A spatiotemporal change-surface model for the onset of deposition at Bourewa

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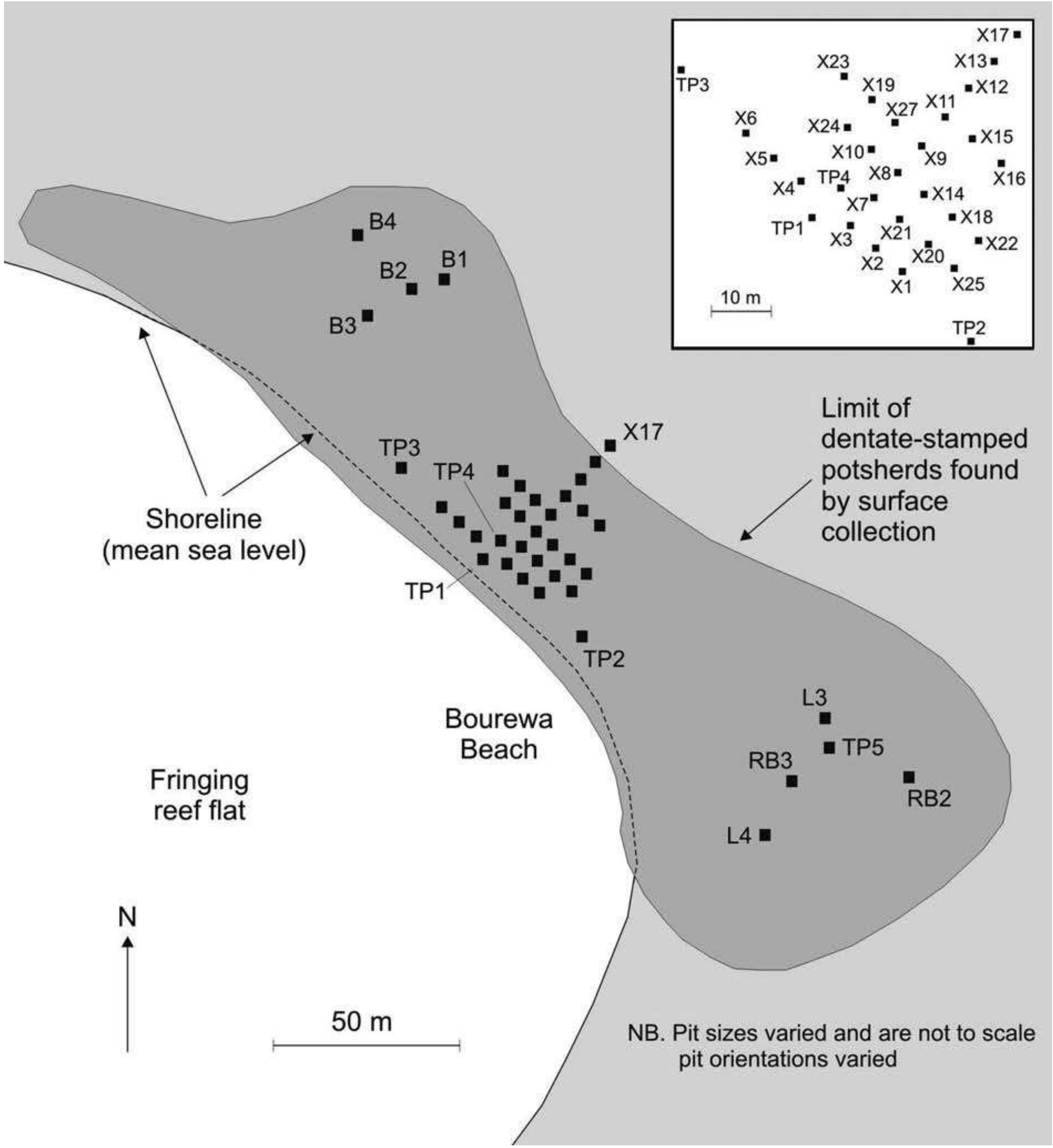
