Equivalence Concepts for Social Networks

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Outline

Structural Equivalence

Regular Equivalence

Stochastic Equivalence

Block modeling

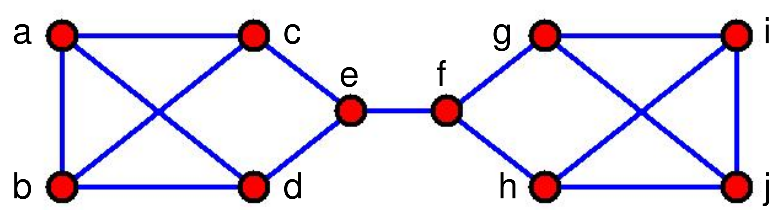
The idea of block modeling is to bring out
some main features of the network
by dividing (partitioning) the nodes into categories
of ‘equivalent’ nodes.

The big question is what are meaningful types of equivalence.

Lorrain and White (1971) defined that
nodes *a* and *b* are structurally equivalent,
if they relate to other nodes in the same way.

The Borgatti-Everett Network

The following is a network
proposed by Borgatti and Everett (1991).
Which nodes are structurally equivalent?



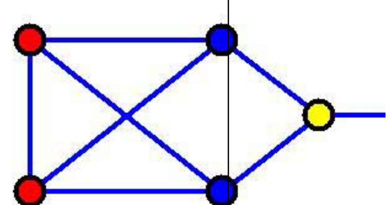


Image matrix

For a coloring / partition for a structural equivalence,
the *image matrix* is the corresponding adjacency matrix.

The 0 and 1 diagonal entries are meaningful,
unless the equivalence class has only one element.

Usually, the vertices have to be rearranged
so that each color indicates a set of successive nodes;
then the adjacency matrix shows a *block structure*.

Image matrix

$$
\begin{pmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & . & 1 & 0 & 0 \\
0 & 0 & 1 & . & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 1
\end{pmatrix}
$$

Adjacency matrix

$$
\left(\begin{array}{cc|cc|c|c|cc|cc}
. & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & . & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
1 & 1 & . & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & . & 1 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 1 & 1 & . & 1 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 1 & . & 1 & 1 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 1 & . & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & . & 1 & 1 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & . & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & .
\end{array}\right)
$$

The adjacency matrix has
a block structure:
all blocks are either
all – 0 or all – 1.

Approximate structural equivalence

For most empirically observed networks, hardly any nodes are structurally equivalent.
However, there may be groups of nodes that are *approximately structurally equivalent*.

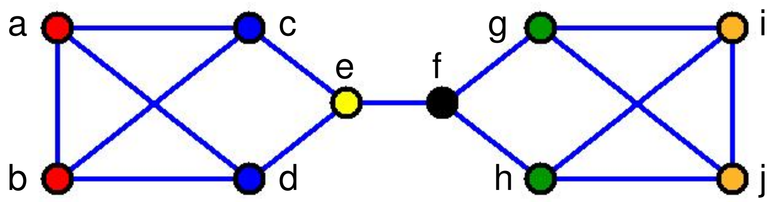
This is elaborated by defining the elements of the image matrix as the *proportion* of ties in the corresponding block of the adjacency matrix;
and striving for an image matrix with elements all of which are as close as possible to either 0 or 1.

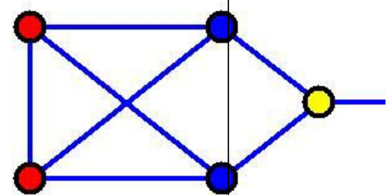
As an exercise, you can run "Operations – Blockmodeling" in Pajek for Doreian's data set of 14 political actors, and find approximate structural equivalence classes.

Uncheck the "short report" option, and ask for 4 classes.
In the output, `com` means 'complete'.

Other equivalences

Here is the Borgatti and Everett (1991) network again:





This is the structural equivalence coloring.

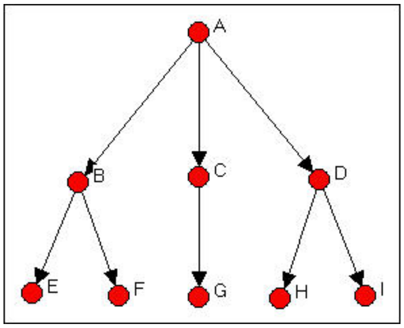
Do you see other possibilities of equivalence? What about this coloring?

