum convergence ratio 0.12.

Conclusions:
men more popular
(minority!)
program similarity.

Extended model specification

1. Creation and maintenance effects

tie creation is modeled by
the sum evaluation function + creation function;

tie maintenance is modeled by
the sum evaluation function + maintenance function.

('maintenance function' = 'endowment function')

Estimating the distinction between creation and maintenance requires a lot of data.

Add maintenance effect of reciprocated tie

Effect	par.	(s.e.)
Rate 1	5.36	(0.97)
Rate 2	4.13	(0.74)
outdegree	–1.68	(0.37)
reciprocity: evaluation	1.27	(0.50)
reciprocity: maintenance	3.58	(1.02)
transitive triplets	0.55	(0.10)
transitive reciprocated triplets	–0.59	(0.22)
indegree - popularity	–0.14	(0.06)
sex alter	0.65	(0.26)
sex ego	–0.21	(0.28)
same sex	0.39	(0.23)
program similarity	0.83	(0.25)
smoke similarity	0.37	(0.18)

convergence t ratios all < 0.06.

Overall maximum convergence ratio 0.16.

Transitive ties effect omitted.

Evaluation effect reciprocity: 1.27
Maintenance reciprocated tie: 3.58

The maintenance effect is significant.

The overall (combined) reciprocity effect was 2.35.
With the split between the evaluation and maintenance effects,
it appears now that the value of reciprocity
for creating a tie is 1.27,
and for withdrawing a tie 1.27 + 3.58 = 4.85.

Thus, there is a very strong barrier
against the dissolution of reciprocated ties.

Extended model specification

2. Non-constant rate function $\lambda_i(\alpha, \rho, x)$.

This means that some actors change their ties
more quickly than others,
depending on covariates or network position.

Dependence on covariates:

$$\lambda_i(\alpha, \rho, x) = \rho_m \exp\Big(\sum_h \alpha_h \, v_{hi}\Big) .$$

ρ_m is a period-dependent base rate.

(Rate function must be positive; $\Rightarrow$ exponential function.)

Dependence on network position:
e.g., dependence on out-degrees:

$$\lambda_i(\alpha, \rho, x) = \rho_m \exp(\alpha_1 \, x_{i+}) \; .$$

Also, in-degrees and $\sharp$ reciprocated ties of actor *i* may be used.

Dependence on out-degrees can be useful especially if there are large 'size' differences between actors, e.g., organizations; then the network may have different importance for the actors as indicated by their outdegrees.

Now the parameter is $\theta = (\rho, \alpha, \beta, \gamma)$.

Continuation example

Rate function depends on out-degree:
those with higher out-degrees
also change their tie patterns more quickly.

Keep the maintenance function depending on tie reciprocation:
Reciprocity operates differently for tie initiation than for tie withdrawal.

Parameter estimates model with rate and maintenance effects

Effect	par.	(s.e.)
Rate 1	4.382	(0.781)
Rate 2	3.313	(0.582)
outdegree effect on rate	0.027	(0.027)
outdegree (density)	–1.611	(0.394)
reciprocity: evaluation	1.320	(0.514)
reciprocity: maintenance	3.439	(1.100)
transitive triplets	0.518	(0.101)
transitive reciprocated triplets	–0.569	(0.219)
indegree - popularity	–0.145	(0.062)
sex alter	0.629	(0.272)
sex ego	–0.207	(0.283)
same sex	0.395	(0.235)
program similarity	0.859	(0.260)
smoke similarity	0.386	(0.185)

convergence t ratios all < 0.18.

Overall maximum convergence ratio 0.21.

Conclusion:

non-significant tendency that actors with higher out-degrees change their ties more often ($t = 0.027/0.027 = 1.0$), and all other conclusions remain the same.

2. Non-directed networks

The actor-driven modeling is less straightforward
for non-directed relations,
because two actors are involved in deciding about a tie.

See chapter by Snijders & Pickup in
Oxford Handbook of Political Networks (2017).

Various modeling options are possible:

1. Forcing model:
   one actor takes the initiative and unilaterally imposes
   that a tie is created or dissolved.

2. Unilateral initiative with reciprocal confirmation:
   one actor takes the initiative and proposes a new tie
   or dissolves an existing tie;
   if the actor proposes a new tie, the other has to confirm,
   otherwise the tie is not created.
3. Pairwise conjunctive model:
   a pair of actors is chosen and reconsider whether a tie
   will exist between them; a new tie is formed if both agree.
4. Pairwise disjunctive (forcing) model:
   a pair of actors is chosen and reconsider whether a tie
   will exist between them;
   a new tie is formed if at least one wishes this.

5. Pairwise compensatory (additive) model:
   a pair of actors is chosen and reconsider whether a tie will exist between them; this is based on the sum of their utilities for the existence of this tie.

Option 1 is close to the actor-driven model for directed relations.

In options 3–5, the pair of actors (i, j) is chosen depending on the product of the rate functions $\lambda_i \lambda_j$ (under the constraint that $i \neq j$).

The numerical interpretation of the ratio function differs between options 1–2 compared to 3–5.

The decision about the tie is taken on the basis of the objective functions $f_i,\ f_j$ of one or both actors.

3. Estimation

Suppose that at least 2 observations on $X(t)$ are available, for observation moments $t_1,\ t_2$.
(Extension to more than 2 observations is straightforward.)

How to estimate θ?

Condition on $X(t_1)$:
the first observation is accepted as given,
contains in itself no observation about θ.

No assumption of a stationary network distribution.

Thus, simulations start with $X(t_1)$.

3A. Method of moments

Choose a suitable statistic $Z = (Z_1, \ldots, Z_K)$,
i.e., K variables which can be calculated from the network;
the statistic Z must be *sensitive* to the parameter θ
in the sense that higher values of θ_k
lead to higher values of the expected value $\mathrm{E}_\theta(Z_k)$;

determine value $\hat{\theta}$ of $\theta = (\rho, \beta)$ for which
observed and expected values of suitable Z statistic are equal:

$$E_{\hat{\theta}}\{Z\} = z \, .$$

Questions:

- What is a suitable (K-dimensional) statistic?
  Corresponds to objective function.
- How to find this value of θ?
  By stochastic approximation (Robbins-Monro process)
  based on repeated simulations of the dynamic process,
  with parameter values
  getting closer and closer to the moment estimates.

Suitable statistics for method of moments

Assume first that $\lambda_i(x) = \rho = \theta_1$,
and 2 observation moments.

This parameter determines the expected “amount of change”.

A sensitive statistic for $\theta_1 = \rho$ is

$$C = \sum_{\substack{i,j=1 \\ i \neq j}}^{g} \mid X_{ij}(t_2) - X_{ij}(t_1) \mid ,$$

the “observed total amount of change”.

For the weights β_k in the evaluation function

$$f_i(\beta, x) = \sum_{k=1}^{L} \beta_k \, s_{ik}(x) ,$$

a higher value of β_k means that all actors
strive more strongly after a high value of $s_{ik}(x)$,
so $s_{ik}(x)$ will tend to be higher for all i, k.

This leads to the statistic

$$S_k = \sum_{i=1}^{n} s_{ik}(X(t_2)) .$$

This statistic will be sensitive to β_k :
a high β_k will to lead to high values of S_k.

Moment estimation will be based on the
vector of statistics

$$Z = (C, S_1, ..., S_{K-1}) .$$

Denote by z the observed value for Z.
The moment estimate $\hat{\theta}$ is defined as the parameter value
for which the expected value of the statistic
is equal to the observed value:

$$\mathrm{E}_{\hat{\theta}}\{Z\} = z .$$

Robbins-Monro algorithm

The moment equation $E_{\hat{\theta}}\{Z\} = z$ cannot be solved by
analytical or the usual numerical procedures, because

$$\mathrm{E}_{\theta}\{Z\}$$

cannot be calculated explicitly.

However, the solution can be approximated by the
Robbins-Monro (1951) method for stochastic approximation.

Iteration step:

$$\hat{\theta}_{N+1} = \hat{\theta}_N - a_N D^{-1}(z_N - z) , \qquad (1)$$

where z_N is a simulation of Z with parameter $\hat{\theta}_N$,
D is a suitable matrix, and $a_N \to 0$.

