Stochastic Simulation
Exercises and Problems, Sheet 4

Michaelmas Term 2002

Dr. Gesine Reinert

1. Consider as queueing system in which customers arrive, according to a Poisson process with rate λ, at a system having two servers. Upon arrival the customer will join the queue if both servers are busy, enter service with server 1 if that server is free, or enter service with server 2 otherwise. When the customer completes service with a server (no matter which one), that customer then departs the system and the customer that has been in the queue the longest (if there are any customers in queue) enters service. The service distribution of server i is $G_i, i = 1, 2$. Give an algorithm to simulate this model, keeping track of the amounts of time spent in the system by each customer, and the number of services performed by each customer.

Solution: Ross (1996), pp.95–97

2. Describe an algorithm to simulate a two-dimensional Poisson process in the region between $[0, T]$ on the x-axis, and a nonnegative, bounded, continuous function $f(x), x \in [0, T]$, on the y-axis.

Solution: Ross (1996), pp.255

3. Describe how to generate n random points from the uniform distribution on a circle with radius 1 centred at the origin, conditional on the event that no two points are within a distance d of each other.

Solution: Ross (1996), pp.226

4. Assume that the following data are an i.i.d. sample from a uniform distribution on $[\theta, 1]$. Test the hypothesis that $\theta = 0$ using a Monte Carlo test, at level $\alpha = 0.05$. To this purpose, generate 99 $\mathcal{U}([0, 1])$-samples of size 10 each, using the SPLUS generator `runif()`.

0563 0.624 0.187 0.811 0.999 0.810 0.809 0.620 0.429 0.478