Doesn't is seem also a good representation of equivalence?

Regular equivalence

A coloring is a *regular equivalence*
(Sailer 1978; White and Reitz 1983)
if vertices with the same color
also have neighbors of the same color.
(*a* and *b* are neighbors if they are tied to each other.)

This definition is a nice mathematical representation
of the sociological concept of *role*:
the color / role determines
to which other colors / roles you should be tied;
what is required is being tied to *some* actors in this role,
not to *all* actors of this role.



Wasserman and Faust (1994) give the following example.

You can look in Hanneman's text (Section 15) for further examples.

One graph can have many different colorings that all are regular equivalences!

Stochastic equivalence

The classical concepts of equivalence in networks can be applied to cases of approximate equivalence by maximizing some measure of adequacy, that measures how well the observed block structure corresponds to what would be predicted in the case of exact equivalence.
All nodes are classified in one of the classes.

Probability models provide another way to express the deviations between observations and the idealized concept of ("exact") equivalence.

For a probability distribution of the ties in a graph,
a coloring is a *stochastic equivalence*
(Fienberg and Wasserman, 1981)
if nodes with the same color have
the same *probability distribution* of ties with other nodes.

More formally:
the probability distribution of the graph must
remain the same when equivalent nodes are exchanged.
Such a distribution is called a *stochastic block model*.

The stochastic block model is a kind of
Latent Structure Analysis (LSA).
The basic idea of LSA, proposed by Lazarsfeld & Henry (1968),
is that there exist latent (i.e. unobserved) variables such that
the observations are *conditionally independent*
given the latent structure (= latent variables).
The *structural model* then specifies the latent variables
and the *measurement model* specifies how the observations
depend on the latent variables.
LSA has been extended to measurement models that specify
not conditional independence, but more generally
also allow simple, restricted, types of dependence.

The stochastic block model is a latent structure model where the latent structure is the node coloring,
which has to be recovered from the observed network;

for each pair of nodes i and j, the colors of these nodes determine the probability of a tie
or (for valued / multivariate networks)
of a certain tie configuration between i and j;
conditional on the coloring, the tie variables are independent.

This is a 'rough' type of network model,
which is useful for bringing out the global structure.

Often we are interested in *cohesive blocks*:
1-blocks on diagonal, 0-blocks off-diagonal:
high within-group density,
low between-group density.

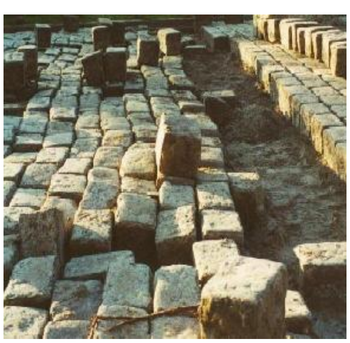
The stochastic block model, however,
is much more general:
any difference between probabilities of ties
within and between groups is permitted.

Methods for estimating stochastic block models
for valued graphs and digraphs
were developed by Nowicki and Snijders (1997, 2001).
By representing multivariate graphs as valued graphs
(reverse to the use of dummy variables in regression)
this can also be applied to multivariate graphs.

E.g., for two binary relations,

represent	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
by	0	1	2	3 .

BLOCKS



These methods are implemented
in the computer program BLOCKS.
BLOCKS gives a Bayesian procedure,
which means that it estimates
a *probability distribution*
for the colors / block structure.

BLOCKS is contained in the
StOCNET package.

Bayesian features of BLOCKS:

1. It is a *Bayesian* procedure:
   i.e., it gives ‘*posterior distributions*’
   (posterior = ‘after looking at the data’)
   for all parameters in the model.
2. In particular, for each pair of nodes it yields
   the posterior probability that these nodes are equivalent
   – have the same color.
3. It does however not give posterior probabilities
   that nodes have a given color
   because the color labels are unidentified
   – there is no a priori meaning to red, blue, white...

