

Networks and Graphs

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Outline

Representations of networks

Graphs with attributes

Complete and incomplete networks

Some literature

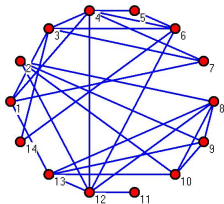
Graphs

A graph is a set of points (vertices, nodes) V
and a set of edges (lines) E ;
the edges are unordered pairs of points.

If $\{i, j\}$ is an edge, i.e., $\{i, j\} \in E$,
then we say that i and j are tied, or related to each other.

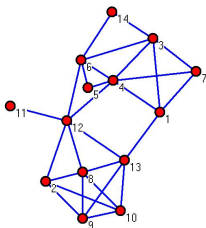
Usually in Social Network Analysis,
the points represent *social actors*;
and the graphs are *non-reflexive*:
 $\{i, i\} \notin E$, points are not tied to themselves.

Doreian's network



An example of a graph
of 14 actors, from Doreian
(Social Networks 1988).
The actors are
the main political actors in a
Midwestern county in the US;
a tie means they can expect
public and private support
from each other
and they are also friends.

Doreian's network



This is the same graph!
Graphical representation
is a science and art of itself.

Digraphs

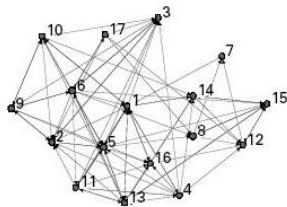
Often there is a directionality inherent in ties:

I like you, but you don't like me...

A directed graph (digraph) is a set of points V
and a set of arcs (directed edges) E ;
the edges are **ordered** pairs of points.

If (i, j) is an arc, i.e., $(i, j) \in E$,
then we say that i is tied (or related) to j .
 i is the *sender* of the tie,
 j the *receiver*.

Paper Factory Graph



July 1997 (time = t_4)

The trust network of employees in a paper factory collected by Rafael Wittek (1999) (van de Bunt, Wittek, de Klepper, Int. Soc. 2005).

Adjacency matrix

A convenient representation of graphs and digraphs (we often just say “graphs” when we also refer to digraphs) is the adjacency matrix:

j is adjacent to i if there is a tie from i to j ;
the adjacency matrix is the matrix (y_{ij}) with

$$y_{ij} = \begin{cases} 1 & \text{if } (i, j) \in E \\ 0 & \text{if } (i, j) \notin E \end{cases}$$

The diagonal of the adjacency matrix will be structurally zero when there are no self-ties.

Adjacency matrix

The adjacency matrix for Doreian's network is

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

On terminology

In mathematical terminology, a *relation* is the same as a graph.

Therefore we do not like to talk about the relation from i to j , but rather about the *tie* from i to j .

Element y_{ij} of the adjacency matrix is called a *tie variable*.

The *density* of a graph is the fraction of pairs of nodes that are tied.

For Doreian's network, there are 28 edges and

$$\frac{14 \times 13}{2} = 91 \text{ unordered pairs, so the density is } \frac{28}{91} = 0.31.$$

Graphs with attributes

Practically always, there is more of interest in a network than merely the structure represented by the graph.

Often the nodes have attributes, or values; also the arcs may have attributes or values.

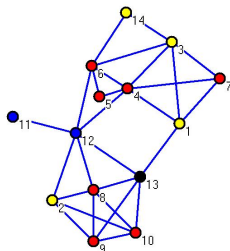
A *valued (di)graph* is a (di)graph where edges have values.

An important instance is the *signed (di)graph*, where edge (arc) values are $-$ and $+$.

Categorical attributes often are referred to as *colors*.

The colored network may tell a more extensive story...

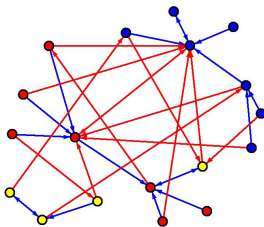
Example: Doreian's network



Colors:

- council members
- officers
- mayor
- former council members

Example: Signed digraph



Sampson's data (1969)
'crisis in a cloister'
at time T4,

"+" blue arcs: best liked
"-" red arcs: least liked.

- young turks
- loyal opposition
- outcasts

Graphs with attributes

Another extension is *multivariate* or *multiplex* networks, where several relations are given on the same set of points; e.g., trust, liking, and cooperation in a work department.

Complete and incomplete networks

Networks where for each pair of actors it is known if they are tied, are called *complete* or *entire* networks.

The term *complete* is more common, but invites confusion with the *complete graph* which is the graph with density 1.

Networks data where for a set of actors, the outgoing ties are known for each actor, are called *personal* or *egocentric* networks.

These can be collected through surveys of a large population.

Complete and incomplete networks

Intermediate forms are less common but very important;

e.g., survey members are asked to indicate ties between all or some of their network members;

or a *snowball sample* is carried out, where some network members themselves are interviewed.

Network sampling is an interesting area, in which only little research has been done.

Introductory books on social network analysis

- ▶ Stanley Wasserman and Katherine Faust ,
Social Network Analysis: Methods and Applications. Cambridge University Press, 1994.
- ▶ Peter Carrington, John Scott, Stanley Wasserman (eds.),
Models and Methods in Social Network Analysis. Cambridge University Press, 2005.
- ▶ Alain Degenne and Michel Forsé,
Introducing Social Networks. Sage, 1999.
- ▶ John Scott,
Social Network Analysis: A Handbook. 2nd edition. Sage, 2000.

Books from complementary viewpoints

- ▶ Duncan Watts,
Six Degrees. The Science of a Connected Age. W.W. Norton, 2003.
(A popular account linking work about networks by physicists and computer scientists with social science work about network analysis.)
- ▶ Gabrielle Demange and Myrna Wooders (eds.),
Group formation in economics. CUP, 2005.
(A reader of papers on networks published in the economics literature.)