Time Series
Exercises and Problems, Sheet 4

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1. Let $Y_t = \epsilon_t + \theta\epsilon_{t-2}$ be a MA(2) process, where $(\epsilon_t)_t$ is $\mathrm{WN}(0, \sigma_\epsilon^2)$.

1. Find the autocovariance and autocorrelation function for this process when $\theta = .8$
2. Compute the variance of the sample mean when $\theta = .8$.
3. Repeat the above when $\theta = -.8$.

2. The Wolf sunspot numbers $\{Y_t, t = 1, \ldots, 100\}$ have sample autocovariances $\hat\gamma_0 = 1382.2, \hat\gamma_1 = 1114.4, \hat\gamma_2 = 591.72$, and $\hat\gamma_3 = 96.215$. Find the Yule-Walker estimates of ϕ_1, ϕ_2, and σ_ϵ^2 in the model

$$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \epsilon_t,$$

where $(\epsilon_t)_t$ is $\mathrm{WN}(0, \sigma_\epsilon^2)$, for the mean-corrected series $Z_t = Y_t - 46.93, t = 1, \ldots, 100$. Find approximate 95% confidence intervals for ϕ_1 and ϕ_2.

3. Suppose you fit an ARIMA(1,1,1) model to 110 observations and calculate the sample autocorrelations of the residuals. The sample autocorrelation function of the residuals for the first nine lags is

Lag h	1	2	3	4	5	6	7	8	9
r_h	0.01	-0.05	-0.11	0.13	-0.05	0.04	0.14	-0.01	-0.02

Calculate the Box-Pierce statistic Q, and interpret the result.

4. Find approximate values for the mean and variance of the periodogram ordinate $I_{200}(\pi/4)$ of the causal AR(1) process

$$Y_t = 0.5Y_{t-1} + \epsilon_t,$$

where $(\epsilon_t)_t$ is WN$(0, \sigma_\epsilon^2)$. Defining

$$\hat{f}(w_j) = \frac{1}{10\pi} \sum_{k=-2}^{2} I_{200}(w_j + w_k), \quad w_j = \frac{2\pi j}{200},$$

use the asymptotic distribution of the periodogram ordinates to approximate

1. the mean and variance of $\hat{f}(\pi/4)$
2. the covariance of $\hat{f}(\pi/4)$ and $\hat{f}(26\pi/100)$
3. $P(\hat{f}(\pi/4) > 1.1 f(\pi/4))$, where f is the spectral density of $(Y_t)_t$.