Specification of Homophily in Actor-oriented Network Models

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Homophily

Homophily, the attraction of like to like,
is a fundamental characteristic of human (and other) relations
(Lazarsfeld & Merton, 1954; McPherson, Smith-Lovin & Cook, 2001).
It is mostly theorized for binary or categorical attributes
(sex, race, ...).
This presentation is about how to express homophily
for continuous (or numerical ordinal) variables
in statistical network models,
focusing on (longitudinal) actor-oriented models.

Multilevel: actor – dyad

Homophily is a concept relating the actor level to the dyadic level,
exemplifying the *multilevel nature* of network analysis,
where concepts and measures of different types of units
meet each other.

(Cf. E. Lazega & T. Snijders,
Multilevel Network Analysis for the Social Sciences, Springer, 2016.)

Usual expressions

Consider a numerical variable V.

A usual way to express tendencies toward homophily
is by letting probabilities of tie existence or creation
depend on the absolute difference between sender i and receiver j,

$$\mid v_i - v_j \mid \quad .$$

This has been proposed for many models (e.g., ERGM: Lusher et al., 2013; SAOM: Snijders, van de Bunt & Steglich, 2010; p_2: van Duijn et al., 2004; other: Powell et al., AJS 2005).

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As an alternative, it has been proposed to make the probability of tie existence or creation dependent on the ego $\times$ alter interaction

$$v_i \, v_j \text{ or } (v_i - \bar{v})(v_j - \bar{v}) \ .$$

Criticism of the usual expressions

For numerical variables, however, these models are quite poor approximations.

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are quite poor approximations.

It is helpful to conceptualize the probabilities of tie creation and existence
as being dependent on a preference or attraction function
indicating how strongly one social actor is attracted to another,
in dependence on their values of the attribute;
for some attributes this function will be unimodal (single-peaked),
for others it may be monotone ('the higher the better'),
or even bimodal (repulsive).

Diverse principles of attraction

For a given numerical attribute V and relations X,
there may be various different principles of attraction:

1. homophily: attraction to others with the same value;
2. aspiration: attraction to others with a high value;
3. conformity: attraction to others with the average/normative value;
4. and all this may be modified
   by the other effects included in the model.

Stokman and Vieth (2004, '*What attracts us when to whom?*')
also mention repulsion; this is left out of consideration here.

These principles will usually be combined.

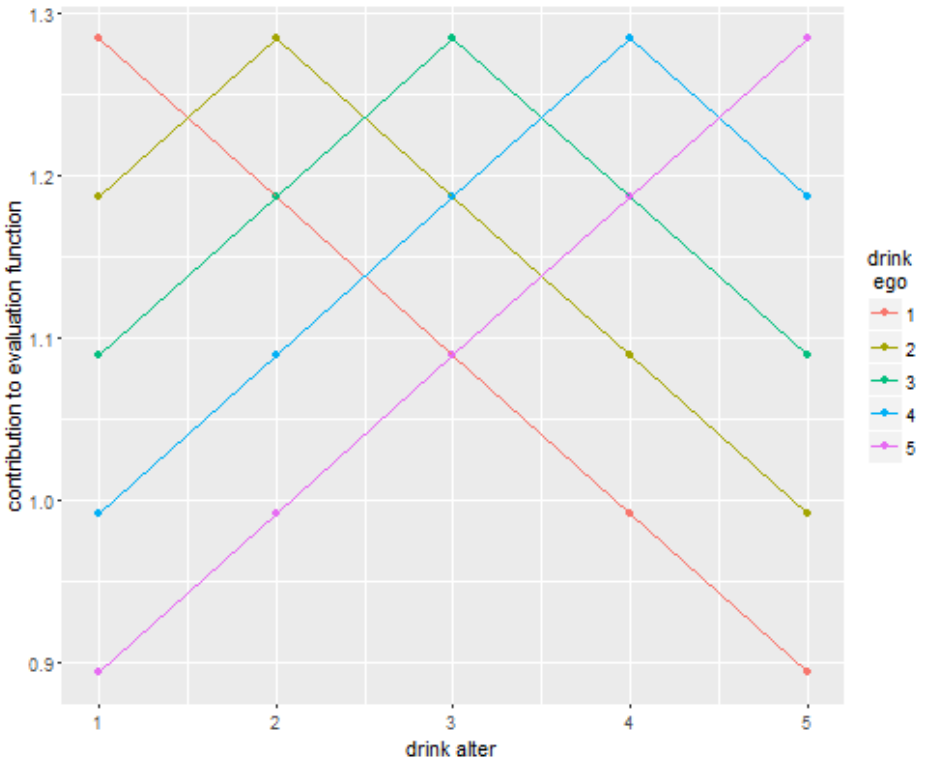
Modeling attraction in SAOMs: absolute difference

In Stochastic Actor-Oriented Models, let us call the part of the evaluation function depending on the actor covariate V the *attraction function*.