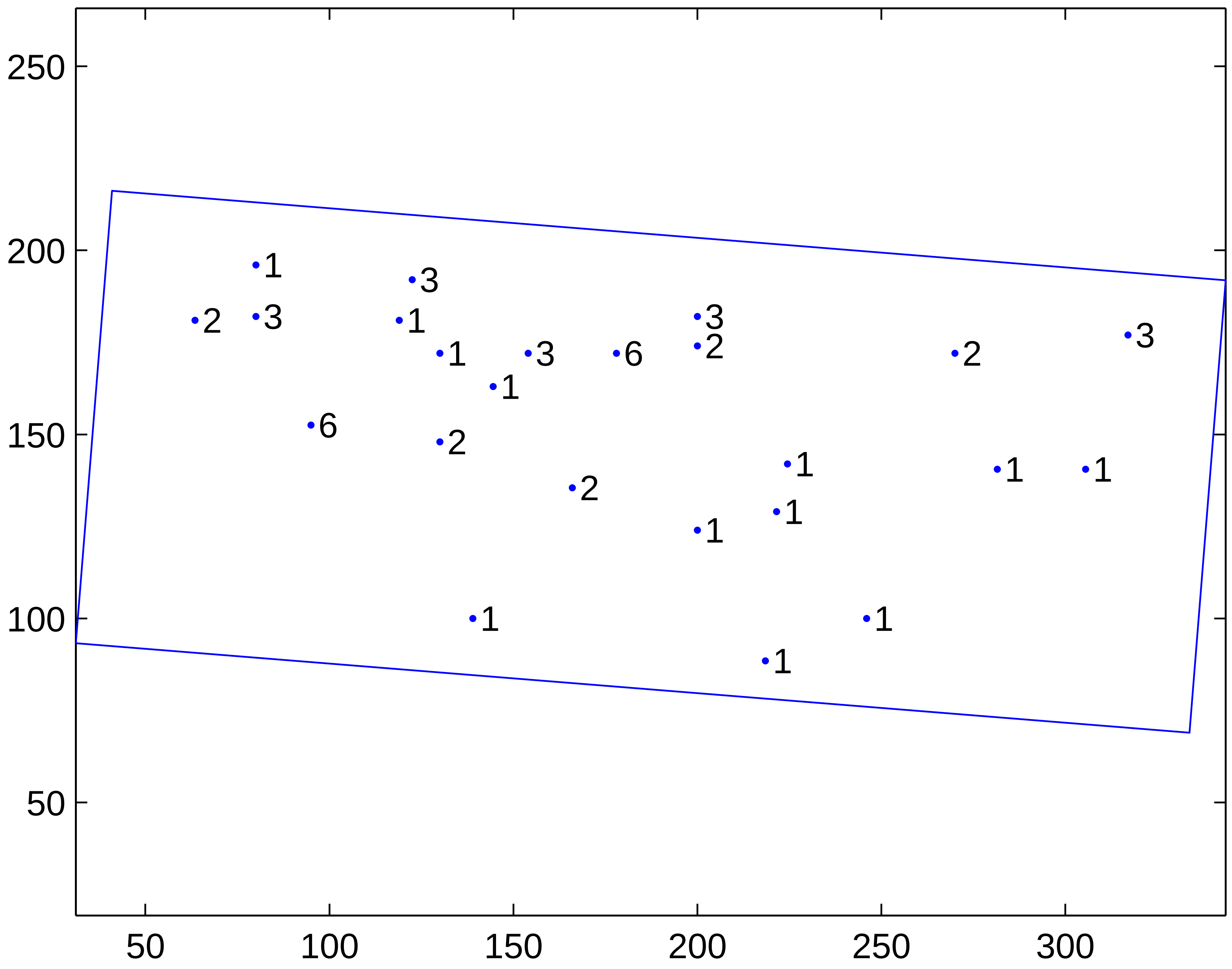
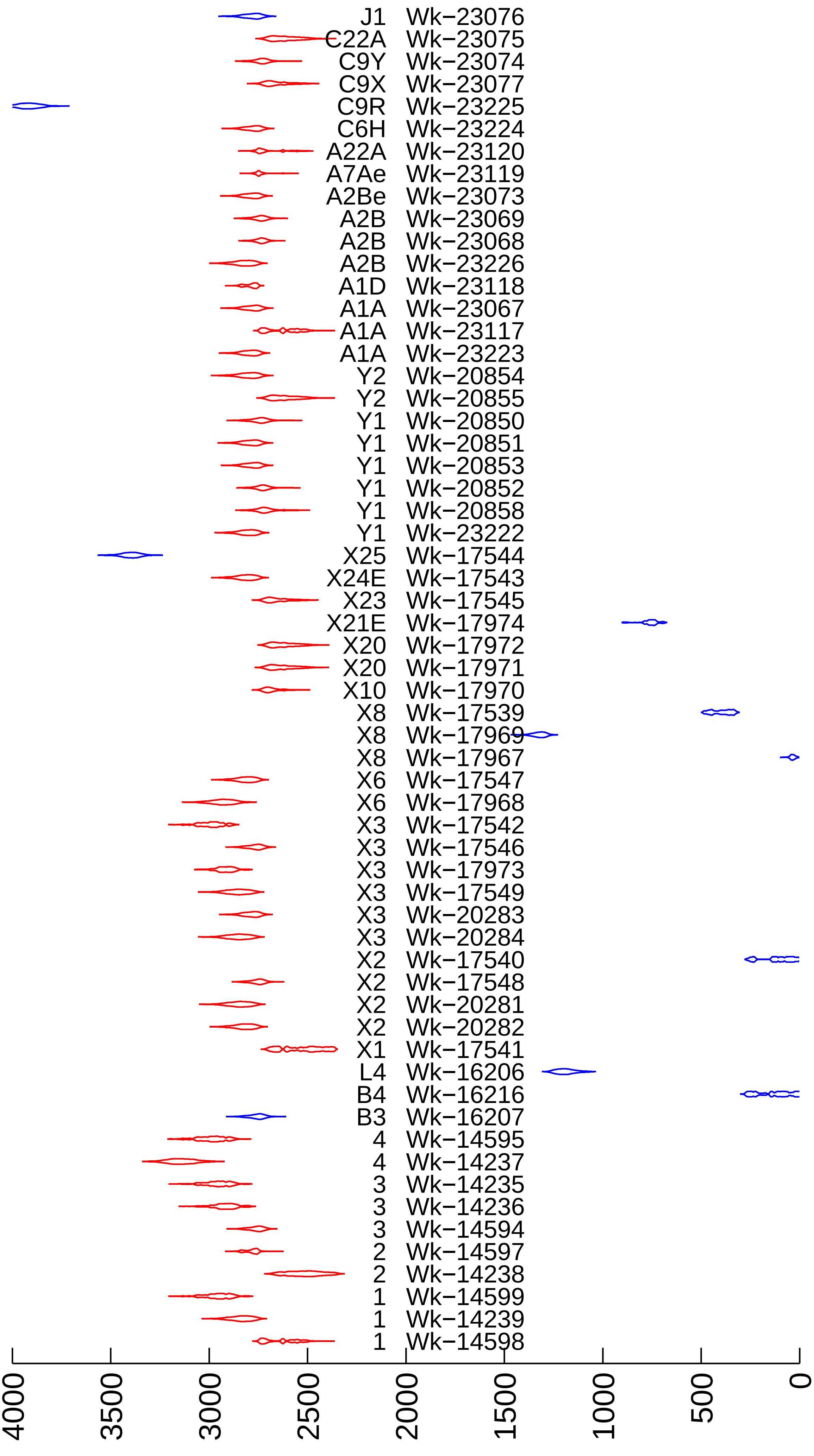
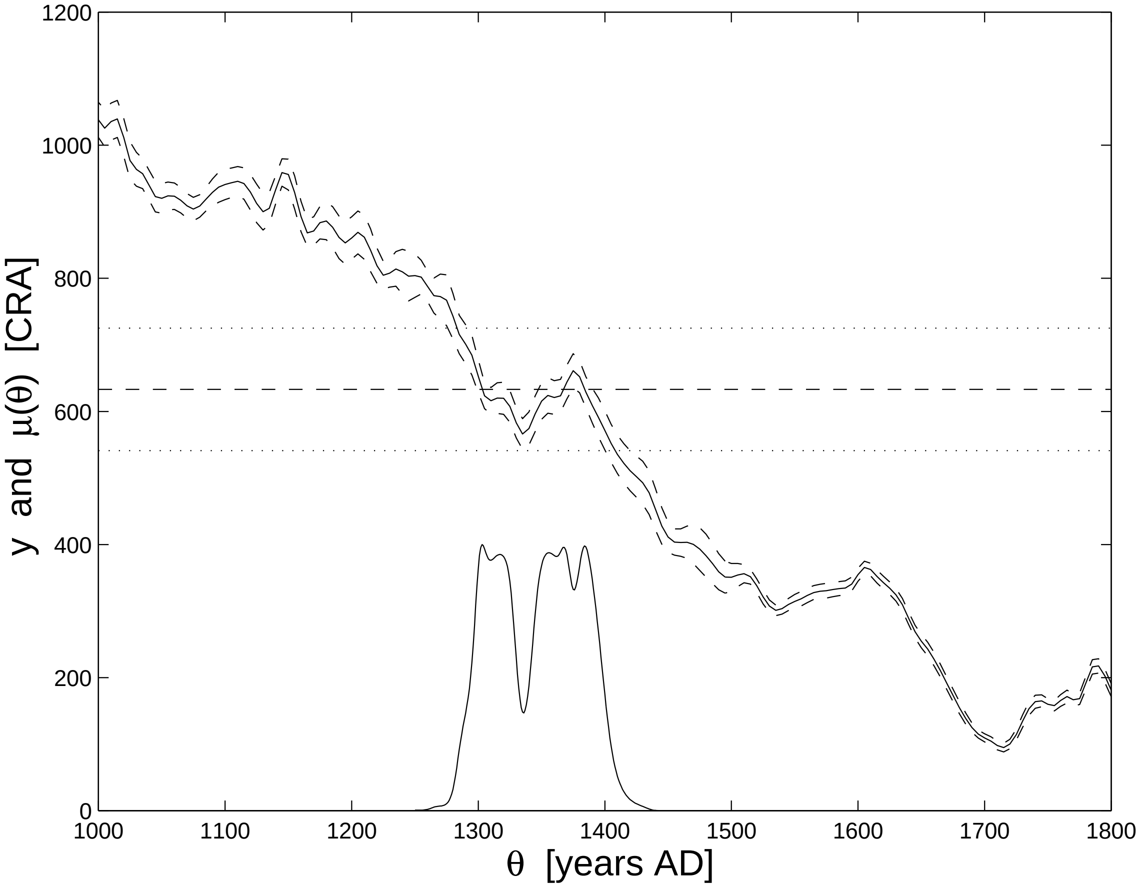
Seminar, Department of Statistics, University of Glasgow 9/12/09