Covariance matrix

The method of moments yields the covariance matrix

$$\mathrm{cov}(\hat{\theta}) \approx D_\theta^{-1}\, \Sigma_\theta\, D_\theta'^{\,-1}$$

where

$$\begin{aligned} \Sigma_\theta &= \mathrm{cov}\{Z \mid X(t_1) = x(t_1)\} \\ D_\theta &= \frac{\partial}{\partial\theta}\mathrm{E}\{Z \mid X(t_1) = x(t_1)\}\,. \end{aligned}$$

Matrices Σ_θ and D_θ can be estimated
from MC simulations with fixed θ.

After the presumed convergence of the algorithm
for approximately solving the moment equation,
extra simulations are carried out

(*a*) to check that indeed $\mathrm{E}_{\hat{\theta}}\{Z\} \approx z$,

(*b*) to estimate Σ_θ,

(*c*) and to estimate D_θ
using a score function algorithm
(earlier algorithm used
difference quotients and common random numbers).

Modified estimation method:

conditional estimation .

Condition on the observed numbers of differences between successive observations,

$$c_m = \sum_{i,j} \mid x_{ij}(t_{m+1}) - x_{ij}(t_m) \mid \ .$$

For continuing the simulations do not mind the values of the time variable t,
but continue between t_m and t_{m+1} until
the observed number of differences

$$\sum_{i,j} \mid X_{ij}(t) - x_{ij}(t_m) \mid$$

is equal to the observed c_m .
This is defined as time moment t_{m+1} .

This procedure is a bit more stable; requires modified estimator of ρ_m .

In practice the differences are small.

Computer algorithm has 3 phases:

1. brief phase for preliminary estimation of $\partial \mathrm{E}_\theta \{Z\} / \partial \theta$ for defining D;
2. estimation phase with Robbins-Monro updates, where a_N remains constant in *subphases* and decreases between subphases;
3. final phase where θ remains constant at estimated value; this phase is for checking that

$$\mathrm{E}_{\hat{\theta}} \{Z\} \approx z \ ,$$

and for estimating D_θ and Σ_θ to calculate standard errors.

Default n3=1000 runs; for publications, more required (e.g., 5000).

Convergence

After running the algorithm, the convergence must be checked before starting to interpret the results.

For each statistic Z_k used for estimation, we define

$$\begin{aligned} \bar{z}_{k\cdot} &= \text{average of simulated values in Phase 3;} \\ \mathrm{sd}(z_{k\cdot}) &= \text{their standard deviation;} \\ z_k &= \text{the target value.} \end{aligned}$$

Ideally,

$$\bar{z}_{k\cdot} = z_k \text{ for all } k \ .$$

The requirement for convergence is

$$\mathrm{tconv}_k = \frac{\left|\bar{z}_{k\cdot} - z_k\right|}{\mathrm{sd}(z_{k\cdot})} \leq 0.1 \text{ for all } k \ .$$

tconv_k is called the *t-ratio for convergence*, and is given in the table with estimation results.

A further criterion is that

$$\text{tconv.max} = \max_{w} \frac{\sum_k w_k \left(\bar{z}_{k\cdot} - z_k\right)}{\text{sd}\Big(\sum_k w_k\, z_{k\cdot}\Big)} \leq 0.25 \; .$$

where the maximum is taken over all vectors w of weights of linear combinations of the Z_k.

tconv.max is called the *overall maximum convergence ratio*.

Obtaining convergent estimation

The default settings of the estimation algorithm are such, that for most data sets and models, convergence is achieved in one run.

If the model is complicated given the information available in the data, and also if some highly correlated parameters are being estimated, it can be necessary to run the estimation again, using the previous estimates as new starting values: the *prevAns* option.

If this still is not successful: consult the manual, Sections 6.2 and 6.3.

Extension: more periods

The estimation method can be extended
to more than 2 repeated observations:
observations $x(t)$ for $t = t_1, ..., t_M$.

Parameters remain the same in periods between observations
except for the basic rate of change ρ
which now is given by ρ_m for $t_m \leq t < t_{m+1}$.

For the simulations,
the simulated network $X(t)$ is reset to the observation $x(t_m)$
whenever the time parameter t passes the observation time t_m .

The statistics for the method of moments are defined as
sums of appropriate statistics calculated per period (t_m, t_{m+1}).

A special property of the SAOM is that
the interpretation of the parameters of the objective function
is not affected by the number of waves (2 or more);
for more periods (a period is the interval between two waves)
the only things added are the rate parameters per period.

However, for two or more periods,
it is necessary to check time homogeneity of the parameters
(function sienaTimeTest).
Note that, even with constant = time homogeneous parameters,
the network still may be systematically changing;
this depends on the combination of parameters with the initial network.

4. Special topic: Effects of numerical actor variables

Actor covariates are defined at the '*monadic*' level,
whereas tie variables are at the *dyadic* level.

The transformation from monadic to dyadic usually implies the necessity of using more than one parameter;
e.g., ego/sender, alter/receiver, and combinations of ego and alter.

This topic is treated in Snijders & Lomi, *Network Science*, 2019.

Here we consider a numerical actor variable V with range $[V^-, V^+]$
(may be ordinal with numerical values 1, 2, 3, ...)
and a network X where ties $i \to j$ may be regarded
as a 'positive' choice by sender ego of receiver alter.

The part of the evaluation function depending on V
is supposed to be given by

$$\sum_j x_{ij}\, a(v_j \mid v_i)$$

where $a(v_j \mid v_i)$ is called the 'attraction function', and expresses the tendency for actors with value v_i to send ties to actors with value v_j.

Four 'mechanisms' are considered for
how V is associated with network X:

1. homophily
   (attraction of ego, with value v_i) , to alters with similar values v_j)
2. aspiration (attraction toward alters with high values v_j)
3. conformity (attraction toward alters with 'normal' values v_j
   where 'normal' is defined by a social norm,
   located at a value V^{norm}).
4. sociability
   (higher tendency to send ties as ego's value v_i is higher).

Modeling attraction in SAOMs

These four mechanisms are expressed jointly by the function

$$a(v_j \mid v_i) \;=\; \theta_1\,(v_j - v_i)^2 \;+\; \theta_2\,v_j^2 \;+\; \theta_3\,v_j \;+\; \theta_4\,v_i$$
$$\sim\; \theta_1\,(v_j - v_i)^2 \;+\; \theta_2\left(v_j + \frac{\theta_3}{2\theta_2}\right)^2 \;+\; \theta_4\,v_i\;.$$

('$\sim$' means the difference is a constant;
this will be absorbed by the outdegree parameter.)
$-\theta_1$ is a weight for homophily,
$-\theta_2$ is a weight for conformity with the normative value

$$V^{\text{norm}} \;=\; -\frac{\theta_3}{2\,\theta_2}\;,$$

parameter θ_4 can be used to express lower or higher sociability.

Location of the optimum

If $\theta_1 + \theta_2 < 0$ this function is unimodal,
with maximum for given v_i attained at the value for v_j given by

$$v_j \;=\; v^{\max}(v_i, \theta) \;=\; \frac{\theta_1 \, v_i \;-\; \theta_3/2}{\theta_1 \;+\; \theta_2} \;=\; \frac{\theta_1 \, v_i \;+\; \theta_2 \, V^{\rm norm}}{\theta_1 \;+\; \theta_2} \;,$$

if this is in the range $[V^-, V^+]$.
If $\theta_1 < 0$ and $\theta_2 < 0$ this is a weighted mean of v_i and $V^{\rm norm}$.

If $V^- \leq V^{\rm norm} = -\theta_3/(2\,\theta_2) \leq V^+$,
the term $\theta_2 \, v_j^2 \;+\; \theta_3 \, v_j$ can be interpreted as conformity, attraction toward the social norm $V^{\rm norm}$.

Aspiration

What is aspiration? Three definitions, from weak to strong:

1. The norm $V^{\rm norm}$ is higher than the mean $\bar{V}$.
   For centered V (i.e., $\bar{V} = 0$), equivalent to $\theta_3 > 0$.
2. The normative contribution
   $$\theta_2 \left(v_j + \frac{\theta_3}{2\theta_2} \right)^2$$
   is increasing in v_j throughout $V^- \leq v_j \leq V^+$.
   If $\theta_2 < 0$, equivalent to $V^{\rm norm} \geq V^+$.
   If $\theta_2 > 0$, equivalent to $-\theta_3/(2\,\theta_2) \leq V^-$.
3. Aspiration trumps homophily for everybody, i.e.,
   $a(v_j \mid v_i)$ is increasing in v_j for all v_i.
   If $\theta_1 < 0,\; \theta_2 < 0$, this is equivalent to $v^{\max}(v_i = V^-, \theta) \geq V^+$,
   and to $V^{\rm norm} \geq \; V^+ \;+\; \theta_1 \big(V^+ \;-\; V^-\big)/\theta_2$.