For the absolute difference, the attraction function is

$$f_1(v_j) = \beta_1 \mid v_i - v_j \mid \ .$$

$(\beta_1 < 0)$



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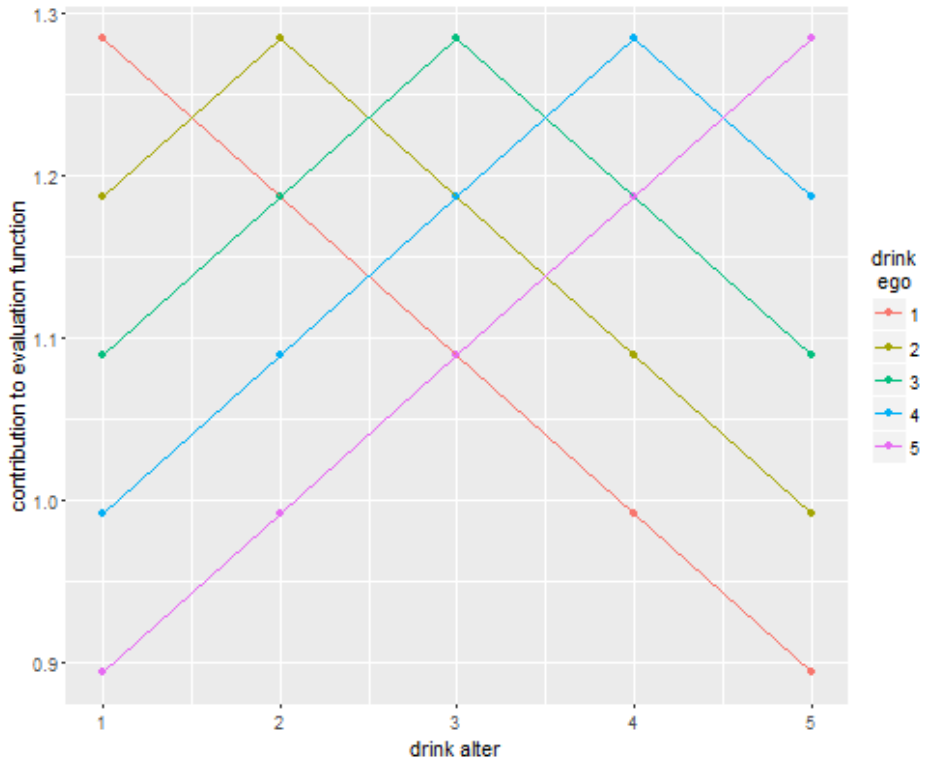
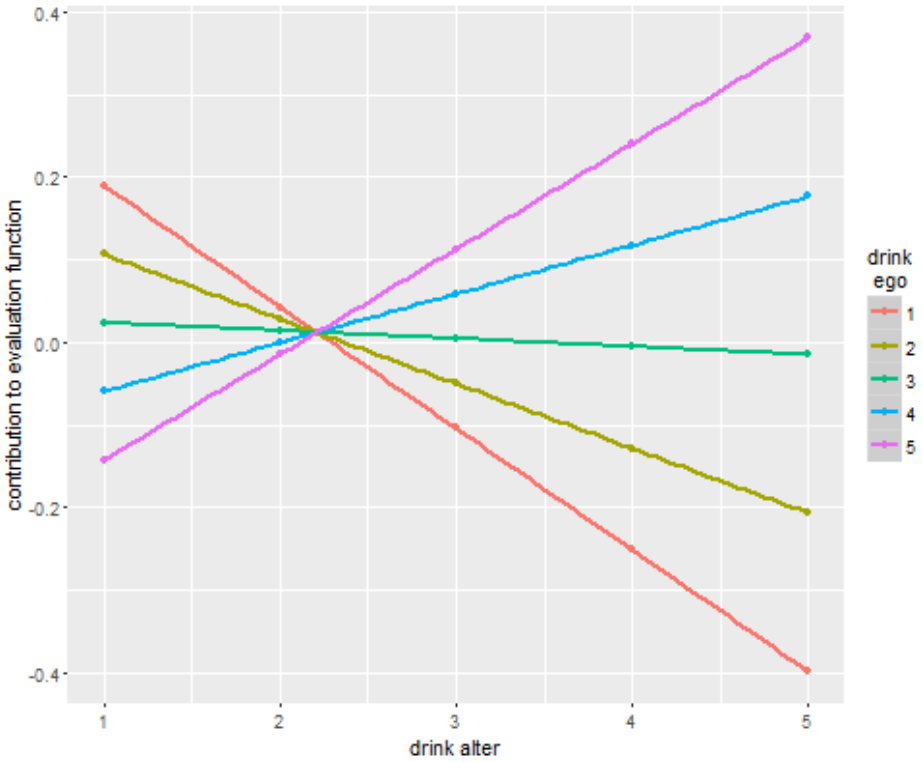


top must be at $v_j = v_i$; not quite flexible

Modeling attraction in SAOMs: ego × alter interaction

For the representation by
the ego × alter interaction,
we also need the two main effects,
and the attraction function is

$$f_2(v_j) = \beta_1 v_i + \beta_2 v_j + \beta_3 v_i v_j$$



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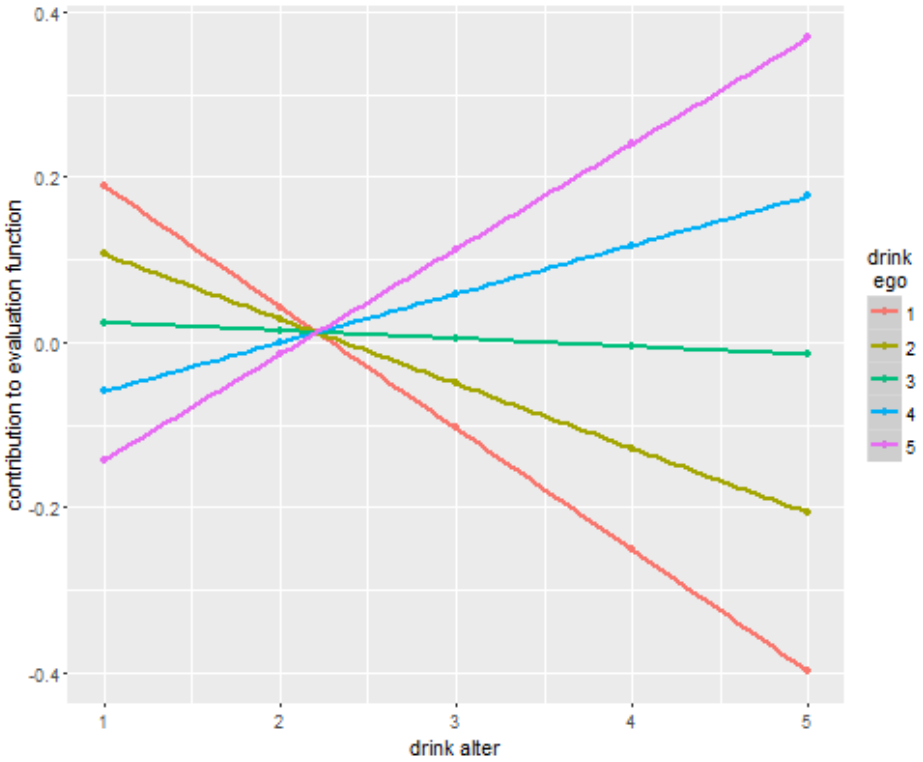


not quite a representation of homophily

Modeling attraction in SAOMs: better model

A combination of attraction to different values
can be mathematically expressed by a quadratic function of v_i and v_j,
because this allows expressing a shifted maximum
by a linear term.

To include the homophily principle, it is convenient to express the optimum as a linear function of ego's value, $v_i^{\max} = \gamma_2 + \gamma_3\, v_i$.