Model features of BLOCKS:

1. The number of latent classes (colors) is predefined;
   the user can make a choice based on the balance
   between fit and good separation between the classes.
2. The methods accounts for reciprocity effects
   by considering not probabilities of ties but of dyads:
   conditional on the coloring, *dyads* are independent.
3. In case of multivariate networks,
   dependence between ties of different types is represented by
   coding all combinations as separate values:
   arbitrary associations between the networks.

For example, BLOCKS could distinguish between
two latent groups, where probabilities of ties are the same
between and within all groups, but between-group ties
are more often reciprocal than within-group ties.

The generality of handling reciprocity and multivariate relations
comes at the price of requiring a large number of parameters.
The program works well for a relatively small number of groups.

Thus, nodes have probabilities for certain colors,
and a perfect block structure is obtained
if all these probabilities are 0 or 1.

These are called *posterior probabilities*
because they are estimated *after* looking at the data.

An advantage is, that nodes which ‘fall between the classes’
can be represented as such:
they have positive probabilities for several colors,
rather than one color with probability (almost) 1.

In the technique of stochastic block modeling, there is an *identifiability problem*:

the partition of nodes into equivalence classes is identifiable, but the coloring not because color labels are arbitrary.

This leads to complications in estimation.
It is meaningful to say

vertices i and j have the same color

but not

vertex i has color 1.

The solution chosen in BLOCKS is that the results are not given primarily as colors of nodes, but as:

1. matrix of posterior probabilities of color equality: probabilities that vertices i and j have same color
2. probability distribution of the tie variables (Y_{ij}, Y_{ji}) between vertices i, j, given the colors.

In a post-processing step, this result is transformed to a partial coloring of the nodes, with a rest category of nodes with unclear classification.

In block modeling, like in any type of cluster analysis,
there is the problem of finding a suitable number of colors.

In BLOCKS, the number of classes / colors
is determined from two numbers:

1. Information I_y ("surprise index") in observed tie pattern,
   given the coloring: 'badness of fit';
   indicates how poorly the observed pattern of ties conforms
   to the ideal pattern of ties for this probability distribution;
2. clarity H_x of block structure;
   indicates the ambiguity with which nodes can be colored.

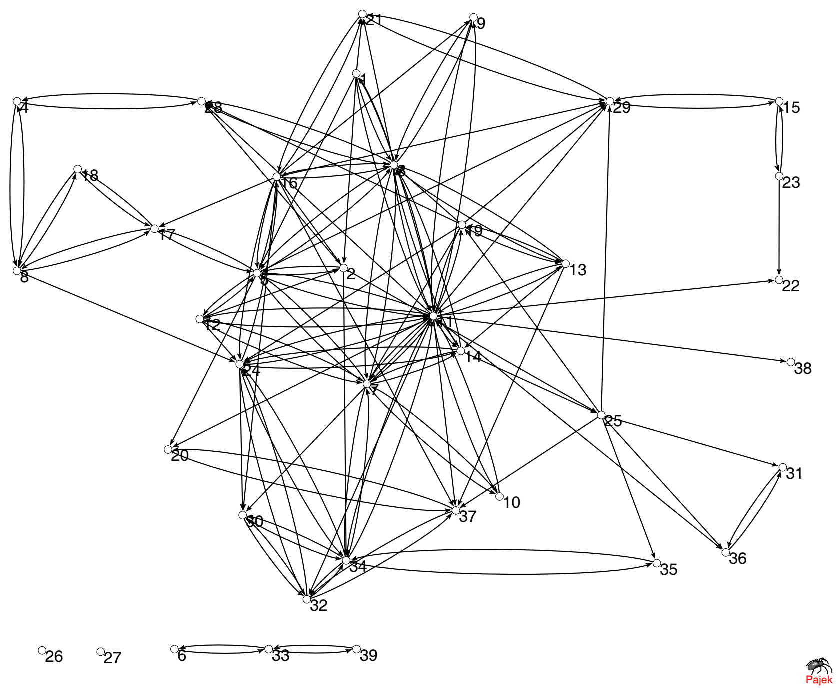
These should both be low.

Example: Kapferer's Tailor Shop

Kapferer (1972) studied the interactions
between $n = 39$ workers in tailor shop in Zambia (1972),
changing patterns of alliance among the workers:
two measurements in 1972 in period of unrest
(extended wage negotiations).
Relations studied: work-related assistance and friendship.

