Longitudinal Analysis of Multilevel Networks

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Overview

⇒ Types of combination multilevel $\leftrightsquigarrow$ networks

⇒ Analysis of multilevel networks (multiple node sets!): examples of representation possibilities in RSiena.

Multilevel concepts

Two kinds of combinations multilevel—networks:

Type 1: MAN = *Multilevel Analysis of Networks*:
the study of a set of 'parallel' networks according to a common model, like in Hierarchical Linear Modeling etc.;

Type 2: AMN = *Analysis of Multilevel Networks*
(Wang, Robins, Pattison, Lazega, 2013):
is the study of networks with nodes of several types, distinguishing between types of ties according to types of nodes they connect.

Type 3: AMNN = *Analysis of Networks with a Multilevel Node Set*:
one network with a node set with hierarchically structured node set.

A longitudinal network data structure can be analyzed using Stochastic Actor-oriented Models / Siena.

Analysis of Multilevel Networks (Type 2)

Multilevel network (Wang, Robins, Pattison, Lazega, *Social Networks* 2013):

Network with nodes of several types,
distinguishing between types of ties
according to types of nodes they connect.

Thus, if types of nodes are A, B, C,
distinguish between $A - A$ and $B - B$ and $C - C$ ties, (*one-mode*)
and $A - B$ and $A - C$, etc., ties (*two-mode*).

Some may be networks of interest,
others may be fixed constraints,
still others may be non-existent or non-considered.

This generalizes two-mode networks
and multivariate one mode – two mode combinations.

Emmanuel Lazega and Tom A.B. Snijders (eds)., *Multilevel Network Analysis for the Social Sciences*. Cham: Springer, 2016.

Special issue of *Social Networks* ‘Multilevel Social Networks’,
edited by Alessandro Lomi, Garry Robins, and Mark Tranmer,
vol. 44 (January 2016).

Co-evolution one-mode – two-mode:
Tom A.B. Snijders, Alessandro Lomi, and Vanina Torlò (2013),
A model for the multiplex dynamics of two-mode and one-mode networks,
with an application to employment preference, friendship, and advice.
Social Networks, 35, 265–276.

Thus, multilevel networks generalize two-mode networks.

Perhaps a better name would be multi-mode networks.

We stick to the nomenclature of Wang et al. (2013).

Two-sided agency

In the basic RSiena setup,
for two-mode networks only the first mode has agency.

‘Actors choose their activities, activities do not choose the actors.’

This is not always a good representation of the social world;
sometimes the choice is two-sided.

In the implementation of RSiena,
two-sided choices are implemented for symmetric networks:
modelType = 3.

Interlude: non-directed networks

For RSiena for non-directed networks,
see Snijders & Pickup (2016) and the manual.

`modelType = 3`, proposal–confirmation:

1. Ego proposes a tie change, based on a multinomial choice just like in the directed case.
2. If the proposal is a new tie, alter decides in a binary choice whether or not to accept.

For one-mode networks, this represents tie dynamics where both actors involved in the tie have agency (cf. Jackson & Wolinsky, *J. Economic Theory*, 1996).

This is implemented in RSiena as `modelType = 3`.

Two-mode networks represented as one-mode

This can be exploited using the RSiena feature of structural zeros,
which are elements in the adjacency matrix / tie variables
that are bound to be 0.

There are many possibilities for adapting the specification of networks in RSiena using blocks of structural zeros,
with blocks differentiating actor sets.

A basic option is the embedding of a two-mode network
in a bipartite one-mode network.

Two-mode network :

two node sets ('modes') A and B, ties only $A - B$;

Bipartite network :

one node set $N = A \cup B$ with $A \cap B = \emptyset$, ties only $A - B$.

These are subtly different.

The following page represents the two-mode network X
in two ways as a bipartite network:
directed $A \Rightarrow B$ and non-directed.

directed

$$\begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} A \quad\quad\quad\quad B \\ \begin{pmatrix} 0 & \text{two-mode } A \times B \\ 0 & 0 \end{pmatrix} \\ \quad\quad\quad\quad \text{network } X \end{array}$$

contains the same information as

non-directed

$$\begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} A \quad\quad\quad\quad B \\ \begin{pmatrix} 0 & \text{two-mode } A \times B \\ \text{two-mode } B \times A & 0 \end{pmatrix} \\ \text{network } t(X) \quad\quad \text{network } X \end{array}$$

where $t(X) = \text{transpose}(X)$.

see scripts `RscriptSienaTwoModeAsOneMode.R` and `TwoModeAsSymmetricOneMode_Siena.R` on the Siena website

It is necessary to define dummy variables representing the two types of node: 1_A and 1_B, respectively. Although these nodal variables are linearly dependent, it is convenient to have them both available.

When the rate effect of 1_B is –100 (practically $-\infty$), only the A nodes will make changes.