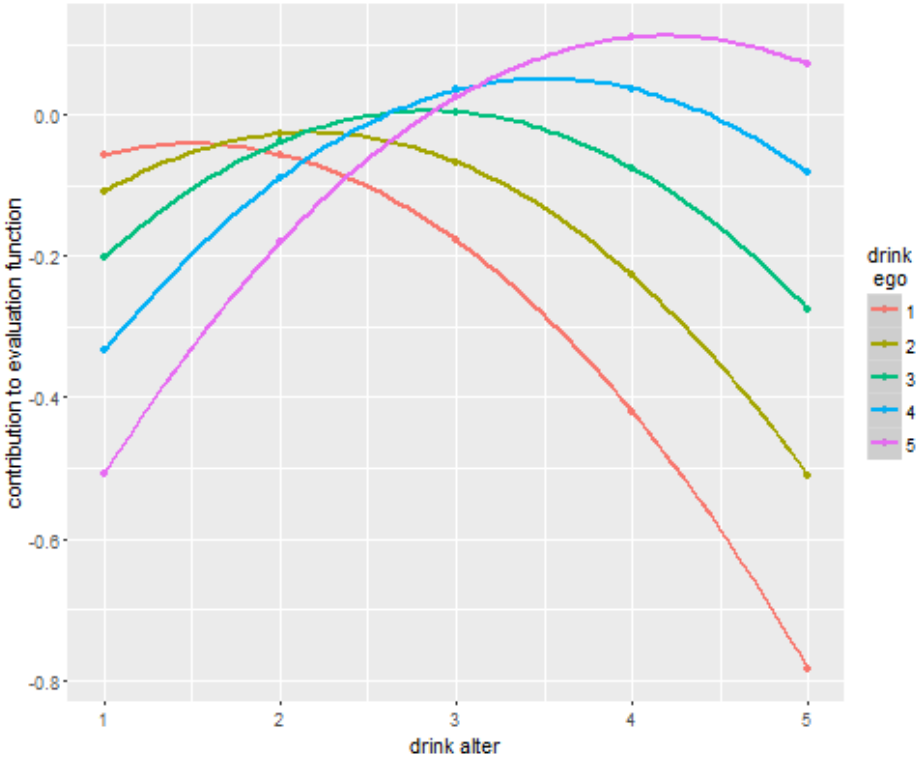
This leads to

$$\begin{aligned} f(v_j) &= \gamma_1 \left(v_j - \gamma_3\, v_i - \gamma_2\right)^2 \\ &= -2\,\gamma_1\gamma_2\, v_j + \gamma_1\, v_j^2 - 2\gamma_2\gamma_3\, v_i + \gamma_1\gamma_3^2\, v_i^2 - 2\gamma_1\gamma_3\, v_i v_j + \gamma_1\gamma_2^2\,. \end{aligned}$$

Modeling attraction in SAOMs: better model

Turning this into a function that is linear in all parameters yields

$$f_3(v_j) = \beta_1 v_j + \beta_2 v_j^2 + \beta_3 v_i + \beta_4 v_i^2 + \beta_5 v_i v_j \ .$$



Parameter interpretation

For the function

$$f_3(v_j) = \beta_1 v_j + \beta_2 v_j^2 + \beta_3 v_i + \beta_4 v_i^2 + \beta_5 v_i v_j \ :$$

1. The function is unimodal if $\beta_2 < 0$.
2. Higher values for ego go with attraction toward higher values of alter if $\beta_5 > 0$.
3. The optimum is obtained, for a given v_i and $\beta_2 < 0$, when

$$v_i^{\max} = \frac{\beta_1 + \beta_5 v_i}{-2\beta_2} \ . \tag{1}$$

When this is outside the range of V,

the function will be increasing or decreasing over the entire range.

Expectations

This means that we expect:

1. unimodality: $\beta_2 < 0$ (effect V^2 alter)
2. homophily: $\beta_5 > 0$ (effect V alter $\times$ V ego)
3. if there is an element of aspiration:

$$\frac{\beta_1 + \beta_5\,\bar{v}}{-2\beta_2} > \bar{v}$$

where $\bar{v}$ is the mean of V: on average, the attraction is to values higher than the mean.

For $\beta_2 < 0$, this is the same as $\beta_1 + (\beta_5 + 2\,\beta_2)\bar{v} > 0$.

For centered V, this is $\beta_1 > 0$ (effect V alter)

Note that these interpretations and expectations are formulated
without thinking of the rest of the model,
and accordingly the expectations may be off
if the rest of the model already has implications
(e.g., by correlation of effects)
for how ties depend on the values of V for tie senders and receivers.

Example: Vanina Torló's students

International MBA program in Italy;

75 students; 3 waves distributed over one year.

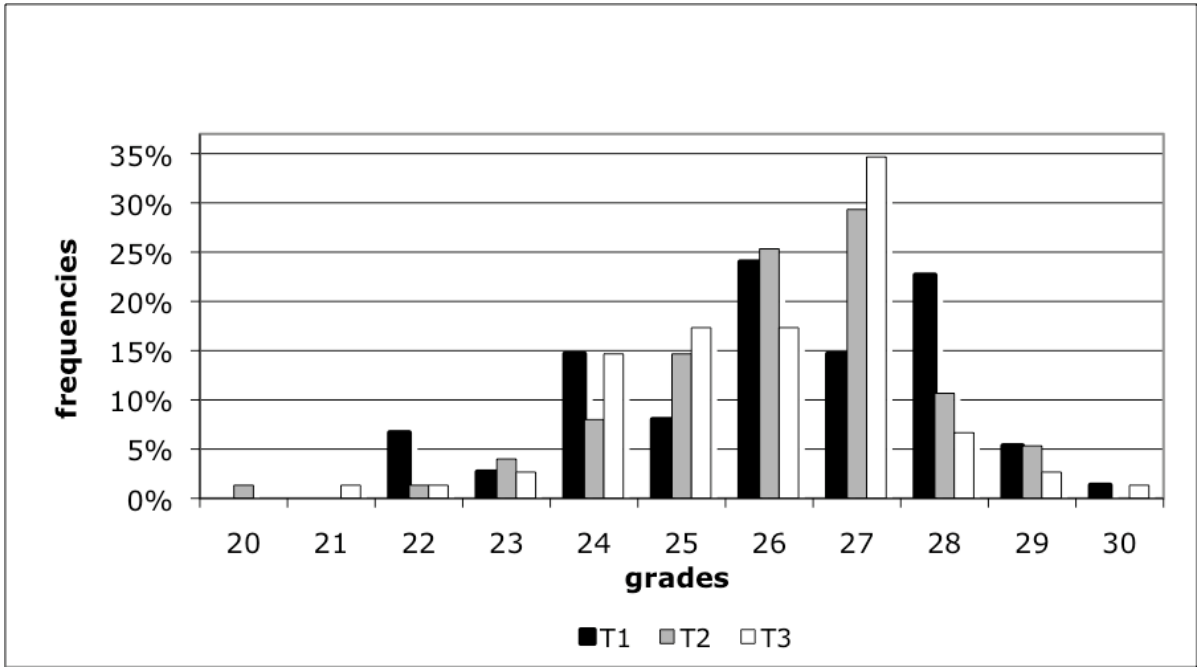
Two dependent networks:

1. *Friendship*: meaningful relations outside program context.
2. *Advice*: help, support on program-related tasks.

Relevant covariate: *Achievement*: average exam grades.