Here: post-strike relations.

Following digraph shows *friendship relations*
at second time point.



Multivariate network: two types of interaction:

1. work- and assistance-related relationship *A* (nondirected, density 0.30)
2. friendship interactions *F* (directed, density 0.20).

These ties have positive association (log odds ratio 1.11).

Which actors are stochastically equivalent?

For multivariate networks, the set of values of combinations of tie variables in both directions (node i to node j *and* node j to node i) is referred to as the *alphabet* of ties.

The alphabet $\mathcal{A}$ here consists of 4 symmetric tie configurations

$$\mathcal{A}_0 = \{(0,0),\, (A,A),\, (F,F),\, (AF,AF)\}$$

and 4 nonsymmetric tie configurations

$$\mathcal{A}_1 = \{(0,F),\, (A,AF),\, (F,0),\, (AF,A)\}\ .$$

There are two non-redundant nonsymmetric configurations.

a_1	a_2			
	0	A	F	AF
0	493	–	19	–
A	–	153	–	24
F	19	–	6	–
AF	–	24	–	46

Frequencies of dyadic ties $(a_1, a_2) \in \mathcal{A}$.

(– : structural zeros because A is nondirected.)

How many classes ?

Use information I_y and block structure clarity H_x.

c	I_y	H_x
2	0.94	0.24
3	0.91	0.21
4	0.89	0.26
5	0.89	0.27

Parameters for the class structure for Kapferer's data set.

Conclusion: $c = 3$ attractive.

The following table presents (over two pages)

the matrix of posterior probabilities of color equality.

1	–	9	9	9	9	6	9	9	9	8	9	6	9	8	8	7	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
2	9	–	9	9	9	6	9	9	9	8	9	6	9	8	8	7	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
3	9	9	–	9	9	6	9	9	9	9	9	6	9	8	8	7	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
5	9	9	9	–	9	6	9	9	9	9	9	6	9	8	8	7	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
7	9	9	9	9	–	6	9	9	9	8	9	6	9	8	8	7	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
9	6	6	6	6	6	–	6	6	6	5	5	7	6	4	5	5	4	4	4	5	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	0
12	9	9	9	9	9	6	–	9	9	8	9	6	9	8	8	7	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
13	9	9	9	9	9	6	9	–	9	9	9	6	9	8	8	7	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
14	9	9	9	9	9	6	9	9	–	8	9	6	9	8	8	7	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
16	8	8	9	9	8	5	8	9	8	–	9	5	9	9	7	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1
19	9	9	9	9	9	5	9	9	9	9	–	5	9	9	7	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1
21	6	6	6	6	6	7	6	6	6	5	5	–	6	4	7	6	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	0
24	9	9	9	9	9	6	9	9	9	9	9	6	–	8	8	7	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
25	8	8	8	8	8	4	8	8	8	9	9	4	8	–	7	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
30	8	8	8	8	8	5	8	8	8	7	7	7	8	7	–	8	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0
34	7	7	7	7	7	5	7	7	7	6	6	6	7	6	8	–	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	0
4	0	0	0	0	0	4	0	0	0	0	0	4	0	0	2	3	–	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	0
6	0	0	0	0	0	4	0	0	0	0	0	4	0	0	2	3	9	–	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	0