Sociability

Tendency toward sociability for an actor i as depending on v_i can be expressed by maximum of attraction function

$$a^{\max}(v_i) = \max_{v_j} a(v_j \mid v_i) .$$

When this is increasing in v_i,
V may be said to have a positive sociability dimension.

Full quadratic model

To treat incoming and outgoing ties similarly, a quadratic ego effect may be added:

$$\theta_1 (v_j - v_i)^2 + \theta_2 v_j^2 + \theta_3 v_j + \theta_4 v_i + \theta_5 v_i^2$$
$$= \theta_1 (v_j - v_i)^2 + \theta_2 \left(v_j + \frac{\theta_3}{2\theta_2}\right)^2 + \theta_4 v_i + \theta_5 v_i^2 .$$

Include θ_5 if there are good reasons for it (empirical or theoretical).

Effects: diffSqX, altSqX, altX, egoX, egoSqX.

Summary: four confounded mechanisms / dimensions

$$\theta_1\,(v_j - v_i)^2 \;+\; \theta_2\left(v_j + \frac{\theta_3}{2\theta_2}\right)^2 \;+\; \theta_4\,v_i \qquad \left(\;+\;\theta_5\,v_i^2\right)$$

1. Test homophily by $-\theta_1$ (one-sided).
2. Test conformity by $-\theta_2$ (one-sided).
3. Test / express aspiration by checking the three definitions involving θ_3, θ_2, and the distribution of V.
   Note that aspiration is a special case of conformity:
   aspiration = all agree that high v_j values are desirable.
4. Express sociability by looking at the function $a^{\max}(v_i)$,
   to which θ_4 and θ_5 have important contributions.

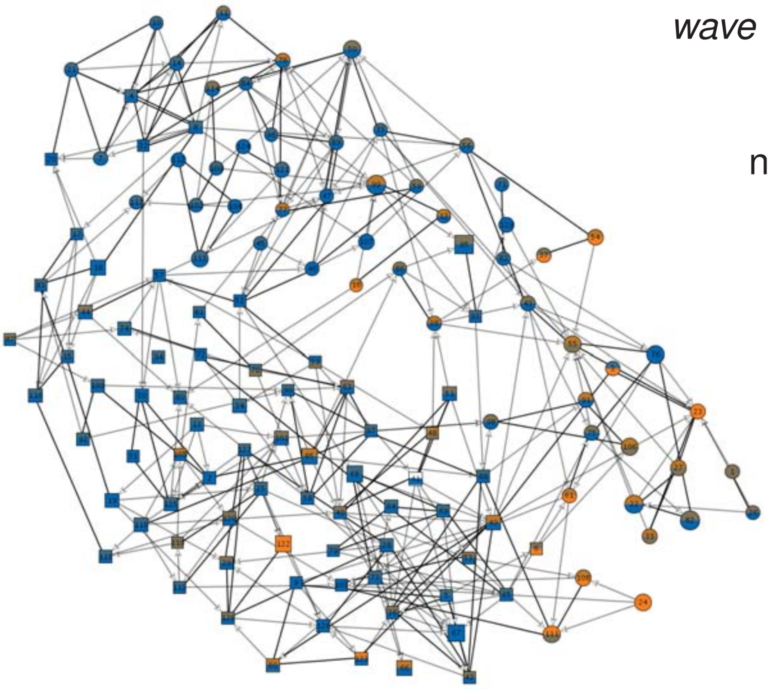
Example :

Study of smoking initiation and friendship
(following up on earlier work by P. West, M. Pearson & others)
(Steglich, Snijders & Pearson, *Sociological Methodology*, 2010).

One school year group from a Scottish secondary school
starting at age 12-13 years, was monitored over 3 years;
total of 160 pupils, of which 129 pupils present at all 3 observations;
with sociometric & behaviour questionnaires at three moments, at
appr. 1 year intervals.

Smoking: values 1–3;
drinking: values 1–5;

covariates:
gender, smoking of parents and siblings (binary), pocket money.

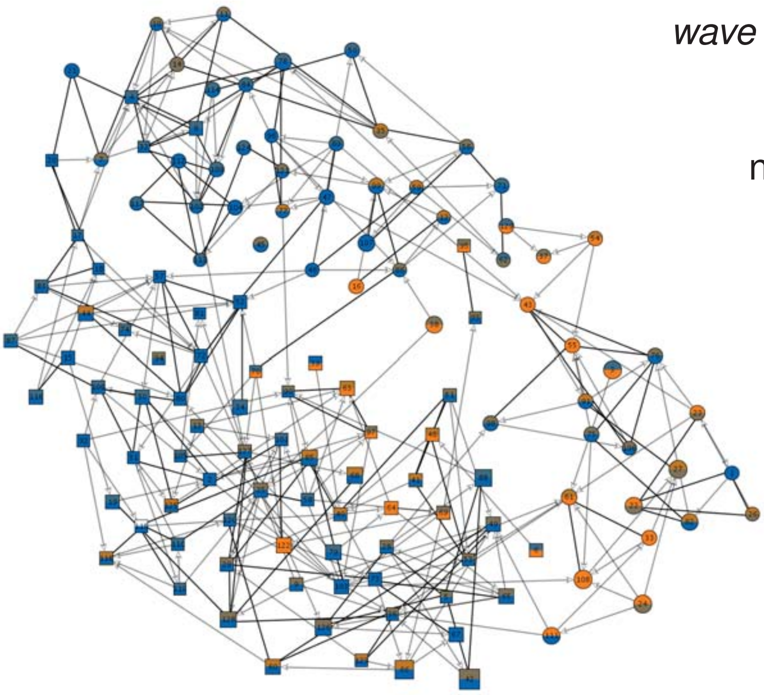


wave 1

girls: circles
boys: squares

node size: pocket money

color: top = drinking
bottom = smoking
(orange = high)

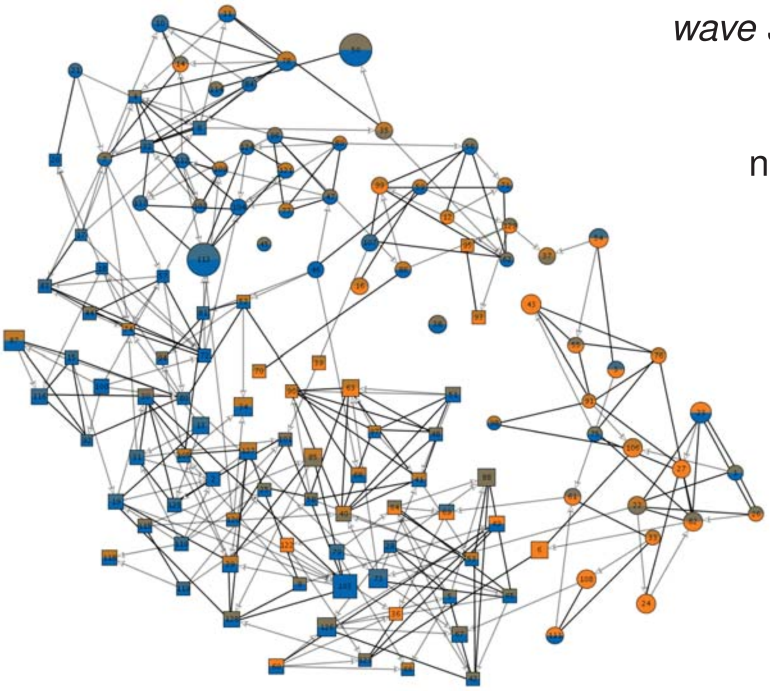


wave 2

girls: circles
boys: squares

node size: pocket money

color: top = drinking
bottom = smoking
(orange = high)