Descriptives

	Friendship			Advice		
	T_1	T_2	T_3	T_1	T_2	T_3
Av. degree	9.9	9.2	9.3	4.1	4.9	4.5
Reciprocity	0.58	0.54	0.57	0.29	0.33	0.33
Transitivity	0.44	0.40	0.38	0.24	0.24	0.26



Results

Effect	par.	(s.e.)	par.	(s.e.)
	Friendship		*Advice*	
outdegree (density)	–2.676***	(0.178)	–3.225***	(0.261)
reciprocity	1.696***	(0.145)	0.641**	(0.202)
transitive triplets	0.254***	(0.024)	0.310***	(0.046)
transitive recipr. triplets	–0.179***	(0.034)	–0.104	(0.081)
indegree - popularity	0.046***	(0.011)	0.052***	(0.007)
outdegree - popularity	–0.065***	(0.014)	–0.040	(0.031)
outdegree - activity	0.001	(0.005)	0.013	(0.012)
gender alter	0.017	(0.074)	0.008	(0.104)
gender ego	–0.123	(0.081)	–0.203*	(0.100)
same gender	0.233**	(0.078)	–0.004	(0.095)
same natio	0.286**	(0.102)	0.371*	(0.147)
grades alter	–0.028	(0.024)	0.112**	(0.035)
grades squared alter	–0.028**	(0.009)	–0.027†	(0.014)
grades ego	–0.097***	(0.025)	–0.037	(0.032)
grades squared ego	–0.019*	(0.008)	0.010	(0.010)
grades ego × alter	0.034**	(0.011)	0.044**	(0.016)

...continued...

Results (continued)

Effect	par.	(s.e.)	par.	(s.e.)
	Friendship		*Advice*	
advice	1.563***	(0.247)	—	
friendship	—		1.628***	(0.265)
reciprocity with advice	0.660**	(0.209)	—	
reciprocity with friendship	—		0.335	(0.249)
indegree advice popularity	–0.044***	(0.011)	—	
indegree friendship popularity	—		–0.024*	(0.011)
outdegree advice activity	–0.030†	(0.016)	—	
outdegree friendship activity	—		–0.053***	(0.012)

Attraction function for grades

The effect of grades on friendship is important.

Joint Wald test: $\chi^2_5 = 33.8,\ p < 0.0001$.

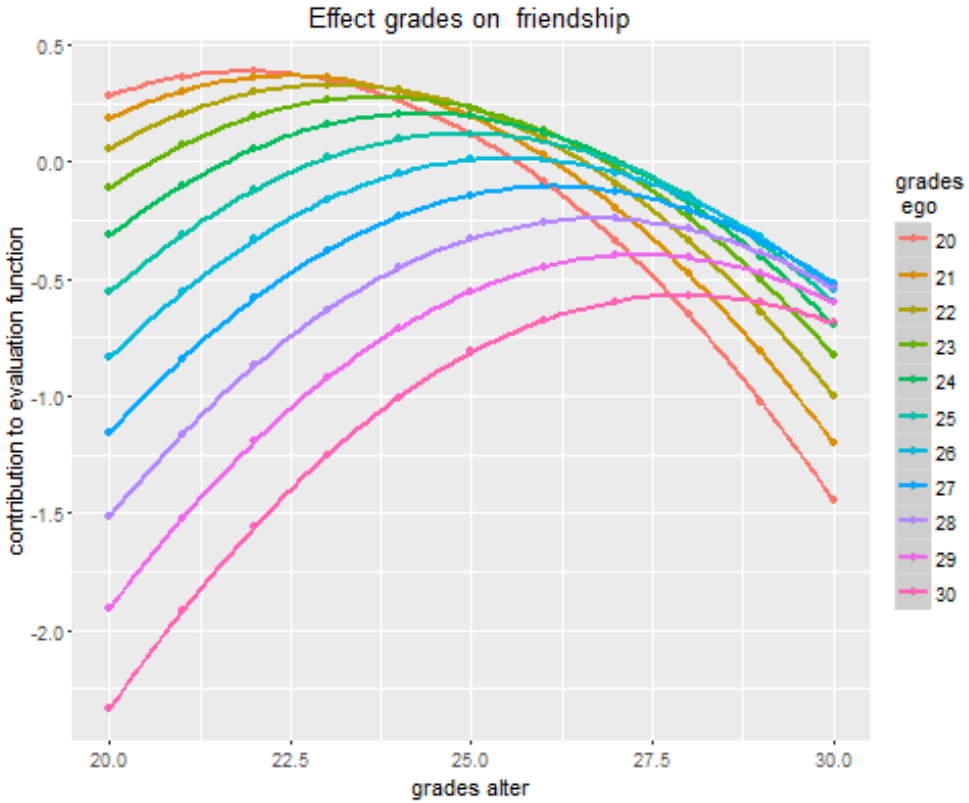
The effect of grades on advice also is significant, but less strongly:

Joint Wald test: $\chi^2_5 = 16.9,\ p < 0.001$.

Attraction function of friendship based on grade is

$$f(v_j) = -0.028\, v_j - 0.028\, v_j^2 - 0.097\, v_i - 0.019\, v_i^2 + 0.034\, v_i v_j\ ,$$

where $v = \text{grade} - 26.1$.



Attraction point

Main expectations are borne out: $\hat{\beta}_2 < 0$, $\hat{\beta}_5 > 0$; note $\hat{\beta}_1 < 0$ (n.s.).

The maximum is attained for

$$v_i^{\max} = \frac{\hat{\beta}_1 + \hat{\beta}_5\, v_i}{-2\hat{\beta}_2} = \frac{(-0.028) + 0.034\, v_i}{-2 \times (-0.028)} = -0.5 + 0.6 v_i \ ,$$

indeed increasing with v_i;
the intercept $\hat{\beta}_1/(-2\hat{\beta}_2) = -0.5$ has s.e. $= 0.4$ (delta method),
not significantly negative.

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not significantly negative.

How should we interpret that the plot shows
generally lower values for higher v_i (ego) ?

How is this reflected in observed friendships?

grade alter	≤ 23	24	25	26	27	28	≥ 29	total
grade ego								
≤ 23	.06	.25	.14	.36	.13	.06	.01	9.9
24	.09	.14	.12	.21	.27	.14	.03	13.2
25	.05	.13	.12	.26	.25	.16	.03	17.6
26	.03	.08	.12	.29	.25	.19	.03	15.9
27	.03	.11	.11	.31	.23	.15	.06	15.8
28	.01	.05	.15	.33	.27	.10	.08	13.1
≥ 29	.03	.09	.14	.24	.22	.22	.06	19.5

Rowwise proportions of ties depending on grades of alter, given ego's grade; and average outdegrees ('total'); added for waves 2 and 3.