8	0	0	0	0	0	4	0	0	0	0	0	4	0	0	2	3	9	9	–	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	0
10	1	1	1	1	1	5	1	1	1	0	0	4	1	0	2	3	9	9	9	–	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	0
15	0	0	0	0	0	4	0	0	0	0	0	4	0	0	2	3	9	9	9	9	–	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	0
17	0	0	0	0	0	4	0	0	0	0	0	4	0	0	2	3	9	9	9	9	9	–	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	0
18	0	0	0	0	0	4	0	0	0	0	0	4	0	0	2	3	9	9	9	9	9	9	–	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	0
20	0	0	0	0	0	4	0	0	0	0	0	4	0	0	2	3	9	9	9	9	9	9	9	–	9	9	9	9	9	9	9	9	9	9	9	9	9	9	0
22	0	0	0	0	0	4	0	0	0	0	0	4	0	0	2	3	9	9	9	9	9	9	9	9	–	9	9	9	9	9	9	9	9	9	9	9	9	9	0
23	0	0	0	0	0	4	0	0	0	0	0	4	0	0	2	3	9	9	9	9	9	9	9	9	9	–	9	9	9	9	9	9	9	9	9	9	9	9	0
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27	0	0	0	0	0	4	0	0	0	0	0	4	0	0	2	3	9	9	9	9	9	9	9	9	9	9	9	–	9	9	9	9	9	9	9	9	9	9	0
28	0	0	0	0	0	4	0	0	0	0	0	4	0	0	2	3	9	9	9	9	9	9	9	9	9	9	9	9	–	9	9	9	9	9	9	9	9	9	0
29	0	0	0	0	0	4	0	0	0	0	0	4	0	0	2	3	9	9	9	9	9	9	9	9	9	9	9	9	9	–	9	9	9	9	9	9	9	9	0
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32	1	1	1	1	1	4	1	1	1	1	1	4	1	0	2	3	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	–	9	9	9	9	9	9	0
33	0	0	0	0	0	4	0	0	0	0	0	4	0	0	2	3	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	–	9	9	9	9	9	0
35	0	0	0	0	0	4	0	0	0	0	0	4	0	0	2	3	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	–	9	9	9	9	0
36	0	0	0	0	0	4	0	0	0	0	0	4	0	0	2	3	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	–	9	9	9	0
37	0	0	0	0	0	4	0	0	0	0	0	4	0	0	2	3	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	–	9	9	0
38	0	0	0	0	0	4	0	0	0	0	0	4	0	0	2	3	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	–	9	0
39	0	0	0	0	0	4	0	0	0	0	0	4	0	0	2	3	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	–	0
11	0	0	0	0	0	0	0	0	0	1	1	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–

Look at the block structure (for the reordered nodes)!

The following table presents (again over two pages)
the matrix of posterior probabilities
of the probability of assistance and mutual friendship
given the vertex colors for two-way ties of type (AF, AF).

1	–	2	2	2	2	1	2	2	2	2	2	1	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
2	2	–	2	2	2	1	2	2	2	2	2	1	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
3	2	2	–	2	2	1	2	2	2	2	2	1	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
5	2	2	2	–	2	1	2	2	2	2	2	1	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
7	2	2	2	2	–	1	2	2	2	2	2	1	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
9	1	1	1	1	1	–	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
12	2	2	2	2	2	1	–	2	2	2	2	1	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
13	2	2	2	2	2	1	2	–	2	2	2	1	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
14	2	2	2	2	2	1	2	2	–	2	2	1	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
16	2	2	2	2	2	1	2	2	2	–	2	1	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
19	2	2	2	2	2	1	2	2	2	2	–	1	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
21	1	1	1	1	1	1	1	1	1	1	1	–	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
24	2	2	2	2	2	1	2	2	2	2	2	1	–	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
25	2	2	2	2	2	1	2	2	2	2	2	1	2	–	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
30	1	1	1	1	1	1	1	1	1	1	1	1	1	2	–	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
34	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	–	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	0	0	0	0	0	0	0	0	0	0	0	0	1
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	0	0	0	0	0	0	0	0	0	0	0	1
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	0	0	0	0	0	0	0	0	0	0	1
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	0	0	0	0	0	0	0	0	0	1
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	0	0	0	0	0	0	0	0	1
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	0	0	0	0	0	0	0	1
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	0	0	0	0	0	0	1
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	0	0	0	0	0	1
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	0	0	0	0	1
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	0	0	0	1
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	0	0	1
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	0	1
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	1
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	1
11	4	4	4	4	4	3	4	4	4	4	4	3	4	4	4	3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	–

Similar tables exists for the other elements of the alphabet.

With this clear block structure, now turn to the colors provided by the post-processing.