X17
X13
X12
X23
TP3
X19
X27
X11
X6
X24
X15
X10
X9
X5
TP4
X8
X16
X4
X14
X7
X18
TP1
X3
X21
X22
X2
X20
X1
X25
10 m
TP2
B4
B1
B2
B3
X17
Limit of dentate-stamped potsherds found by surface collection
TP3
TP4
Shoreline (mean sea level)
TP1
TP2
L3
Bourewa Beach
TP5
RB3
RB2
Fringing reef flat
L4
N
50 m
NB. Pit sizes varied and are not to scale
pit orientations varied



250
200
150
100
50
50
100
150
200
250
300
1
3
2
3
1
3
2
1
3
6
2
2
3
1
6
2
1
1
1
2
1
1
1
1
1


J1 Wk−23076
C22A Wk−23075
C9Y Wk−23074
C9X Wk−23077
C9R Wk−23225
C6H Wk−23224
A22A Wk−23120
A7Ae Wk−23119
A2Be Wk−23073
A2B Wk−23069
A2B Wk−23068
A2B Wk−23226
A1D Wk−23118
A1A Wk−23067
A1A Wk−23117
A1A Wk−23223
Y2 Wk−20854
Y2 Wk−20855
Y1 Wk−20850
Y1 Wk−20851
Y1 Wk−20853
Y1 Wk−20852
Y1 Wk−20858
Y1 Wk−23222
X25 Wk−17544
X24E Wk−17543
X23 Wk−17545
X21E Wk−17974
X20 Wk−17972
X20 Wk−17971
X10 Wk−17970
X8 Wk−17539
X8 Wk−17969
X8 Wk−17967
X6 Wk−17547
X6 Wk−17968
X3 Wk−17542
X3 Wk−17546
X3 Wk−17973
X3 Wk−17549
X3 Wk−20283
X3 Wk−20284
X2 Wk−17540
X2 Wk−17548
X2 Wk−20281
X2 Wk−20282
X1 Wk−17541
L4 Wk−16206
B4 Wk−16216
B3 Wk−16207
4 Wk−14595
4 Wk−14237
3 Wk−14235
3 Wk−14236
3 Wk−14594
2 Wk−14597
2 Wk−14238
1 Wk−14599
1 Wk−14239
1 Wk−14598
4000
3500
3000
2500
2000
1500
1000
500
0



Likelihood:

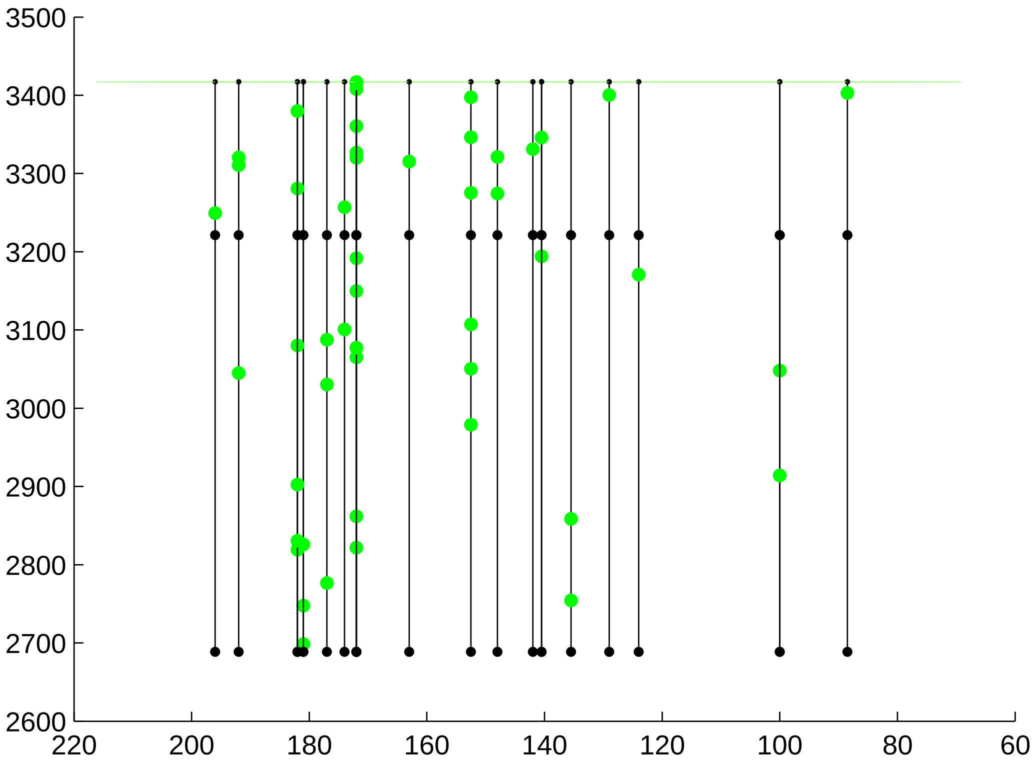
θ_i specimen age (cal),

y_i radiocarbon age (uncal),

$\mu(\theta)$ and $\sigma(\theta)$ defined by look-up table.

$$\ell(\theta; y) \propto \prod_{i=1}^{K} (\sigma_i^2 + \sigma(\theta_i)^2)^{-1/2} \exp\left(-\frac{(\mu(\theta_i) - y_i)^2}{2(\sigma_i^2 + \sigma(\theta_i)^2)} \right)$$

$McCormac$ et al. (2004).
SHCal04 Southern Hemisphere calibration 0 - 11.0 cal kyr bp.
Radiocarbon 46, 1087–1092.



Priors: M Phase boundaries,

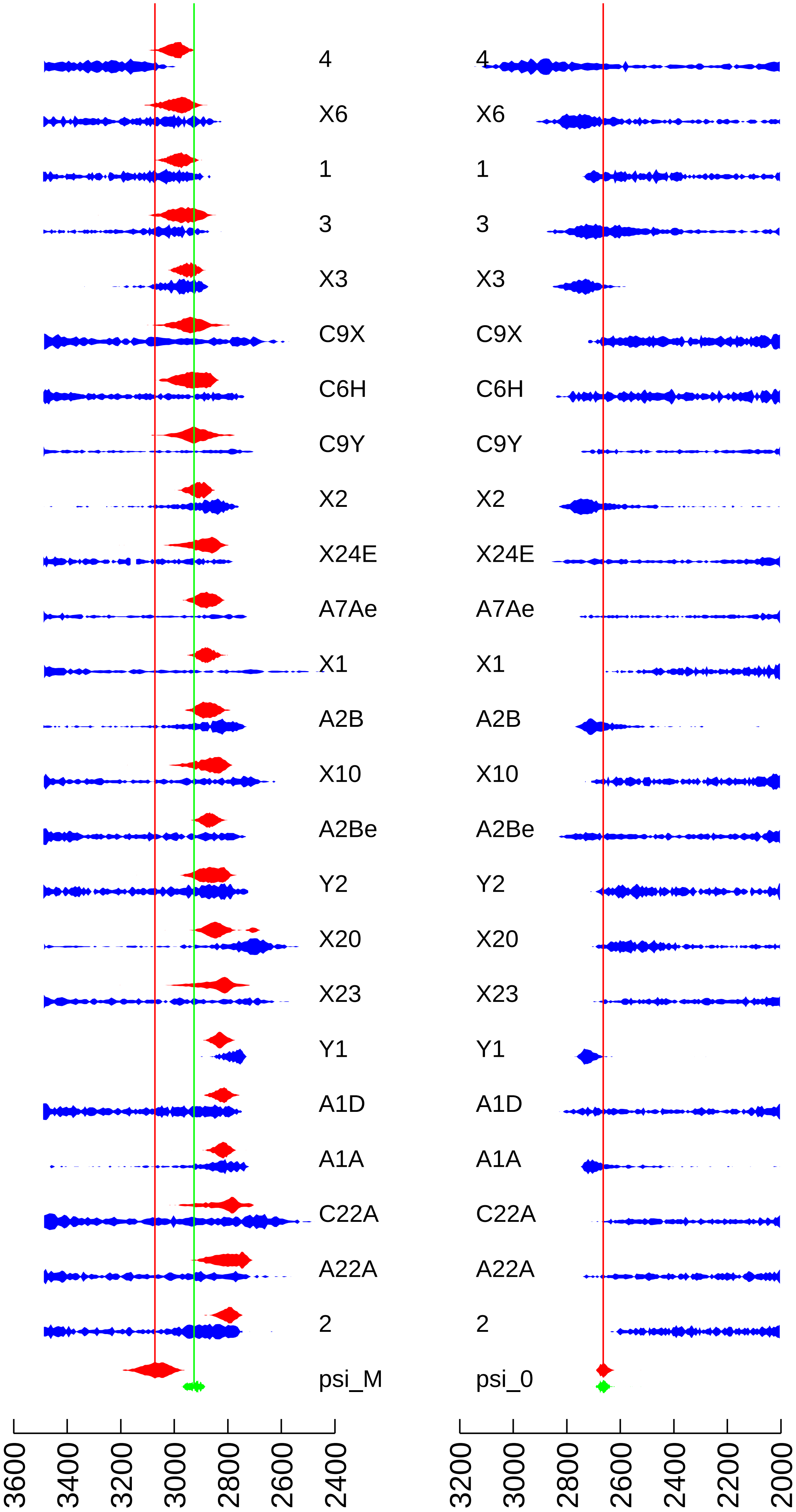
$\lambda(x,t) = \lambda_X(x)\lambda_{m(t)}$

$\mathbf{m}(i)$ = known phase of the i'th dated specimen, of K.

$$\begin{aligned} p(\psi,\theta|M,\mathbf{m}) &= p(\psi|M)p(\theta|\psi,\mathbf{m}) \\ &= p_{0,M}(\psi_0,\psi_M)\frac{1}{(\psi_M-\psi_0)^{M-1}}\prod_{i=1}^{K}(\psi_{\mathbf{m}(i)-1}-\psi_{\mathbf{m}(i)})^{-1} \end{aligned}$$