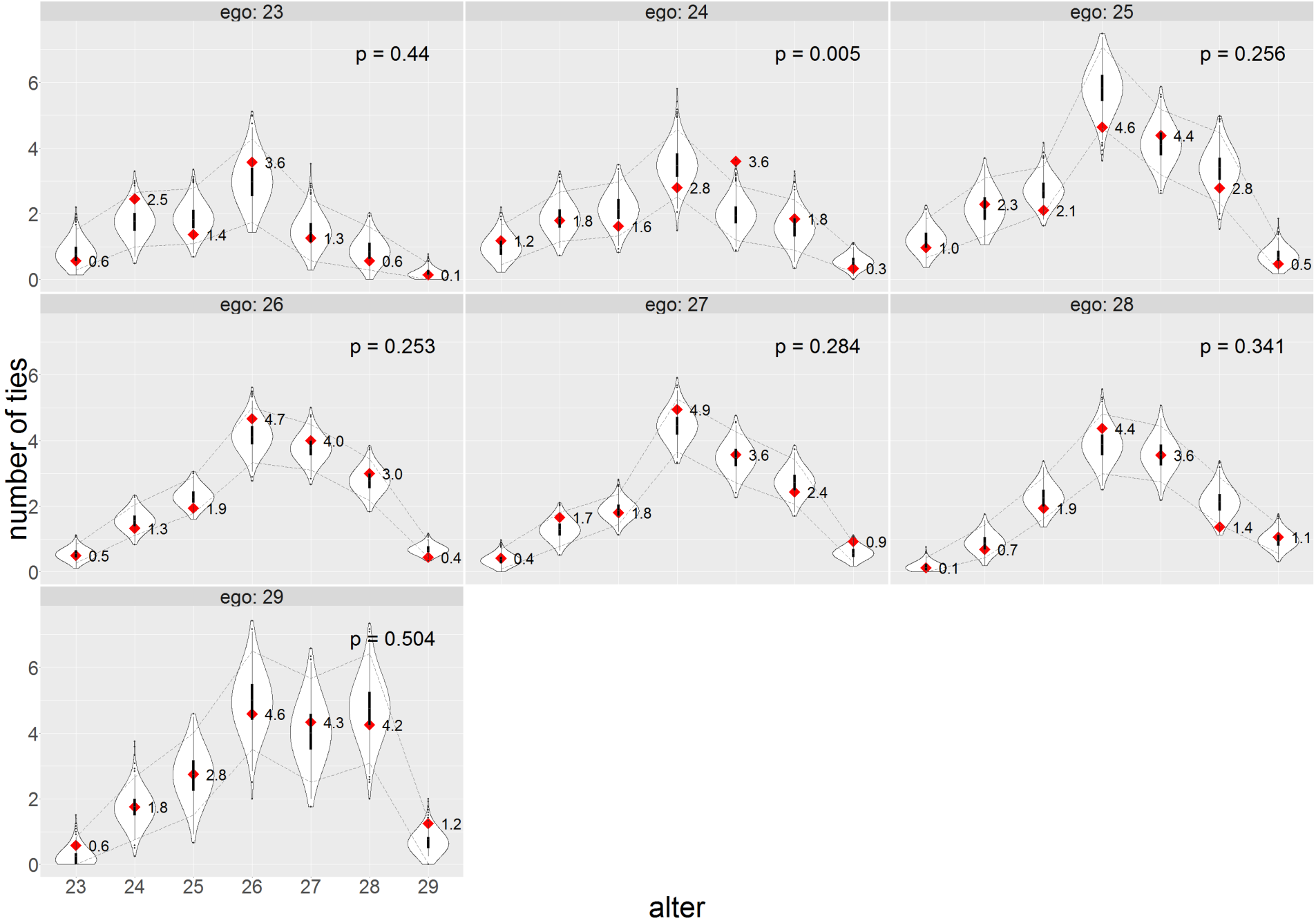
Distribution gradually, slightly, shifting upward. No trend in outdegrees.

How is the fit for observed friendships by grade?

Comparison observed - modeled dependence on grades:
study goodness of fit by `sienaGOF`.

Set of uxiliary statistic is defined as average number of ties,
for egos with a given grade,
to others with grades (respectively) 20-23, 24, 25, 26, 27, 28, and 29-30,
added for the second and third waves (7×7 combinations of values).

Overall Mahalanobis combination $p = 0.045$.
This is acceptable; see plot next page.


ego: 23
p = 0.44
0.6
2.5
1.4
3.6
1.3
0.6
0.1
ego: 24
p = 0.005
1.2
1.8
1.6
2.8
3.6
1.8
0.3
ego: 25
p = 0.256
1.0
2.3
2.1
4.6
4.4
2.8
0.5
ego: 26
p = 0.253
0.5
1.3
1.9
4.7
4.0
3.0
0.4
ego: 27
p = 0.284
0.4
1.7
1.8
4.9
3.6
2.4
0.9
ego: 28
p = 0.341
0.1
0.7
1.9
4.4
3.6
1.4
1.1
ego: 29
p = 0.504
0.6
1.8
2.8
4.6
4.3
4.2
1.2
number of ties
0
2
4
6
23
24
25
26
27
28
29
alter

Students with higher grades
do not have lower outdegrees.
But look again at the
selection table / attraction function.

