Stochastic Simulation
Exercises and Problems, Sheet 1

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1. If $x_0 = 5$ and

$$x_n = 3x_{n-1} \ (\text{mod } 150),$$

find $x_1, x_2, \ldots, x_{10}$.

2. If $x_0 = 3$ and

$$x_n = 5x_{n-1} + 7 \ (\text{mod } 300),$$

find $x_1, x_2, \ldots, x_{10}$.

3. Prove that for the Fibonacci recursion

$$U_i = U_{i-1} + U_{i-2} \ (\text{mod } 1)$$

that $U_{i-2} < U_i < U_{i-1}$ never occurs, whereas this event has probability $\frac{1}{6}$ for random independent uniform $\mathcal{U}([0,1])$ numbers.

4. Without actually computing any iterations, determine which of the following mixed congruential generators have full period:

$$
\begin{aligned}
(a) \quad & X_i &=& \ 13X_{i-1} + 13 \ (\text{mod } 16) \\
(b) \quad & X_i &=& \ 12X_{i-1} + 13 \ (\text{mod } 16) \\
(c) \quad & X_i &=& \ 13X_{i-1} + 12 \ (\text{mod } 16) \\
(a) \quad & X_i &=& \ X_{i-1} + 12 \ (\text{mod } 13).
\end{aligned}
$$

5. The following numbers can be found in Ripley (1987). Test whether they are i.i.d. $\mathcal{U}([0,1])$ distributed, using different tests from lecture.

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0.563   0.478   0.218   0.396   0.455
0.624   0.527   0.163   0.527   0.692
0.187   0.005   0.382   0.923   0.147
0.811   0.531   0.545   0.450   0.839
0.999   0.536   0.926   0.373   0.986
0.810   0.067   0.471   0.824   0.825
0.809   0.603   0.397   0.197   0.811
0.620   0.671   0.867   0.020   0.635
0.429   0.274   0.264   0.217   0.446
0.049   0.945   0.132   0.238   0.082
```