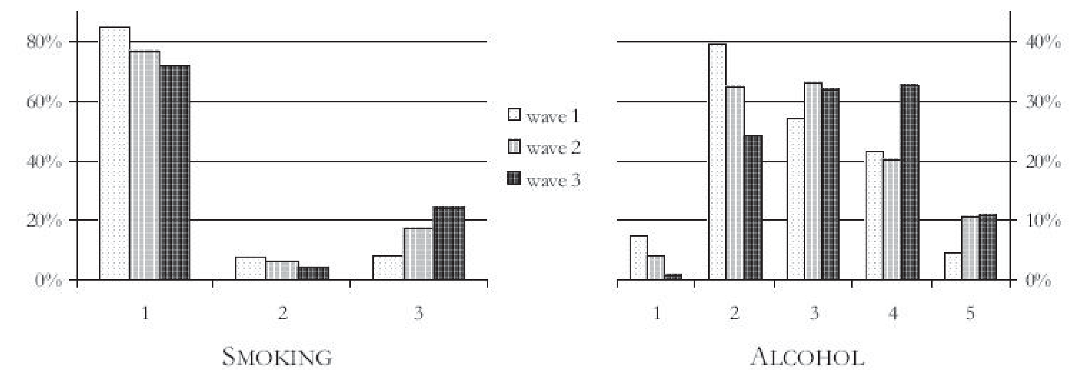
wave 3

girls: circles
boys: squares

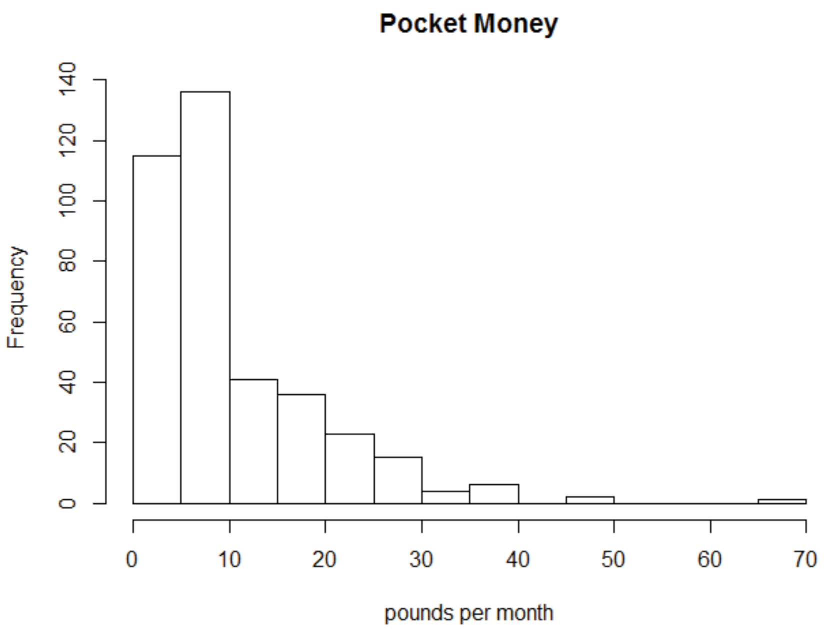
node size: pocket money

color: top = drinking
bottom = smoking
(orange = high)

FIGURE 2. — OBSERVED DISTRIBUTION OF SUBSTANCE USE IN THE THREE WAVES.



Histogram of available pocket money.



Estimation results: structural and sex effects.

Effect	par.	(s.e.)
Rate 1	11.756	(1.116)
Rate 2	9.528	(0.879)
outdegree	–2.984***	(0.255)
reciprocity	3.440***	(0.302)
GWESP-FF ($\alpha = 0.3$)	2.442***	(0.127)
indegree - popularity	–0.045*	(0.020)
outdegree - activity	0.046	(0.041)
reciprocal degree - activity	–0.146*	(0.071)
indegree - activity	–0.122**	(0.043)
sex alter	–0.091	(0.095)
sex ego	0.014	(0.102)
same sex	0.555***	(0.083)
reciprocity $\times$ GWESP-FF	–0.942***	(0.245)

Estimation results: effects of numerical actor variables.

Effect	par.	(s.e.)
drinking alter	–0.002	(0.042)
drinking squared alter	–0.039	(0.036)
drinking ego	0.094†	(0.049)
drinking e–a difference squared	–0.033†	(0.018)
smoking alter	0.114	(0.072)
smoking ego	–0.086	(0.076)
smoking similarity	0.305*	(0.123)
money/10 alter	0.102	(0.069)
money/10 squared alter	0.062†	(0.037)
money/10 ego	–0.074	(0.060)
money/10 e–a difference squared	–0.068**	(0.024)

† $p < 0.1$; * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$;

convergence t ratios all < 0.05; Overall maximum convergence ratio 0.11.

For smoking (values 1-2-3), the quadratic model was not helpful and the simpler model with ego, alter, and similarity effects was satisfactory.

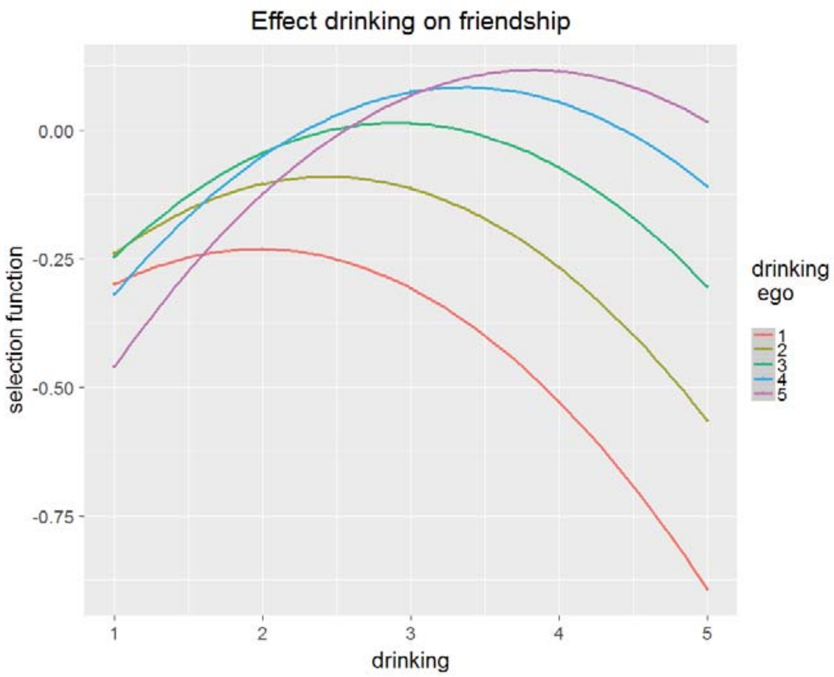
For drinking as well as for pocket money,
the squared ego effect was non significant and therefore dropped.

Joint effect of drinking: $\chi^2_4 = 11.3, p = 0.01$.
Joint effect of smoking: $\chi^2_3 = 10.5, p = 0.02$.
Joint effect of pocket money: $chi^2_4 = 16.7, p < 0.005$.

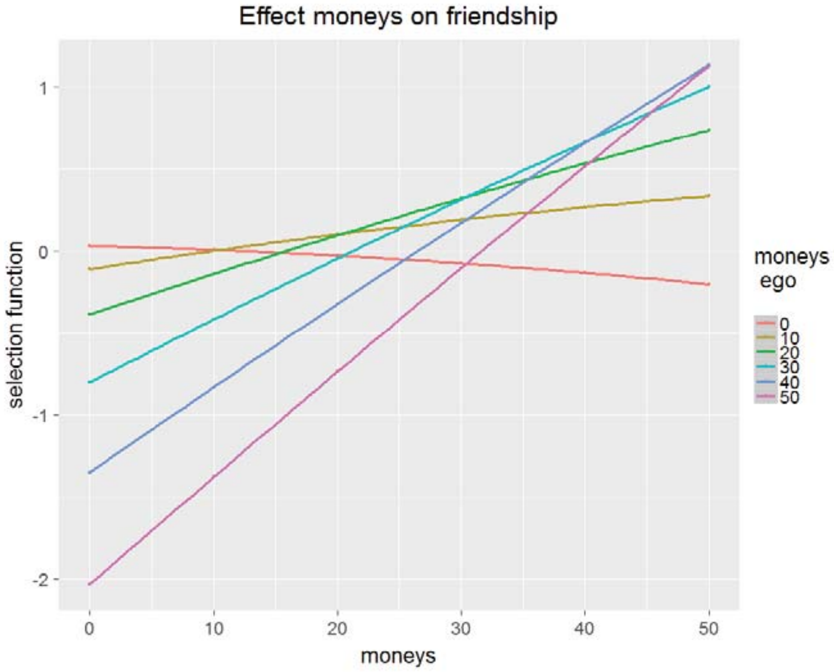
The parameters for each actor variable can be interpreted jointly.
The following pages plot the values of $a(v_j \mid v_i)$
for the various actor variables

(as a function of v_j; separate curves for several v_i).