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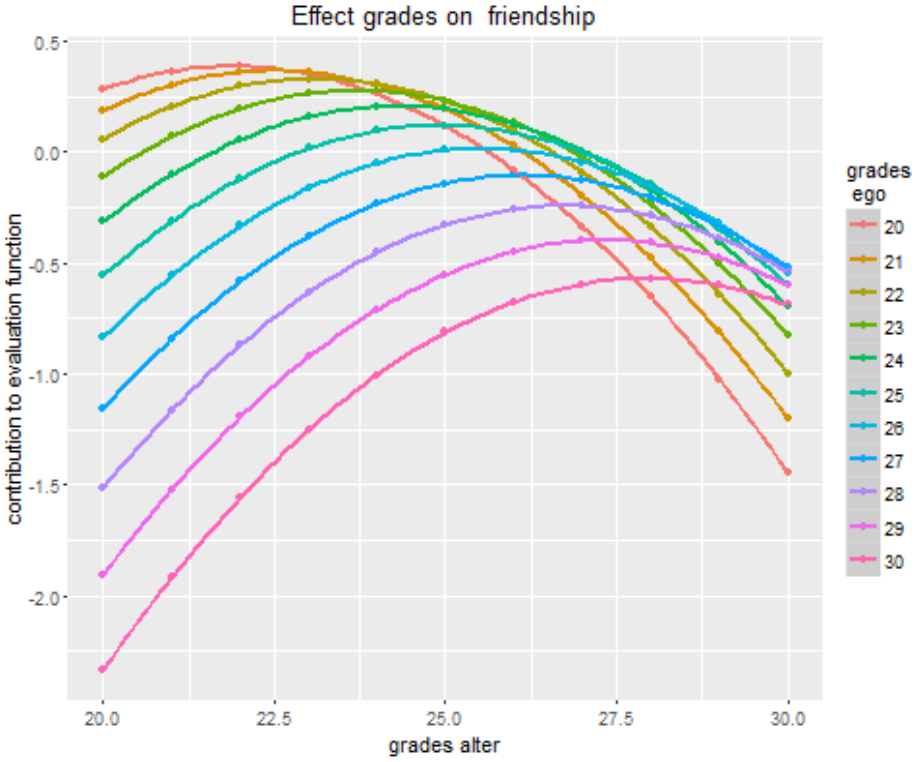
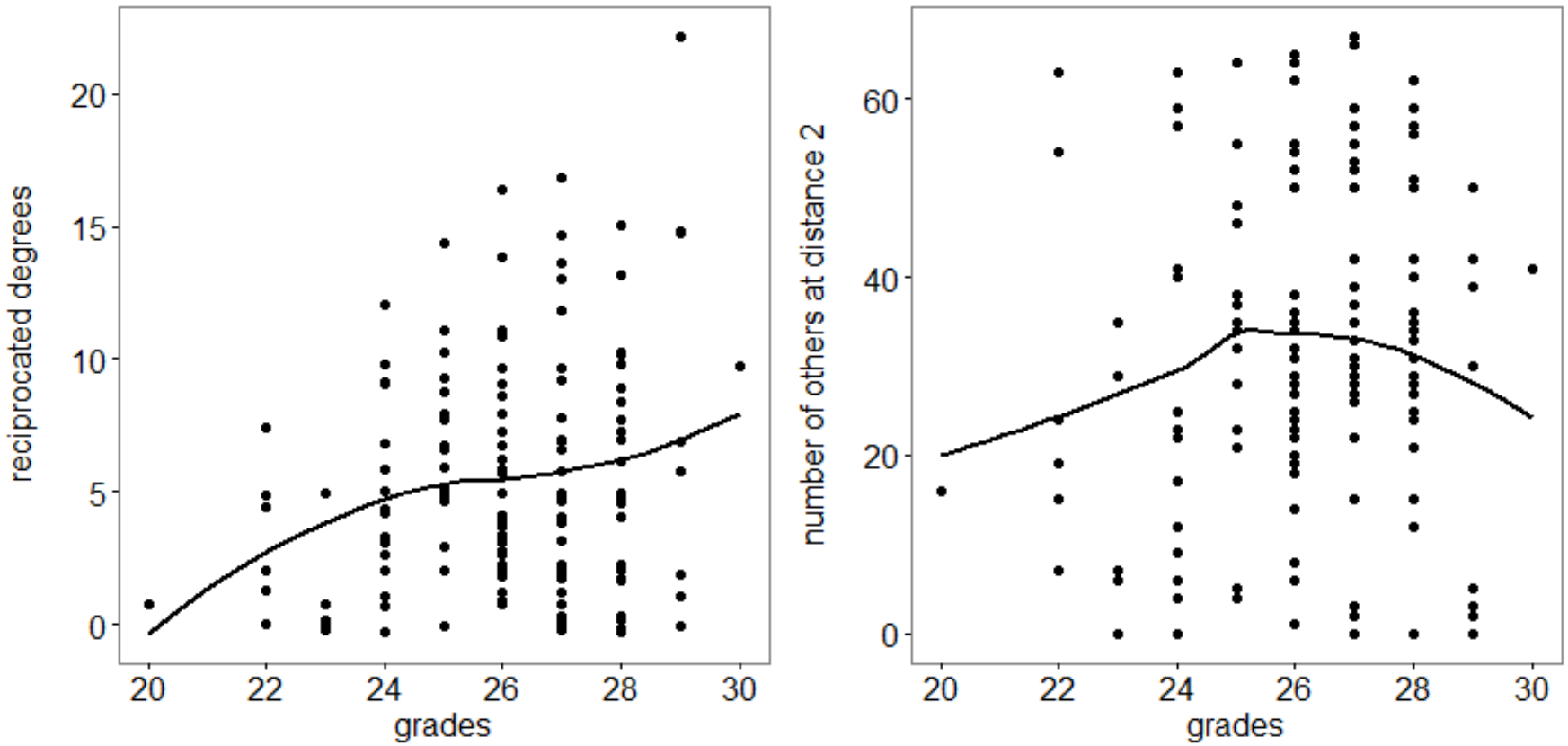


Lower curves for higher grades of ego should not be interpreted as a revealed preference for fewer friends!

Number of ties determined not only by attraction function,
but also by distribution of ties at the previous wave,
'supply' of others with given grades, and other network influences.
Main other network influences are reciprocation and transitivity.

Network embeddedness as a function of grade



Left: reciprocated degree as function of grade.

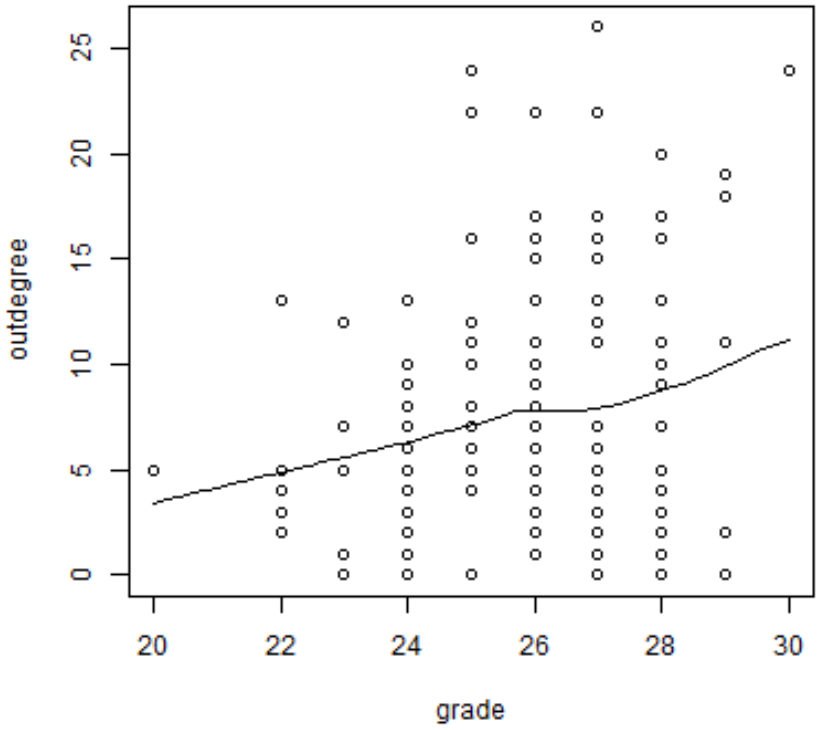
Right: number of others at directed distance two as function of grade.

Those with higher grades have more reciprocated ties and (especially) more open networks $\sim$ more candidates for transitive closure.