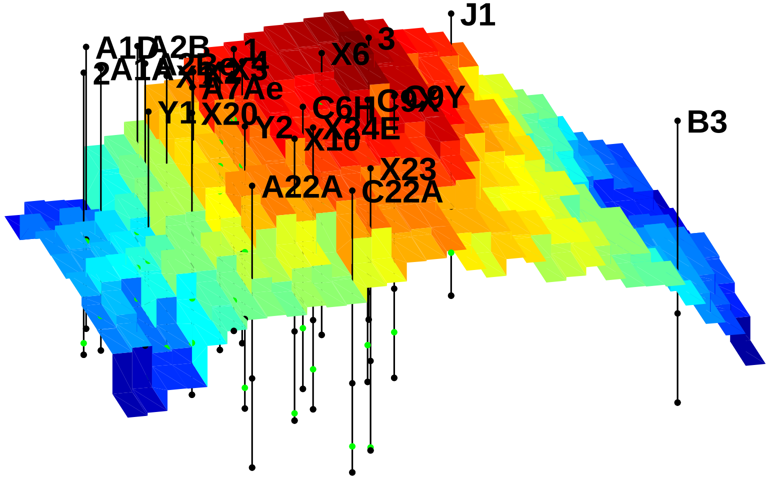
Posterior:

$$p(\psi,\theta|y,\mathbf{m},M) = \ell(\theta;y)p(\psi,\theta|M,\mathbf{m}).$$


4
X6
1
3
X3
C9X
C6H
C9Y
X2
X24E
A7Ae
X1
A2B
X10
A2Be
Y2
X20
X23
Y1
A1D
A1A
C22A
A22A
2
psi_M
psi_0
3600
3400
3200
3000
2800
2600
2400
3200
3000
2800
2600
2400
2200
2000

Shrinkage

Spatial onset-field



$\lambda(x, t)$-estimation confounded by uneven post-depositional thinning.

Assume absence of dates is informative of zero deposition rate. A single well studied site - selectively acting erosion and haphazard excavation excluded.

Model and fit the onset-field - estimate for each location across the site the time at which deposition of datable specimen began.

Ammerman, A. and L. Cavalli-Sforza (1971). Measuring the rate of spread of early farming in Europe. *Man 6*, 674–688.

Davison, K., P. Dolukhanov, G. Sarson, A. Shukurov, and G. Zaitseva (2009). Multiple sources of the European Neolithic: Mathematical modelling constrained by radiocarbon dates. *Quaternary International 203*, 10–18.

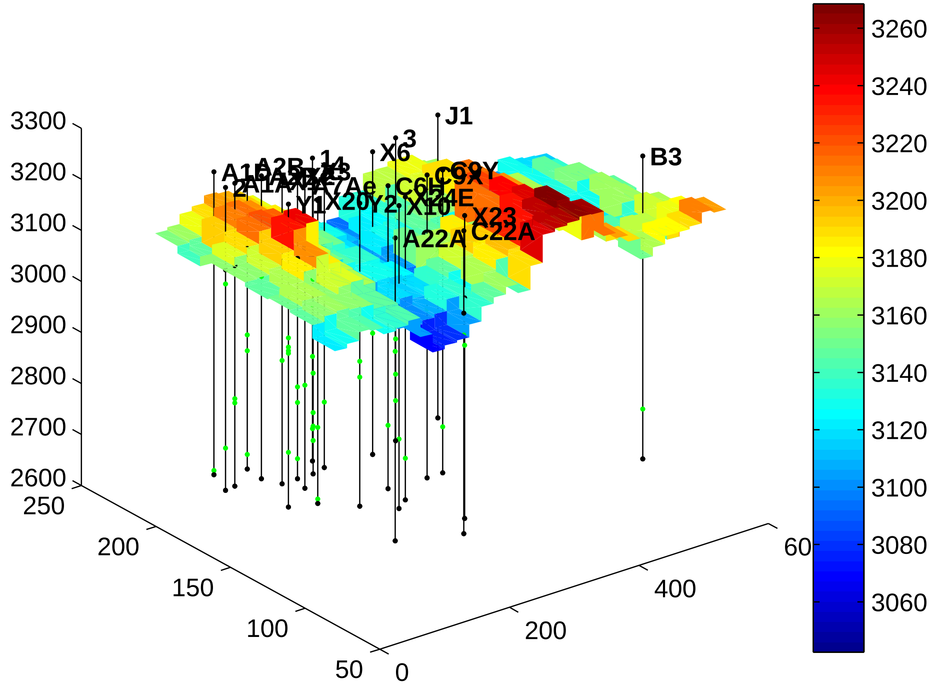
The onset-field ϕ is a realization of the following immigration-migration process: cell c is occupied by immigration at rate α_c; when cell c becomes occupied, the arrival rate for occupation at each of its neighbors $c' \in \mathcal{N}(c)$ is *increased* by $\beta_{c,c'}$.

$$
\begin{aligned}
p(\phi|\alpha,\beta,\psi) &= \prod_{c=1}^{C} \exp\left(-\alpha_c(\psi_M - \phi_c) - \frac{1}{2} \sum_{c' \in \mathcal{N}(c)} \beta_{c,c'} |\phi_c - \phi_{c'}| + \log(\rho(\phi_{\mathcal{N}^+(c)})) \right) \\
\rho(\phi_{\mathcal{N}^+(c)}) &= \alpha_c + \sum_{c' \in \mathcal{N}(c)} \beta_{c,c'} \mathbb{I}_{\phi_{c'} > \phi_c}
\end{aligned}
$$

Normalized - inference for α and β possible.

ϕ is a Markov Random Field, cliques $\{c\}, \{c, c'\}_{c' \in \mathcal{N}(c)}$ and $N^+(c)$.

Richardson growth model $\alpha = 0$.
Johnson-Mehl tesselation.
Contact process, $\alpha = 0$, cell abandoned at rate 1.



