# Stochastic Simulation
# Exercises and Problems, Sheet 2

## Michaelmas Term 2002

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**1.** Let $X$ be real-valued with a continuous cumulative distribution function $F$. Show that $F(X)$ has distribution $\mathcal{U}([0,1])$.

**2.** Use the inversion method to find an algorithm for sampling from the geometric distribution with parameter $p$.

**3.** Consider the problem of simulating from the probability density function given by

$$f(x) = \frac{1}{\pi\sqrt{1-x^2}}, \quad -1 \leq x \leq 1.$$

Show that this is the probability density function of the random variable $X = \cos(\pi U)$, where $U \sim \mathcal{U}([0,1])$. Use this result to devise an acceptance-rejection method for generating from $f(x)$, based on the first quadrant of a circle and similar to the polar Marsaglia method.

**4.** By considering an envelope function based on

$$g(x) = \frac{\lambda \mu x^{\lambda-1}}{(\mu + x^{\lambda})^2} \quad , x \geq 0,$$

with $\mu = \alpha^{\lambda}, \lambda = \sqrt{2\alpha - 1}$ for $\alpha \geq 1$, and $\lambda = \alpha$ for $\alpha < 1$, devise an acceptance-rejection methods for generating from the Gamma $\Gamma(\alpha, 1)$ distribution. If $X \sim \Gamma(\alpha, 1)$, find the distribution of $Y = \frac{1}{\beta}X$ and hence suggest a method for generating random quantities from a $\Gamma(\alpha, \beta)$ distribution. This method is called the *log-logistic method.*

**Hints.** Show that the function $h(x) = \frac{g(x)}{f(x)}$ takes its maximum at $x = \alpha$; here, $f(x)$ is the Gamma density. Then show that $Y = \alpha \left(\frac{U}{1-U}\right)^{\frac{1}{\lambda}}$ has density $g$, where $U \sim \mathcal{U}([0,1])$.

**5.** Explain how the ratio-of-uniforms method could be used to generate random quantities from a standard Student's $t$-distribution with $n$ degrees of freedom, $n \geq 2$.