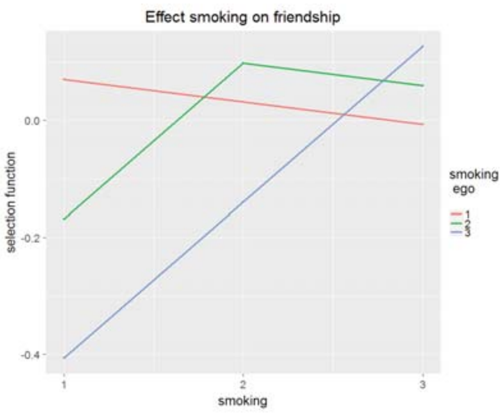
See the manual: section *Ego-alter selection tables*.

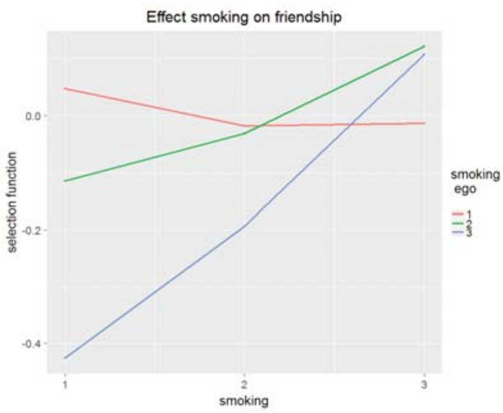


Mainly homophily.



Aspiration, except for those who have no money themselves.





Left: ego – alter – similarity;
Right: quadratic model.

Both seem to fit well.

In both models, the contrast between the values for (ego=2, alter=2) and (ego=2 , alter=3) is non-significant.

The procedures are implemented in the R package

R
S imulation
I nvestigation for
E mpirical
N etwork
A nalysis

(frequently updated) with the website

http://www.stats.ox.ac.uk/siena/.

(programmed by Tom Snijders, Ruth Ripley, Krists Boitmanis, Felix Schönenberger; contributions by many others).

5. Networks as dependent and independent variables

Co-evolution

Simultaneous endogenous dynamics of networks and behaviour: e.g.,

- individual humans & friendship relations:
  attitudes, behaviour (lifestyle, health, etc.)
- individual humans & cooperation relations:
  work performance
- companies / organisations & alliances, cooperation:
  performance, organisational success.

Two-way influence between networks and behaviour

Relational embeddedness is important
for well-being, opportunities, etc.

Actors are influenced in their behaviour, attitudes, performance
by other actors to whom they are tied
e.g., network resources (social capital), social control.

(N. Friedkin, *A Structural Theory of Social Influence*, C.U.P., 1998).

In return, many types of tie
(friendship, cooperation, liking, etc.)
are influenced positively by
similarity on relevant attributes: *homophily*
(e.g., McPherson, Smith-Lovin, & Cook, *Ann. Rev. Soc.*, 2001.)

More generally, actors choose relation partners
on the basis of their behaviour and other characteristics
(similarity, opportunities for future rewards, etc.).

Influence, network & behaviour effects on *behaviour*;
Selection, network & behaviour effects on *relations*.

Terminology

relation = network = pattern of ties in group of actors;
behaviour = any individual-bound changeable attribute
(including attitudes, performance, etc.).

Relations and behaviours are endogenous variables
that develop in a simultaneous dynamics.

Thus, there is a feedback relation in the dynamics
of relational networks and actor behaviour / performance:
macro $\Rightarrow$ micro $\Rightarrow$ macro $\cdot\ \cdot\ \cdot\ \cdot$

(although network perhaps is meso rather than macro)

The investigation of such social feedback processes is difficult:

- Both the *network* $\Rightarrow$ *behaviour*
and the *behaviour* $\Rightarrow$ *network* effects
lead ‘network autocorrelation’:
“friends of smokers are smokers”
“high-reputation firms don’t collaborate
with low-reputation firms”.
It is hard to ascertain the strengths
of the causal relations in the two directions.
- For many phenomena
quasi-continuous longitudinal observation is infeasible.
Instead, it may be possible to observe
networks and behaviours at a few discrete time points.

Data

One bounded set of actors
(e.g. school class, group of professionals, set of firms);

several discrete observation moments;

for each observation moment:

- network: who is tied to whom
- behaviour of all actors

Aim: disentangle effects *networks* $\Rightarrow$ *behaviour*
from effects *behaviour* $\Rightarrow$ *networks*.

Notation:

Integrate the *influence* (dep. var. = behaviour)
and *selection* (dep. var. = network) processes.

In addition to the network X, associated to each actor i
there is a vector $Z_i(t)$ of actor characteristics
indexed by $h = 1, \ldots, H$.
Assumption: ordered discrete
(simplest case: one dichotomous variable).

Actor-driven models

Each actor "controls" not only his outgoing ties,
collected in the row vector $\big(X_{i1}(t), ..., X_{in}(t)\big)$,
but also his behaviour $Z_i(t) = \big(\ Z_{i1}(t), ..., Z_{iH}(t)\ \big)$
(H is the number of dependent behaviour variables).

Network change process and behaviour change process
run simultaneously, and influence each other
being each other's changing constraints.

At stochastic times
(*rate functions* λ^X for changes in network,
λ^{Z_h} for changes in behaviour h),
the actors may change a tie or a behaviour.

Probabilities of change are increasing functions of
objective functions of the new state,
defined specifically for network, f^X,
and for each behaviour, f^{Z_h} .

Again, only the smallest possible steps are allowed:
change one tie variable,
or move one step up or down on a behaviour variable.

For network change, change probabilities are as before.

For the behaviours, the formula of the change probabilities is

$$p_{ihv}(\beta, z) = \frac{\exp\big(f(i,h,v)\big)}{\sum_u \exp\big(f(i,h,u)\big)}$$

where $f(i,h,v)$ is the objective function calculated
for the potential new situation after a behaviour change,

$$f(i,h,v) = f_i^z\big(\beta, z(i, h \rightsquigarrow v)\big) \ .$$

Again, multinomial logit form.
The summation in the denominator extends
over the 2 or 3 options of permitted changes in $\{-1, 0, +1\}$.

Again, an 'optimizing' interpretation is possible.

Micro-step for change in network:

Remember: at random moments occurring at a rate λ_i^X,
actor i is designated
to make a change in one tie variable:
the *micro-step* (on $\Rightarrow$ off, or off $\Rightarrow$ on.)

micro-step for change in behaviour:

At random moments occurring at a rate $\lambda_i^{Z_h}$,
actor i is designated to make a change in behaviour h
(one component of Z_i, assumed to be ordinal):
the *micro-step* is a change to an adjacent category; or stay the same.

Again, many micro-steps can *accumulate* to big differences.

Optimizing interpretation:

When actor i 'may' change an outgoing tie variable to some actor j,
he/she chooses the 'best' j by maximizing
the evaluation function $f_i^X(\beta, X, z)$ of the situation obtained
after the coming network change
plus a random component representing unexplained influences;

and when this actor 'may' change behaviour h,
he/she chooses the "best" change (up, down, nothing)
by maximizing the evaluation function $f_i^{Z_h}(\beta, x, Z)$ of the situation
obtained after the coming behaviour change
plus a random component representing unexplained influences.

There is no comparison network — behaviour.

Dynamics of networks and behaviour

Optimal network change:

The new network is denoted by $x^{(\pm ij)}$.
The attractiveness of the new situation
(evaluation function plus random term)
is expressed by the formula

$$f_i^X(\beta, x^{(\pm ij)}, z) \quad + \quad \underset{\substack{\Uparrow \\ \text{random component}}}{U_i^X(t, x, j)}\,.$$

(Note that the network is also permitted to stay the same.)

Optimal behaviour change:

Whenever actor i may make a change in variable h of Z,
he changes only one behaviour, say z_{ih}, to the new value v.
The new vector is denoted by $z(i, h \rightsquigarrow v)$.
Actor i chooses the "best" h, v by maximizing the objective function of the situation obtained after the coming behaviour change plus a random component:

$$f_i^{Z_h}(\beta, x, z(i, h \rightsquigarrow v)) \quad + \quad \underbrace{U_i^{Z_h}(t, z, h, v)}_{\Uparrow \atop \text{random component}} .$$

(behaviour is permitted to stay the same.)

Specification of the behaviour model

Many different reasons why networks are important for behaviour; e.g.