These two positional characteristics work positively on their outdegrees.

This is counteracted by the lower attraction functions.

The result is a moderate increase of outdegrees as a function of grade.

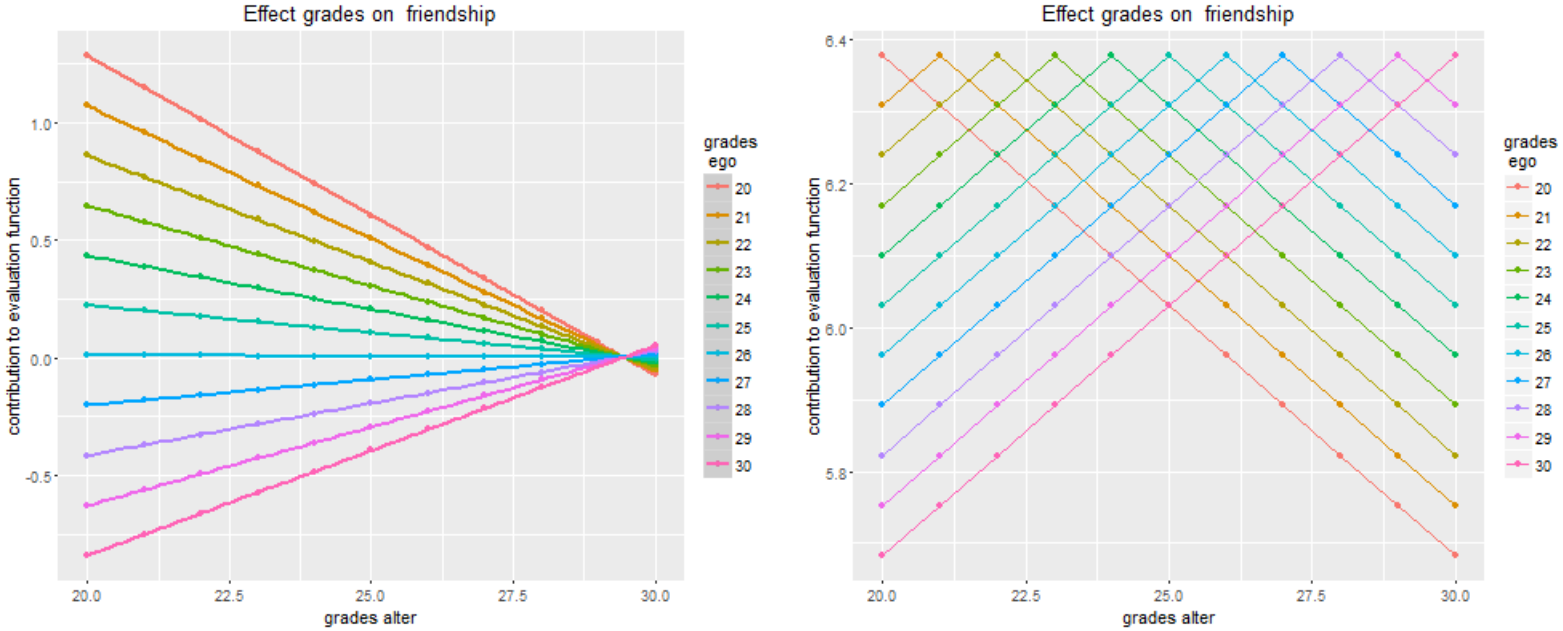


Outdegree as function of grade.

Comparison with other attraction functions

The quadratic ego and alter effects are significant;

this shows that the model with the absolute difference attraction function and that with the ego $\times$ alter attraction function for friendship as a function of grades fit worse than the quadratic model.



Contribution of grades to the evaluation function for friendship.

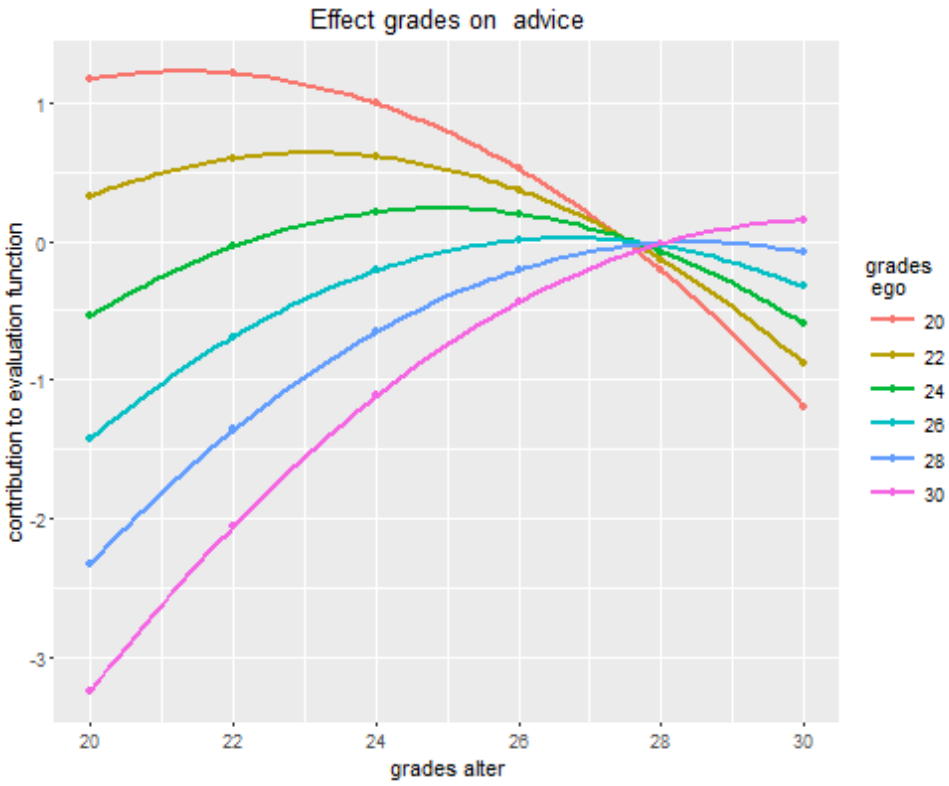
Left: linear interaction specification; right: similarity specification.

The attraction function for advice and grades

The estimated attraction function of advice based on grade is

$$
\begin{aligned}
f(v_j) = & 0.112\, v_j - 0.027\, v_j^2 \\
& - 0.037\, v_i + 0.010\, v_i^2 \\
& + 0.044\, v_i v_j \ .
\end{aligned}
$$

where again $v = \text{grade} - 26.1$.