Class 1 (yellow) :

1 – 3, 5, 7, 12 – 14, 16, 19, 24, 25, 30, 34

higher prestige workers & two cotton boys (30, 34)

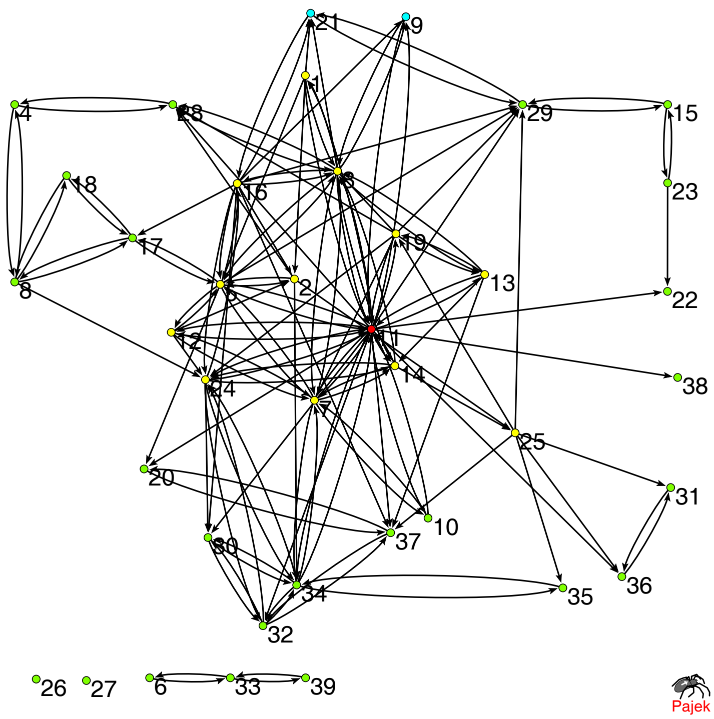
Class 2 (green) :

4, 6, 8, 10, 15, 17, 18, 20, 22, 23, 26 – 29, 31 – 33, 35 – 39

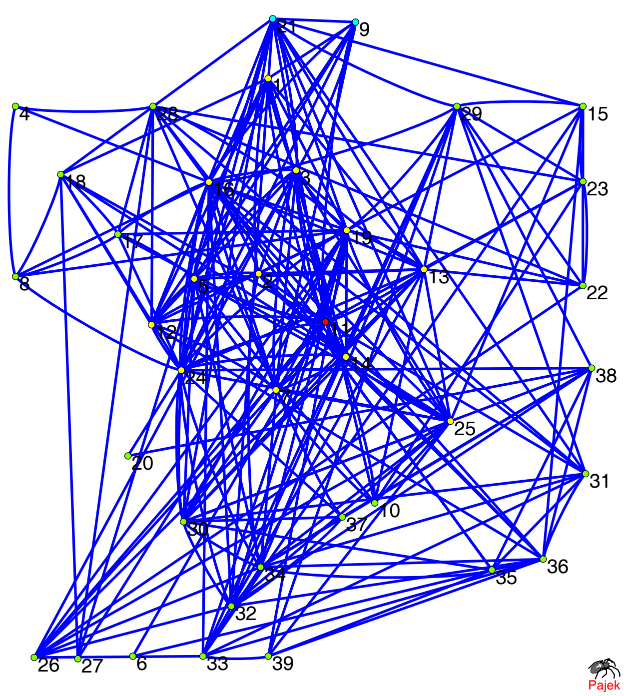
Class 3 (red) :

11 (Lyashi)

Leave out 9, 21 because of the ambiguity of colors of these nodes.



Friendship



Assistance

For this partition into classes, estimate probability distributions.

h	k	$(0,0)$	(A,A)	(F,F)	(AF,AF)	$(0,F)$	$(F,0)$	(A,AF)	(AF,A)
1	1	0.27	0.38	0.03	0.16	0.01	0.01	0.07	0.07
1	2	0.72	0.19	0.01	0.03	0.00	0.03	0.01	0.02
1	3	0.14	0.15	0.07	0.42	0.05	0.04	0.08	0.05
2	1	0.72	0.19	0.01	0.03	0.03	0.00	0.02	0.01
2	2	0.78	0.15	0.01	0.05	0.00	0.00	0.00	0.00
2	3	0.31	0.22	0.04	0.07	0.27	0.03	0.04	0.03
3	1	0.14	0.15	0.07	0.42	0.04	0.05	0.05	0.08
3	2	0.31	0.22	0.04	0.07	0.03	0.27	0.03	0.04

Estimated posterior probabilities of tie configurations,
from actors of color h to actors of color k.

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