

1. Suppose x_1, \dots, x_n are values sampled independently from a Poisson distribution with mean λ . The prior density of λ is *Gamma*(α, β), i.e. $\pi(\lambda) = C(\alpha, \beta)e^{-\beta\lambda}\lambda^{\alpha-1}$, for $\lambda > 0$. What is the normalising constant for the prior density? Find the posterior density of λ .
2. Let X_1, \dots, X_n be a random sample of $\mathcal{N}(\theta, \sigma^2)$, where σ^2 known, and assume the prior distribution $\pi(\theta) \sim \mathcal{N}(\mu, \tau^2)$, where μ and τ^2 are known. Show that the posterior distribution of θ given the data \mathbf{x} is

$$\pi(\theta|x) \sim \mathcal{N}\left(\frac{b}{a}, \frac{1}{a}\right)$$

with

$$a = \frac{n}{\sigma^2} + \frac{1}{\tau^2}; \quad b = \frac{1}{\sigma^2} \sum x_i + \frac{\mu}{\tau^2}.$$

3. * Suppose that, as in Question 2, X_1, \dots, X_n is a random sample of $\mathcal{N}(\theta, \sigma^2)$, where σ^2 known, and assume the prior distribution $\pi(\theta) \sim \mathcal{N}(\mu, \tau^2)$, where μ and τ^2 are known. Show that the predictive distribution for x is $\mathcal{N}(\mu, \sigma^2 + \tau^2)$.
4. Suppose that, given the success probability p , x is negative binomial with parameters 10 and p , i.e. x is the number of Bernoulli trials until there are 10 successes. Suppose that the prior for p is Beta(1/2, 1/2). Give the predictive distribution for x and the posterior density of p .
5. The first apparently reliable datings of particular rock strata were obtained from the K/Ar method (comparing the proportions of Potassium 40 and Argon 40 in the rocks) in the 1960'ss, and these resulted in an estimate of 370 ± 20 million years.

In the late 1970's a newer method gave an age of 421 ± 8 million years.

Suppose that the K/Ar method results in a belief for the age which is normally distributed with mean 370 and standard deviation 20 million years. Suppose that the model for the newer method is that the observed age will be normally distributed with mean the true age and standard deviation 8 million years. How are the initial beliefs revised in the light of the results of the new method?