Time Series
Exercises and Problems, Sheet 3

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1. Consider the periodic process

$$Y_t = A\sin(2\pi\nu t + \phi).$$

Here, A is the amplitude, and ϕ is the phase of the process. Using that $\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$, we obtain

$$Y_t = U_1\sin(2\pi\nu t) + U_2\cos(2\pi\nu t).$$

1. Determine U_1 and U_2 in terms of A and ϕ.

2. Suppose that U_1, U_2 are independent zero-mean random variables with variances σ^2. Calculate the autocovariance function of this process. You could use the identity $\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$.

3. Let Z_1 and Z_2 be independent, standard normal variables. Consider the polar coordinates of the point (Z_1, Z_2), that is,

$$A^2 = Z_1^2 + Z_2^2, \quad \phi = tan^{-1}\left(\frac{Z_2}{Z_1}\right).$$

   Find the joint density of A^2 and ϕ, and, from the result, conclude that A^2 and ϕ are independent random variables, where A^2 is chi-square distributed with 2 df, and ϕ is uniformly distributed on $(0, 2\pi)$.

2. The *normalized* spectral density function $f^*(\omega)$ for a stationary process Y with variance σ_Y^2 is obtained from the spectral density function $f(\omega)$ by

$$f^*(\omega) = \frac{f(\omega)}{\sigma_Y^2}.$$

Assume that the MA(2)-process

$$Y_t = \mu + \epsilon_t + 0.8\epsilon_{t-1} + 0.5\epsilon_{t-2}$$

is weakly stationary, where μ is a constant. Find the acf of $(Y_t)_t$ and show that its normalized spectral density function is given by

$$f^*(\omega) \;=\; \frac{1}{2\pi}(1 + 1.27\cos\omega + 0.53\cos(2\omega)), \quad 0 < \omega < \pi.$$

3. A stationary time series $(Y_t)_t$ has normalized spectral density function

$$f^*(\omega) \;=\; \frac{1}{\pi^2}(\pi - \omega), \quad 0 < \omega < \pi.$$

Calculate its acf.

4. Consider the univariate state-space model given by state conditions $X_0 = W_0, X_t = X_{t-1} + W_t$, and observations $Y_t = X_t + V_t, t = 1, 2, \ldots$, where V_t and W_t are independent, Gaussian, white noise processes with $Var(V_t) = \sigma_V^2$ and $Var(W_t) = \sigma_W^2$. Show that the data follow an ARIMA(0,1,1) model, that is, ∇Y_t follows an MA(1) model.