Isotropy, Homogeneity, Waves, and Speed

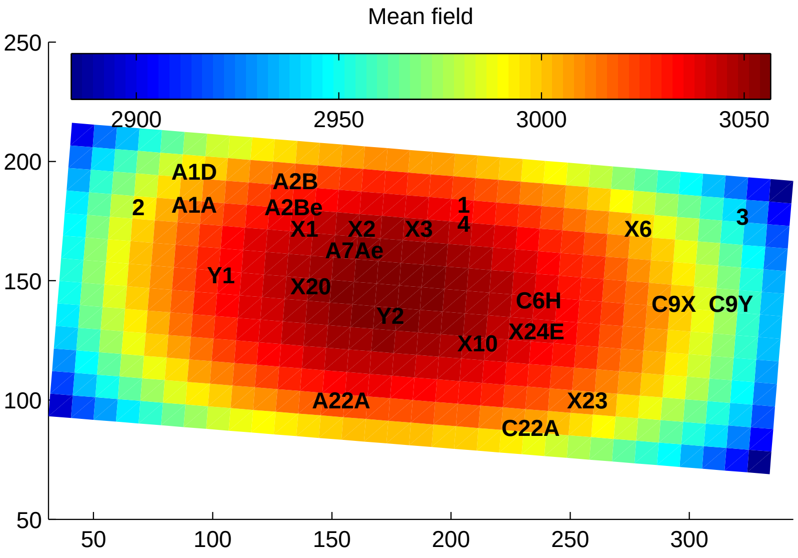
$\int_{(-\infty,\psi_M]^C} p(\phi|\alpha,\beta,\psi)d\phi = 1$ so

$$\mathsf{E}\left(\psi_M - \phi_c\right) = \mathsf{E}\left(\frac{1}{\alpha_c + \sum_{c'\in\mathcal{N}(c)} \beta_{c,c'}\mathbb{I}_{\phi_{c'}>\phi_c}}\right).$$

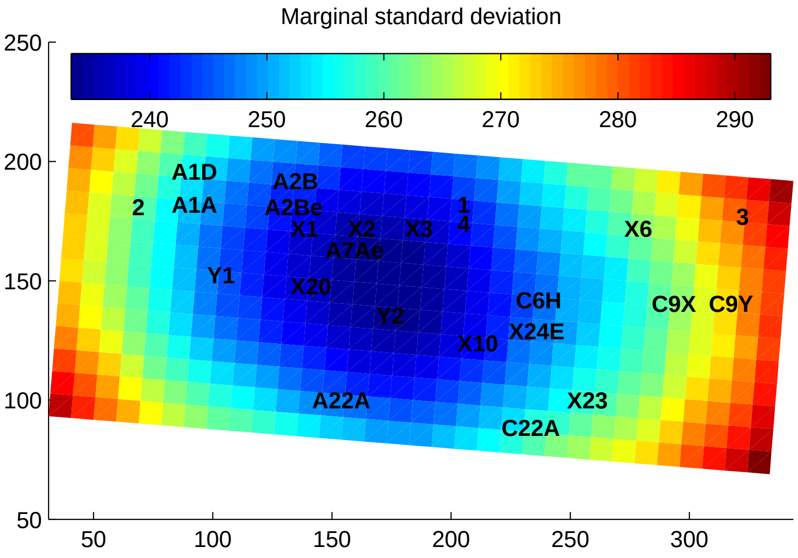
$$\mathrm{cov}\left(\phi_c + \frac{1}{\rho(\phi_{\mathcal{N}^+(c)})}, \phi_{c'} + \frac{1}{\rho(\phi_{\mathcal{N}^+(c')})}\right) = 0$$

for cells $c' \neq c$ so $\phi_c + \rho(\phi_{\mathcal{N}^+(c)})^{-1}$ homogeneous and isotropic to 2nd order.

Edge effect $\mathsf{E}(\Phi_c) < \mathsf{E}(\Phi'_c)$ for c, c' boundary,interior.

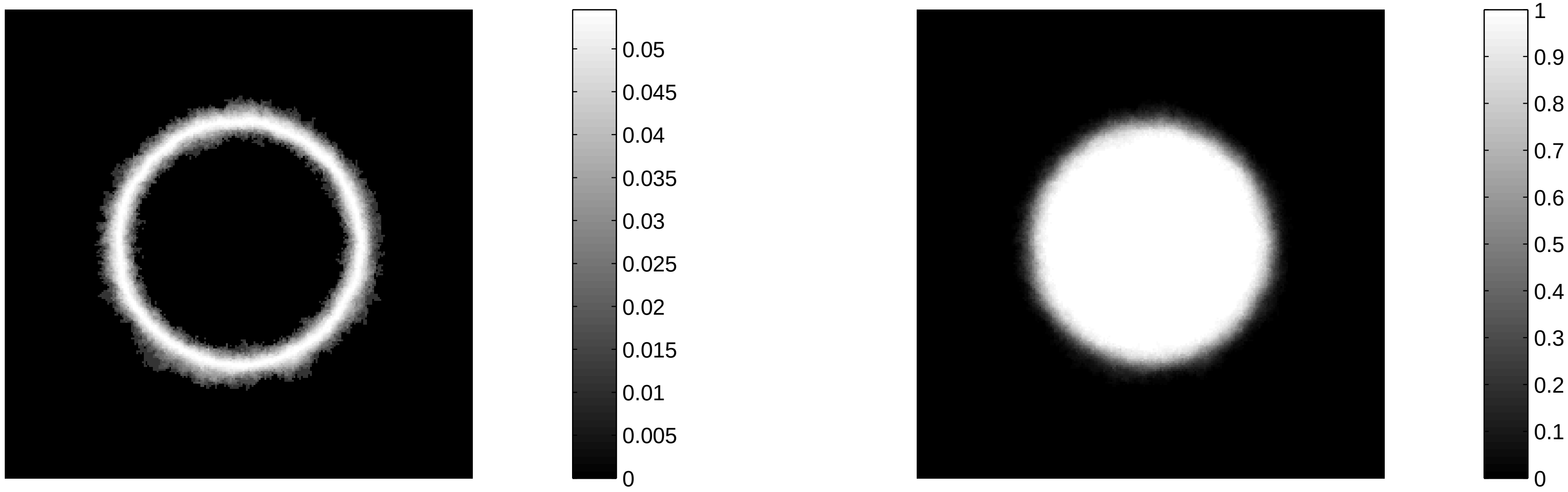


Mean field



Marginal standard deviation

Isotropy of propagation (excluding boundary effect)



mf at 30, (tw-sim,tw-mf), std(I,30), pc at 30

> “The Richardson model, introduced in Richardson (1973), describes growth on $\mathbb{Z}^d$. Sites are either healthy or infected, the dynamics being that a healthy site is infected at a rate proportional to the number of infected neighbors, and that once infected it remains so forever... the existence of an asymptotic shape has been established, but the nature of the shape so far remains unknown”.