The directed and non-directed embeddings of the
two-mode network contain the same information,
but due to the options available in RSiena
for symmetric = non-directed networks,
they can be used in different ways.

For the usual way of modeling two-mode networks in RSiena,
the agency is in the first node set ('A'),
represented by ties being directed $A \Rightarrow B$,
and the nodes in set B 'just have to accept'.

Representing two-mode networks as one-mode networks
gives additional possibilities.

For the dictatorial model option (`modelType = 2`), the results are
identical to the results for the usual two-mode representation.

Construction of two-sided modeling option for two-mode networks

For RSiena for non-directed networks,
see Snijders & Pickup (2016) and the manual.

`modelType = 3`, proposal–confirmation:

1. Ego proposes a tie change, based on a multinomial choice just like in the directed case.
2. If the proposal is a new tie, alter decides in a binary choice whether or not to accept.

`modelType = 6`, tie-based LERGM:

1. A pair of nodes is randomly chosen.
2. For this pair a binary choice is made for the tie, based on the sum of the objective functions.

Thus, the non-directed embedding allows modeling
agency of A nodes combined with agency of B nodes.

However, the possibilities for empirical separation of A and B effects are limited;
e.g., it is impossible to distinguish whether
dispersion of A degrees is due to differential activity of A actors
or differential popularity of A nodes for B actors.

This embedding also allows having dependent actor variables
for both modes (trick by James Hollway)
(use dummy variables for both types of actor,
and give these fixed rate effects equal to –100).

Multilevel network with two node sets

Consider a multilevel network with two node sets, A and B.

Suppose there are two one-mode networks internal to A and B, and two two-mode networks X_1 from A to B; X_2 from B to A.

Specification for RSiena is possible by employing one joint node set $A \cup B$ and two dependent networks:

$$
\begin{array}{c}
\\ A \\ B
\end{array}
\begin{array}{c}
\begin{array}{cc} \quad A \quad & \quad B \quad \end{array} \\
\begin{pmatrix} \text{internal } A & 0 \\ 0 & \text{internal } B \end{pmatrix} \\
\text{networks } A,\ B
\end{array}
\quad
\begin{array}{c}
\begin{array}{cc} \quad A \quad & \quad B \quad \end{array} \\
\begin{pmatrix} 0 & \text{two-mode } A \times B \\ \text{two-mode } B \times A & 0 \end{pmatrix} \\
\text{network } X_2 \qquad \text{network } X_1
\end{array}
$$

There need to be dummy variables 1_A and 1_B
representing the two types of node,
and these can be used to separate the rate functions for A and B,
and must be interacted with all effects in the objective function
to separate this for A and B.

The ‘upper half’ A and A $\times$ B
is just the one-mode – two-mode co-evolution;
in this model this is combined with
the co-evolution of internal B and B $\times$ A.

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the co-evolution of internal B and B $\times$ A.

If there is no $B \times A$ network, one can use $X_2 = t(X_1)$, the transpose,
so that the second network is symmetric
and the options in RSiena for symmetric networks can be used;
this allows double agency,
where A and B actors decide together about $A \times B$ ties.

The separation into two networks is used to separate the A-A choices from A-B choices, and similarly the B-B choices from B-A choices (recall how RSiena handles multivariate networks).

An alternative, avoiding the large number of interactions with dummy variables 1_A and 1_B, is to separate the two internal networks as

$$\begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} \text{internal } A & 0 \\ 0 & 0 \end{pmatrix} \end{array} \quad \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 0 & 0 \\ 0 & \text{internal } B \end{pmatrix} \end{array}$$

network A network B

Note that these constructions also allow
dependent behavioral variables for node sets A as well as B.

A construction with two nested actor sets

A a set of organizations, B a set of individuals,
X_2 is a fixed membership relation (not a dependent network)
X_1 is not there;

if there are only ties between individuals within organizations,
B will be a network of diagonal blocks
and structural zeros between different organizations;

if there are within-organization as well as between-organization
ties of individuals, B can be decomposed in two further distinct networks.

If membership changes endogenously, X_2 can be a dependent network.

Conclusion

Wang et al. (2013) define multilevel networks,
and show how to estimate an ERGM for such networks
by defining the node set as the union of all node sets.

Such constructions are also possible for longitudinal analysis
according to the SAOM, by RSiena.
Dummy variables are used to define the node sets,
and structural zero blocks to define the networks.

Because the SAOM is based on ideas of agency (choices),
this requires distinguishing between node sets with,
and node sets without agency.

To separate choices by actors within a given actor set
according to the various networks in which they are involved,
it is necessary to express the data as a multivariate network.
Two-sided agency can be represented by a symmetric construction.

Multilevel constructions have been used for the co-evolution of
one-mode and two-mode networks
(Snijders, Lomi, & Torló, *Social Networks*, 2013;
and e.g., Fujimoto, Snijders, & Valente, *Network Science*, 2018);
but not yet in published articles
for more complex multilevel network structures.