1. *imitation* :
   individuals imitate others
   (basic drive; uncertainty reduction).
2. *social capital* :
   individuals may use resources of others;
3. *coordination* :
   individuals can achieve some goals
   only by concerted behaviour;

Theoretical elaboration helpful for a good data analysis.

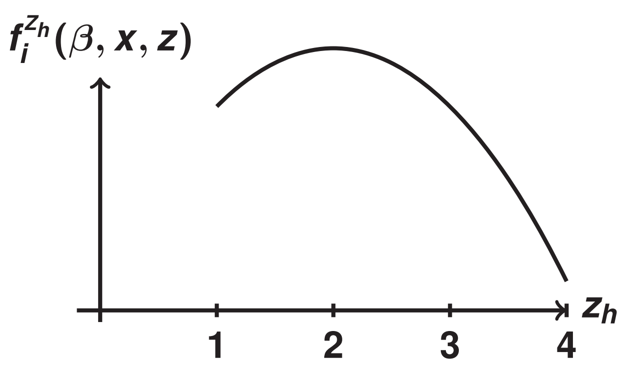
Evaluation function for dynamics of behaviour f_i^Z is again a linear combination

$$f_i^Z(\beta, x, z) = \sum_{k=1}^{L} \beta_k \, s_{ik}(x, z) \, .$$

Basic effects:

1. *linear shape* ,
   $s_{i1}^Z(x, z) = z_{ih}$
2. *quadratic shape*, ‘effect behaviour on itself’,
   $s_{i2}^Z(x, z) = z_{ih}^2$
   Quadratic shape effect important for model fit.

For a negative quadratic shape parameter,
the model for behaviour is a unimodal preference model.



For positive quadratic shape parameters ,
the behaviour objective function can be bimodal
(‘positive feedback’).

3. *behaviour-related average similarity*,
   average of behaviour similarities between i and friends
   $s_{i3}(x) = \frac{1}{x_{i+}} \sum_j x_{ij}\, \mathrm{sim}(z_{ih}, z_{jh})$
   where $\mathrm{sim}(z_{ih}, z_{jh})$ is the similarity between v_i and v_j,
   $$\mathrm{sim}(z_{ih}, z_{jh}) \;=\; 1 - \frac{|z_{ih} - z_{jh}|}{R_{Z^h}} \;,$$
   R_{Z^h} being the range of Z^h;
4. *average behaviour alter* — an alternative to similarity:
   $s_{i4}(x, z) = z_{ih} \frac{1}{x_{i+}} \sum_j x_{ij} z_{jh}$

Effects 3 and 4 are alternatives for each other:
they express the same theoretical idea of influence
in mathematically different ways.
The data, and/or theory, will have to differentiate between them.

Network position can also have influence on behaviour dynamics
e.g. through degrees rather than through behaviour
of those to whom one is tied:

5. *popularity-related tendency*, (in-degree)
   $s_{i5}(x, z) = z_{ih}\, x_{+i}$
6. *activity-related tendency*, (out-degree)
   $s_{i6}(x, z) = z_{ih}\, x_{i+}$

7. *dependence on other behaviours* $(h \neq \ell)$,
   $s_{i7}(x, z) = z_{ih}\, z_{i\ell}$
8. *influence from other characteristics V*
   $s_{i8}(x, z) = z_{ih}\, \frac{1}{x_{i+}} \sum_j x_{ij} v_j$,
   analogous to average alter for behaviour.

For both the network and the behaviour dynamics,
extensions are possible depending on the network position.

Now focus on the *similarity effect* in evaluation function :

sum of absolute behaviour differences between *i* and his friends
$s_{i2}(x, z) = \sum_j x_{ij}\, \mathrm{sim}(z_{ih}, z_{jh})$.

This is fundamental both
to network selection based on behaviour,
and to behaviour change based on network position.

A positive coefficient for this effect means that the actors prefer friends with similar Z_h values (*network autocorrelation*).

Actors can attempt to attain this by changing their own Z_h value to the average value of their friends (*network influence*, *contagion*),

or by becoming friends with those with similar Z_h values (*selection on similarity*).

Statistical estimation: networks & behaviour

Procedures for estimating parameters in this model are similar to estimation procedures for network-only dynamics: Methods of Moments & Stochastic Approximation, conditioning on the first observation $X(t_1), Z(t_1)$.

The two different effects,
networks $\Rightarrow$ behaviour and behaviour $\Rightarrow$ networks,
both lead to network autocorrelation of behaviour;

but they can be (in principle)
distinguished empirically by the time order: respectively association between ties at t_m and behaviour at t_{m+1};
and association between behaviour at t_m and ties at t_{m+1}.

Statistics for use in method of moments:

for estimating parameters in network dynamics:

$$\sum_{m=1}^{M-1} \sum_{i=1}^{n} s_{ik}(X(t_{m+1}), Z(t_m)) \ ,$$

and for the behaviour dynamics:

$$\sum_{m=1}^{M-1} \sum_{i=1}^{n} s_{ik}(X(t_m), Z(t_{m+1})) \ .$$

‘cross-lagged statistics’.

The data requirements for these models are strong:
few missing data; enough change on the behavioural variable.

Currently, work still is going on about good ways
for estimating parameters in these models.

Maximum likelihood estimation procedures
(currently even more time-consuming; under construction...)
are preferable for small data sets.

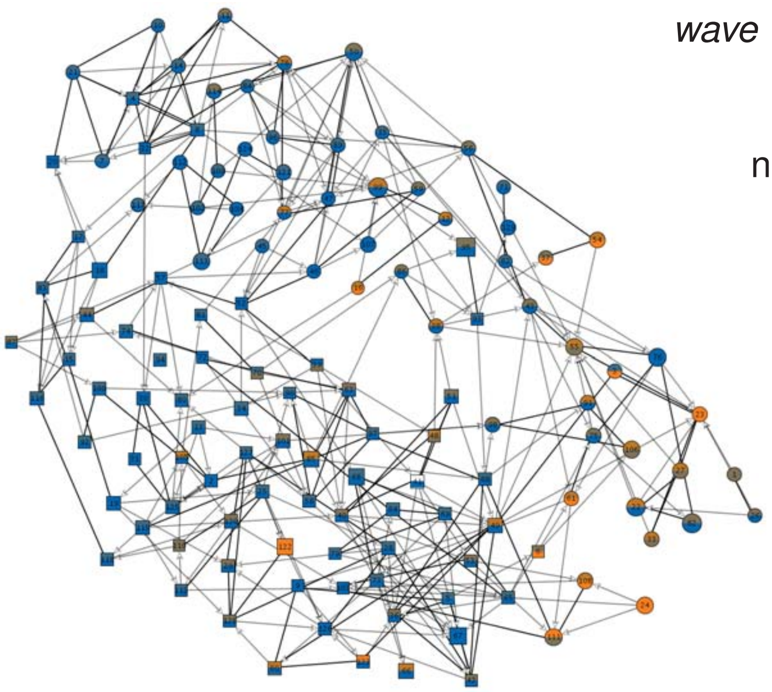
Example :

Study of smoking initiation and friendship
(following up on earlier work by P. West, M. Pearson & others)
(Steglich, Snijders & Pearson, *Sociological Methodology*, 2010).

One school year group from a Scottish secondary school
starting at age 12-13 years, was monitored over 3 years;
total of 160 pupils, of which 129 pupils present at all 3 observations;
with sociometric & behaviour questionnaires at three moments, at
appr. 1 year intervals.

Smoking: values 1–3;
drinking: values 1–5;

covariates:
gender, smoking of parents and siblings (binary), pocket money.