`Adv. Appl. Prob. (SGSA) 36, 973-980 (2004)`
`"COEXISTENCE IN A TWO-TYPE CONTINUUM GROWTH MODEL"`
`MARIA DEIJFEN, OLLE HAGGSTROM`

Mean-field approximation (for $p_c(t) = \Pr(\phi_c > t)$)

Exactly:

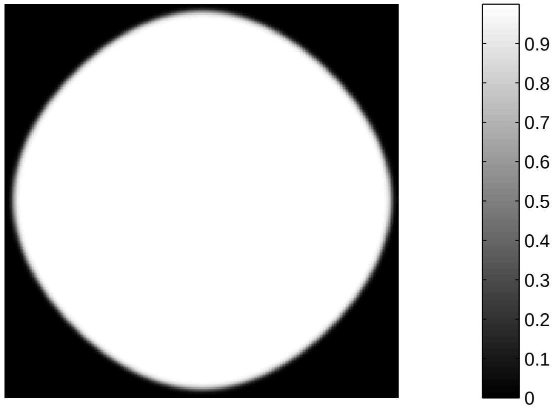
$$\frac{d}{dt} p_c(t) = \alpha(1 - p_c(t)) + \beta \sum_{c' \in \mathcal{N}(c)} \Pr(\Phi_c < t, \Phi_{c'} > t).$$

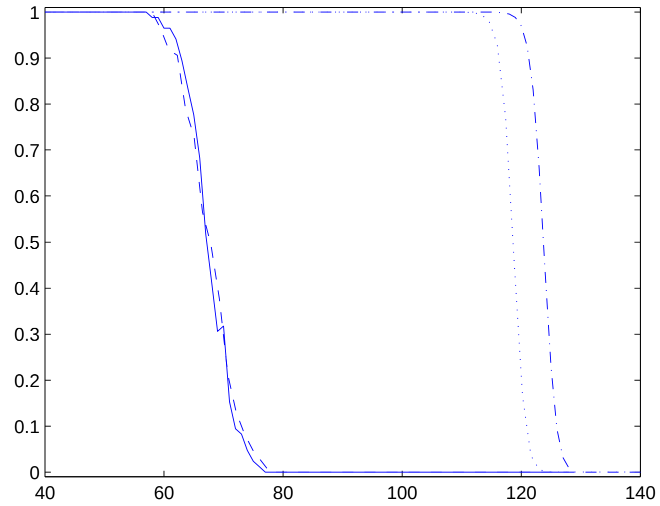
Approximately:

$$\frac{d}{dt} \tilde{p}_c(t) = \alpha(1 - \tilde{p}_c(t)) + \beta(1 - \tilde{p}_c(t)) \sum_{c' \in \mathcal{N}(c)} \tilde{p}_{c'}(t).$$

Naive continuum limit (cf Fisher Equation) at $\alpha = 0$ has traveling wave speed 4β:

$$\frac{\partial \tilde{p}}{\partial t} = (1 - \tilde{p})(\alpha + 4\beta\tilde{p}) + \beta(1 - \tilde{p})\nabla^2 \tilde{p}.$$





Propagation speed about 2 to 3 β (also from E$(|\phi_c - \phi_{c'}|)$).

Onset field coupled to data

The onset field fixes the local change point from zero to non-zero rate for dated-specimen deposition:

$$\psi_{\mathbf{m}(i)-1} < \theta_i < \min(\phi_{\mathbf{c}(x_{\mathbf{h}(i)})}, \psi_{\mathbf{m}(i)}).$$

so new $p(\theta|\psi, \mathbf{m}, ...)$ is

$$p(\theta|\phi, \psi, \mathbf{m}) = \prod_{i=1}^{K} \frac{1}{\min(\phi_{\mathbf{c}(x_{\mathbf{h}(i)})}, \psi_{\mathbf{m}(i)}) - \psi_{\mathbf{m}(i)-1}}.$$

Our prior for α and β is subjective.

For a handful of pure immigration events in $[L, U]$, we need $\mathsf{E}(\alpha) \gtrsim 1/C(U-L)$.

The "speed" of expansion is about 2β [cells/year]. In order to cover the site in $[L, U]$ we need

$$\mathsf{E}(\beta) \gtrsim \frac{\max(C_1, C_2)}{2(L-U)}.$$

Choice $A = 1$, $B = 10$ in

$$\alpha \sim \mathsf{Exp}(A/N(U-L)), \quad \beta \sim \mathsf{Exp}(B \max(C_1, C_2)/2(L-U)).$$

gave plausible range for temporal "roughness" and "peak to valley depth".

Fitting the onset-field for a single phase

1) Dated-pit *location* informs ϕ. Condition $\psi_0 < \phi_c \leq \psi_M$ removes this.

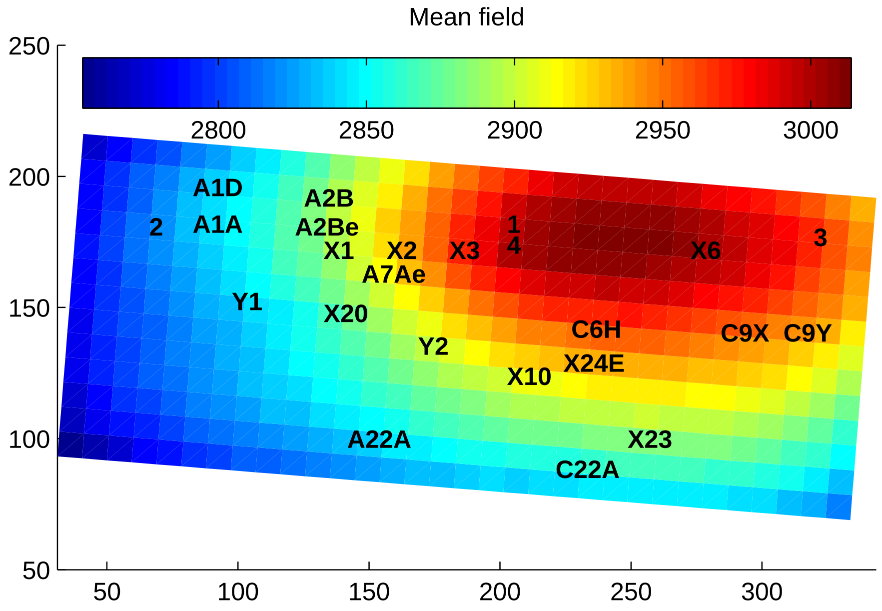
2) Condition on the first onset-field event at ψ_M years BP.