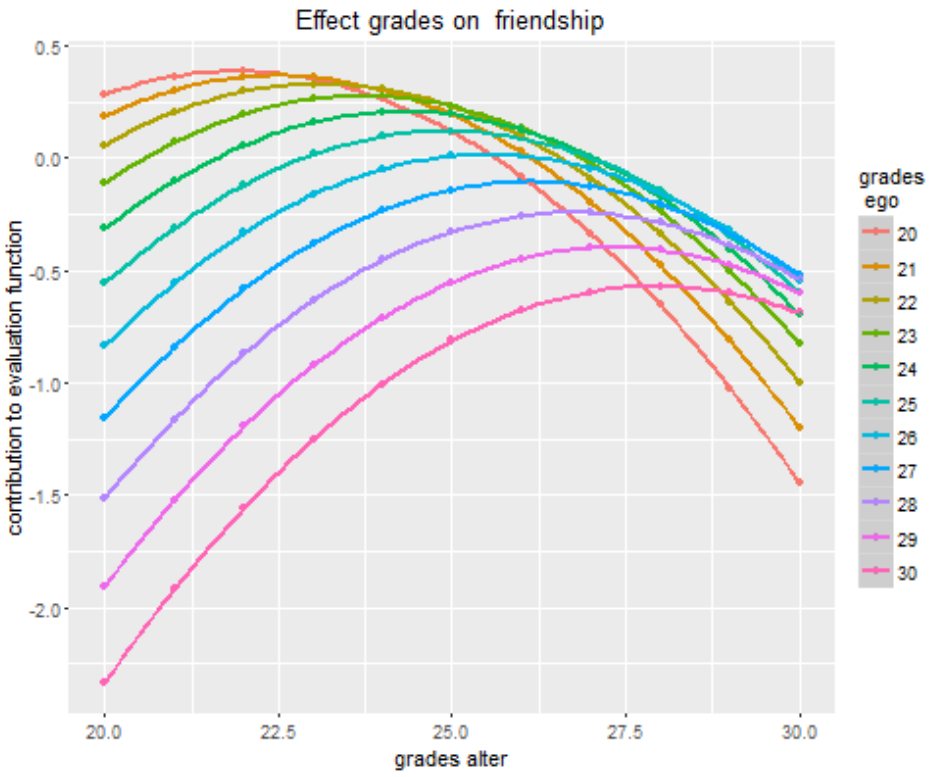


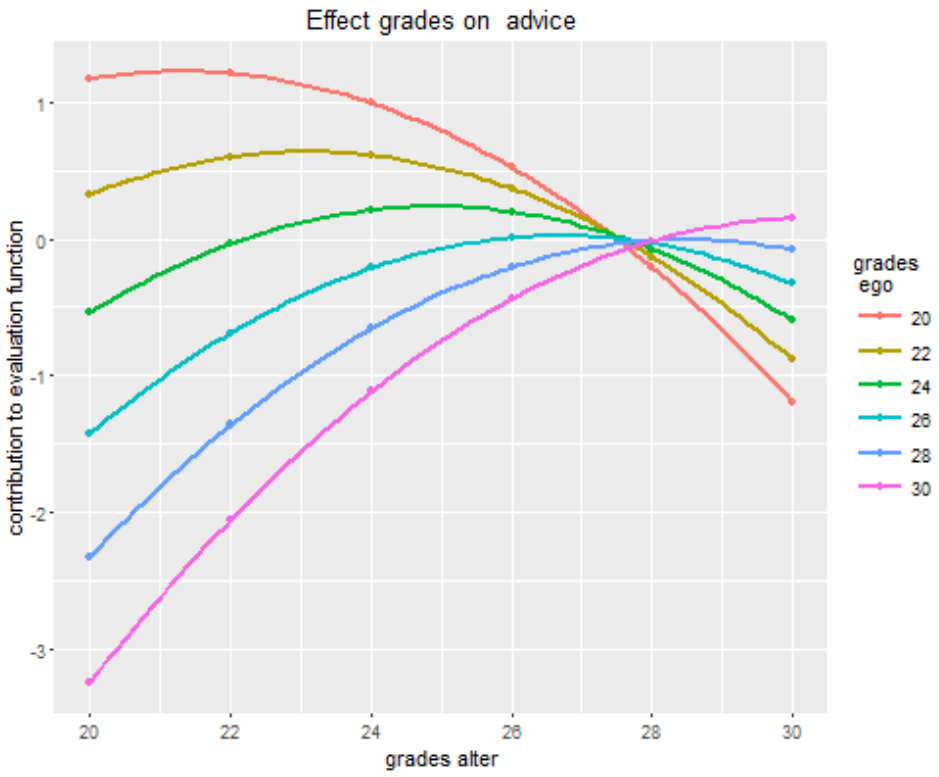
The value for which the optimum is obtained is estimated as

$$v_i^{\max} = \frac{\hat{\beta}_1 + \hat{\beta}_5\, v_i}{-2\hat{\beta}_2} = \frac{0.112 + 0.044\, v_i}{-2 \times (-0.027)} = 2.1 + 0.8 v_i \ . \qquad (2)$$

Standard errors now are, respectively, 0.6 for $\hat{\beta}_1/(-2\hat{\beta}_2)$, and 0.3 for $\hat{\beta}_5/(-2\hat{\beta}_2)$.

The group norm for advice accordingly is estimated as $2.1 + 0.8\,\bar{v} = 28.2$, significantly higher than average grade:
a combination of aspiration with homophily.





Selection functions for grades
on friendship (left) and advice (right).

Discussion

- Tendencies toward homophily may and will coexist
  with aspirations toward high (low) values
  and with tendencies toward a common (average) value.
- Specifications that have been in use in statistical network modeling
  for expressing homophily on numerical actor covariates
  are too rigid (absolute difference)
  or no good representations of homophily (ego $\times$ alter interaction).
  Briefly, they are often misspecifications.
- A quadratic specification of the dependence on ego's and alter's value
  offers an important improvement.
- New effect `egoSqX` implemented in RSiena.

Discussion – continued

- This implies the recommendation to use *five* parameters to represent the effect of a numerical monadic actor variable on probabilities of tie existence and tie change.
  This seems like a lot, but is more meaningful for theoretical modeling than absolute difference or cross-product interaction.
- Three-parameter sub-model is possible, but will be non-linear, which leads to technical complications.
- For actor variables with moderate effects on the network, the extra parameters (quadratic effects) make little difference.
- For actor variables with strong effects on the network, the extra parameters (quadratic effects) may make a large difference.

Discussion – continued

- Homophily component can be tested by the ego-actor interaction in this five-parameter model ('four control effects').
- Aspiration component can be tested by the alter effect if the variable is centered.
- This model can resolve interpretation problems in cases where 'negative homophily' (repulsion) seemed to be found when using an absolute difference or simple interaction model.
- The value for alter where the optimum is assumed (dependent on ego's value) is an interesting descriptive feature.
- The example offers an interesting perspective on the distinction between the evaluation function and a preference function.