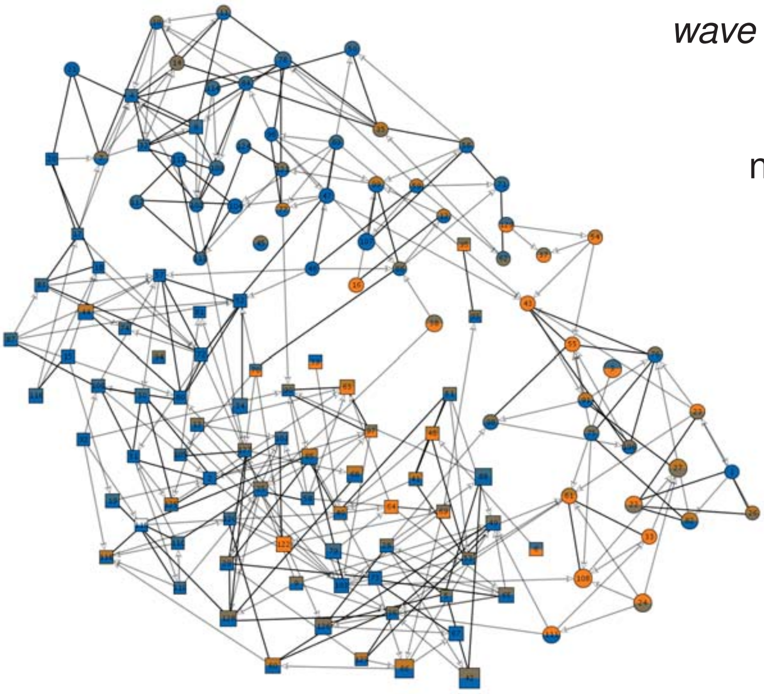


wave 1

girls: circles
boys: squares

node size: pocket money

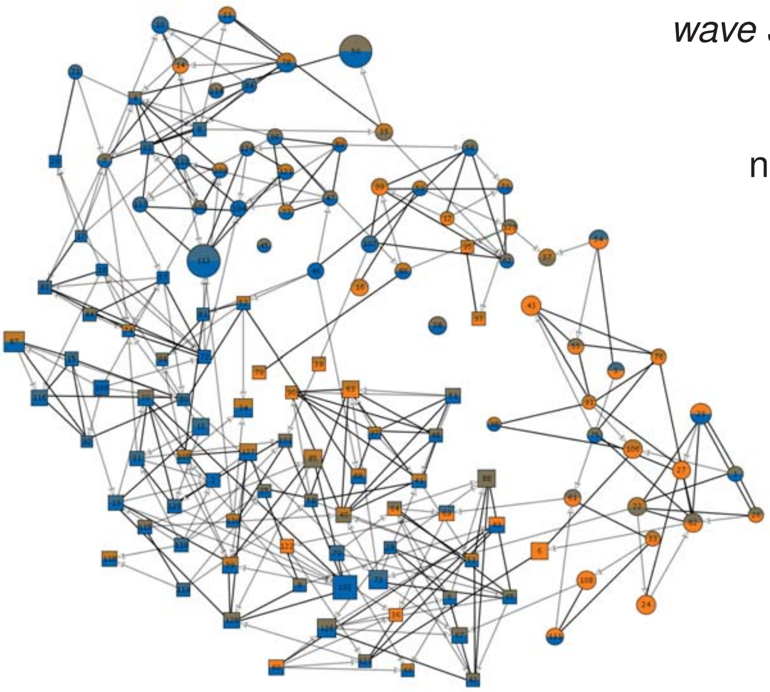
color: top = drinking
bottom = smoking
(orange = high)



wave 2 girls: circles
boys: squares

node size: pocket money

color: top = drinking
bottom = smoking
(orange = high)



wave 3 girls: circles
boys: squares

node size: pocket money

color: top = drinking
bottom = smoking
(orange = high)

Descriptives of covariate change: drinking

Observed changes in alcohol use in the Glasgow data, pooled over periods.

		t_{end}				
		1	2	3	4	5
	1: I don’t drink (alcohol)	3	3	5	1	0
	2: once or twice a year	0	35	27	14	3
t_{begin}	3: about once a month	1	13	31	20	3
	4: about once a week	0	4	10	25	8
	5: more than once a week	0	0	2	4	11

The idea of an underlying process of micro-steps seems reasonable.

Descriptives of covariates: smoking

Observed changes in tobacco use, pooled over periods.

		t_{end}		
		1	2	3
	1: non-smoker	193	9	18
t_{begin}	2: occasional smoker	6	3	9
	3: regular smoker	3	3	27

Not so much variation.

Results

The table of results is distributed over 4 pages:

- structural effects and effect of sex
- friendship: effects of smoking, drinking, pocket money
- drinking
- smoking.

Effect	par.	(s.e.)
Network Dynamics		
constant friendship rate (period 1)	11.403	(1.147)
constant friendship rate (period 2)	9.237	(0.943)
outdegree (density)	–2.693***	(0.312)
reciprocity	3.388***	(0.290)
GWESP-FF ($\alpha = 0.30$)	2.430***	(0.131)
indegree - popularity	–0.053*	(0.024)
outdegree - activity	0.030	(0.044)
reciprocal degree - activity	–0.143*	(0.068)
indegree - activity	–0.120**	(0.046)
sex alter	–0.084	(0.101)
sex ego	0.017	(0.111)
same sex	0.558***	(0.087)
reciprocity $\times$ GWESP-FF	–0.913***	(0.256)

Network Dynamics		
Effect	par.	(s.e.)
Network Dynamics		
drinking alter	–0.016	(0.093)
drinking squared alter	–0.107	(0.096)
drinking ego	0.183†	(0.108)
drinking e–a difference squared	–0.090	(0.058)
smoking alter	0.132	(0.098)
smoking ego	–0.177	(0.116)
smoking similarity	0.437*	(0.179)
money/10 alter	0.105	(0.075)
money/10 squared alter	0.063	(0.040)
money/10 ego	–0.103	(0.076)
money/10 e–a difference squared	–0.067**	(0.025)

Effect	par.	(s.e.)
Behaviour Dynamics: drinking		
rate drinking (period 1)	1.634	(0.336)
rate drinking (period 2)	2.454	(0.534)
drinking linear shape	0.436**	(0.141)
drinking quadratic shape	–0.605**	(0.192)
drinking average alter	1.226*	(0.545)
drinking: effect from sex	0.068	(0.212)
drinking: effect from smoking	–0.096	(0.202)
drinking: effect from moneys	0.021	(0.015)

Effect	par.	(s.e.)
Behaviour Dynamics: smoking		
rate smoking (period 1)	4.389	(1.686)
rate smoking (period 2)	4.162	(1.345)
smoking linear shape	–3.375***	(0.356)
smoking quadratic shape	2.595***	(0.332)
smoking average alter	1.562**	(0.600)
smoking: effect from sex	–0.002	(0.270)
smoking: effect from smoking at home	–0.114	(0.264)
smoking: effect from drinking	–0.113	(0.245)
smoking: effect from moneys	0.016	(0.019)

† $p < 0.1$; * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$;

convergence t ratios all < 0.03. Overall maximum convergence ratio 0.11.

The results for the structural network effects and for the effect of sex and money are almost the same as for the network-only analysis;
the effects of smoking and drinking on friendship are somewhat different, and have smaller standard errors;
their joint effect tests are less strongly significant.

Joint effect of drinking: $\chi^2_4 = 6.2, p = 0.19$.
Joint effect of smoking: $\chi^2_3 = 8.9, p = 0.03$.
Joint effect of pocket money: $\chi^2_4 = 15.3, p < 0.005$.

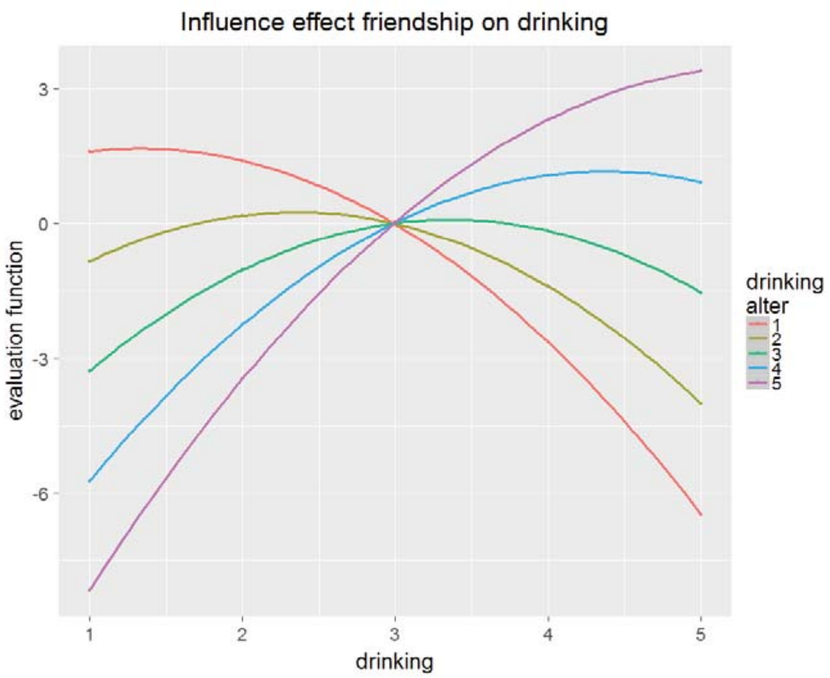
The influence effects for smoking and drinking are significant.

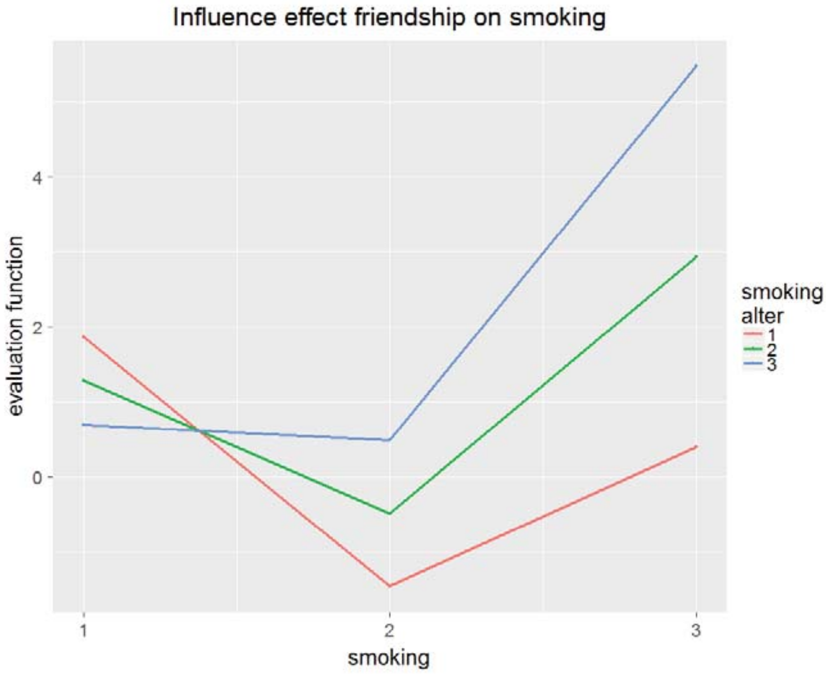
By the way, if for drinking the model is specified as ego, alter, and similarity, then similarity is marginally significant ($t = 1.62,\ p = 0.06$); this illustrates the importance of choosing the model
before looking at results in case of a strict testing approach.

Parameter interpretation for behaviour change

The evaluation function for behaviour can be plotted as a function of Z, the behavior itself, for various different values of the average behaviour of the friends ('average alter').

This is treated in the manual as the *Ego-alter Influence Table*, and the website contains a script `InfluenceTables.r`.





Mind the different shapes of the functions for smoking and drinking:

For drinking, the influence function is concave,
and it is convex for smoking.
This is expressed by
the sign of the coefficient of the quadratic shape effect,
which is the quadratic term in the evaluation function.

Co-evolution, more generally

The idea of '*network-behaviour co-evolution*':

network is considered as one complex variable $X(t)$;

behaviour is considered as one complex variable $Z(t)$;

these are evolving over time in mutual dependence $X(t) \leftrightarrow Z(t)$,
changes occurring in many little steps,
where changes in X are a function of the current values of $\big(X(t), Z(t)\big)$,
and the same holds for changes in Z.

This may be regarded as a 'systems approach',
and is also applicable to more than one network
and more than one behaviour.

Co-evolution of multiple networks

See Snijders – Lomi – Torló in *Social Networks*, 2013.

For example:

friendship and advice;

positive and negative ties.

Co-evolution of one-mode and two-mode networks:

e.g., friendship and shared activities,

6. Conclusion

- These models represent network structure as well as attributes / behaviour.
- Theoretically: they combine agency and structure.
- Available in package RSiena in the statistical system R.
- What was treated here is just the basic structure. Further possibilities, e.g.: multivariate, valued (only for few values!), two-mode, non-directed, continuous behaviour variables.
- Important: model choice, goodness-of-fit.
- The method is in a stage of continuous development: networks are very complicated data structures, we are only starting to understand them.

Discussion (2)

- This approach attempts to tackle peer effects questions by process modeling: data-intensive and potentially assumption-intensive. Cox / Fisher: *Make your theories elaborate.*
- This type of analysis offers a very restricted take an causality: only *time sequentiality*.
- Assessing network effects is full of confounders. Careful theory development, good data are important. Asses goodness of fit of estimated model.

What distinguishes a statistical modeling approach from other kinds of network analysis?

$\Rightarrow$ Direct combination of networks and attributes
and: combination of structure and agency.

$\Rightarrow$ Distinction dependent $\Leftrightarrow$ explanatory variables

$\Rightarrow$ Hypothesis testing,
clearer support of theory development.

$\Rightarrow$ Combination of multiple mechanisms: test theories
while controlling for alternative explanations.

$\Rightarrow$ Assessment of uncertainties in inference.

$\cdots$ . . . but the classical network studies are also important
(positions, equivalence, centrality, blockmodeling, ...) !

Other work (recent, current, near future)

1. Changing composition of node set (Huisman & Snijders, *SMR* 2003).
2. Score-type tests (Schweinberger, *BJMSP* 2011).
3. Time heterogeneity (Lospinoso et al., *ADAC* 2011),
function *sienaTimeTest*.
4. Goodness of fit (Lospinoso & Snijders, *Meth. Innovations* 2019),
function *sienaGOF*.
5. Bayesian estimation; Maximum Likelihood estimation
(Koskinen & Snijders, *J.Stat.Plann.Inf.* 2007);
(Snijders, Koskinen, & Schweinberger, *Ann.Appl.Statist.* 2010).
6. Treatment of missing data
(Krause et al., *Ital. J. Stat.*, 2018); script on website.
7. Explained variation (‘R^2’)
(Snijders, *Math.Soc.Sci*. 2004; Indlekofer, 2014); function *sienaRI*.

Model extensions

1. Non-directed relations. (Snijders & Pickup, 2017)
2. Multivariate relations. (Snijders, Lomi, & Torlò, *SoN* 2013)
3. Two-mode networks. (Koskinen & Edling, *SoN* 2011; and Snijders, Lomi, & Torlò, *SoN* 2013)
4. Multilevel network analysis (meta analysis approach) (function *siena08*; Snijders & Baerveldt, *J.Math.Soc.* 2003).
5. Random effects multilevel network models (function *sienaBayes*; Koskinen, Snijders).
6. Diffusion of innovations. (Greenan 2015).
7. Valued relations (example in Elmer, Boda, Stadtfeld, *Network Science* 2017).
8. Larger networks, dropping assumption of complete information (settings model; Preciado/Snijders).

Further study – keeping updated

1. The version of RSiena at CRAN is not so frequently updated; check website - News whether the R-Forge version is preferable.
2. Basic tutorial: Tom A.B. Snijders, Gerhard G. van de Bunt, Christian E.G. Steglich (2010), Introduction to actor-based models for network dynamics. *Social Networks*, 32, 44–60.
3. The manual (available from website) has a lot of material.
4. Go through the website to see what's there:
   http://www.stats.ox.ac.uk/siena/
   For example, many useful scripts!
5. There is also a user's group:
   http://groups.yahoo.com/groups/stocnet/

Some references about longitudinal models

- Tom A.B. Snijders (2001),
  The Statistical Evaluation of Social Network Dynamics,
  Sociological Methodology, 31, 361–395;
- Tom A.B. Snijders (2005), Models for Longitudinal Network Data, Ch. 11 in P. Carrington, J. Scott, & S. Wasserman (Eds.), *Models and methods in social network analysis*. New York: Cambridge University Press.
- Tom A.B. Snijders (2017), Stochastic actor-oriented models for network dynamics. *Annual Review of Statistics and Its Application*, 4, 343–363.
- Tom A.B. Snijders, Gerhard G. van de Bunt, Christian E.G. Steglich (2010), Introduction to actor-based models for network dynamics. *Social Networks*, 32, 44–60.
- Tom A.B. Snijders, Johan Koskinen, and Michael Schweinberger (2010). Maximum Likelihood Estimation for Social Network Dynamics. *Annals of Applied Statistics*, 4, 567–588.
- See `SIENA` manual and homepage.

Some references about co-evolution and agent-based models.

- Yuval Kalish (2019). Stochastic Actor-Oriented Models for the co-evolution of networks and behavior: an introduction and tutorial. *Organizational Research Methods*, in press.
- Christian E.G. Steglich, Tom A.B. Snijders, and Mike Pearson (2010).
  Dynamic Networks and Behavior: Separating Selection from Influence.
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