$$p(\phi|\alpha, \beta, \psi, \max(\phi) = \psi_M) = \alpha^{-1} p(\phi|\alpha, \beta, \psi)$$

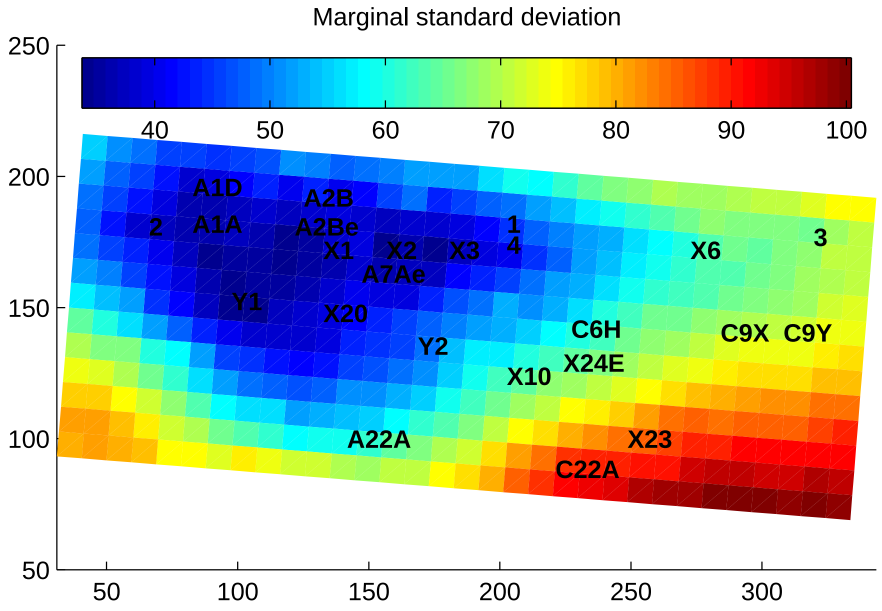
The posterior density, single phase $(M = 1)$, onset-field $\phi \in [\psi_0, \psi_M]^C$

$$p(\alpha, \beta, \phi, \psi, \theta|y, \mathbf{m}, M) \propto \ell(\theta; y) p(\alpha, \beta, \phi, \psi, \theta|\mathbf{m}, M).$$

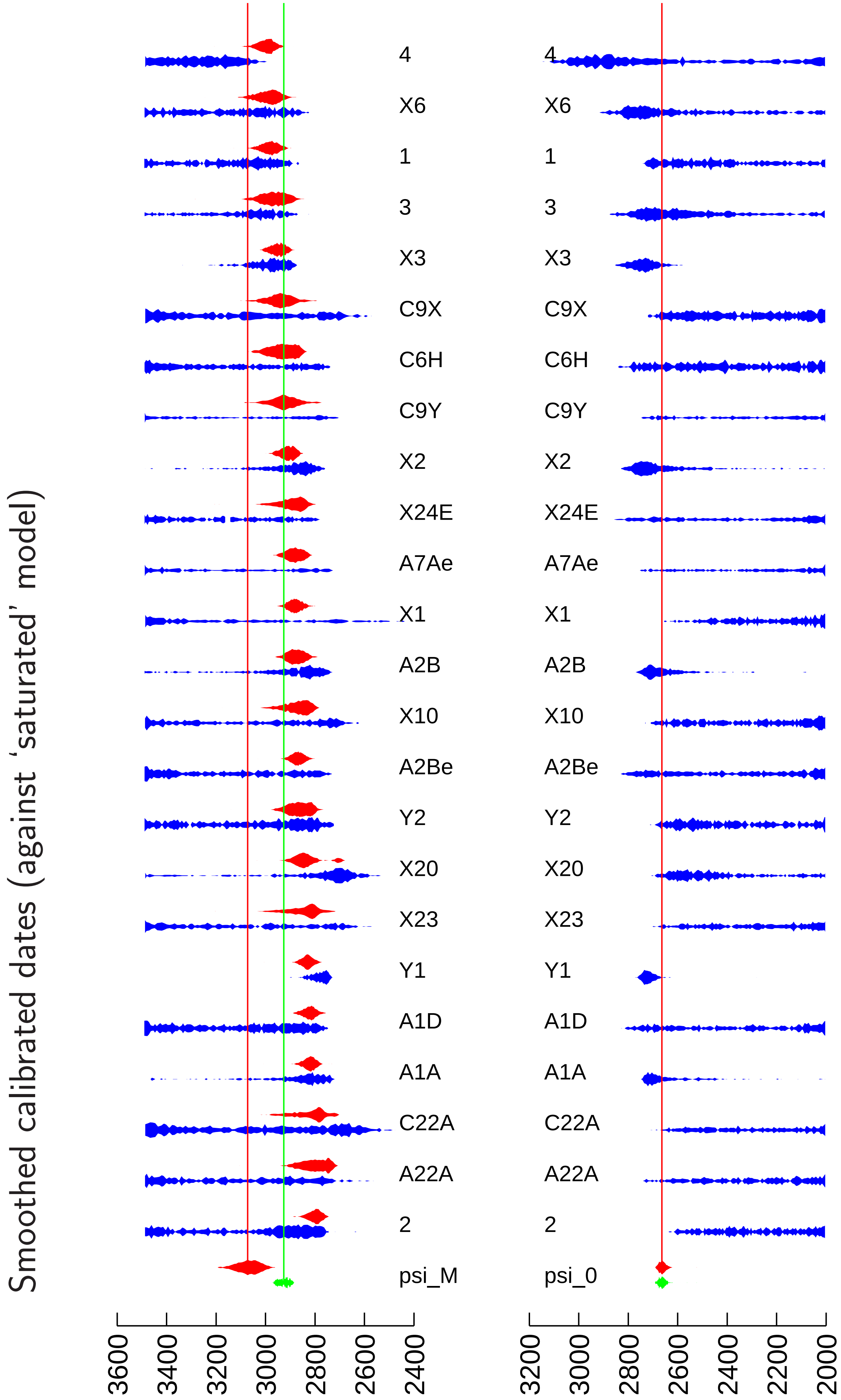
Prior $p(\alpha, \beta, \phi, \psi, \theta|\mathbf{m}, M)$. Fit via MCMC simulation.

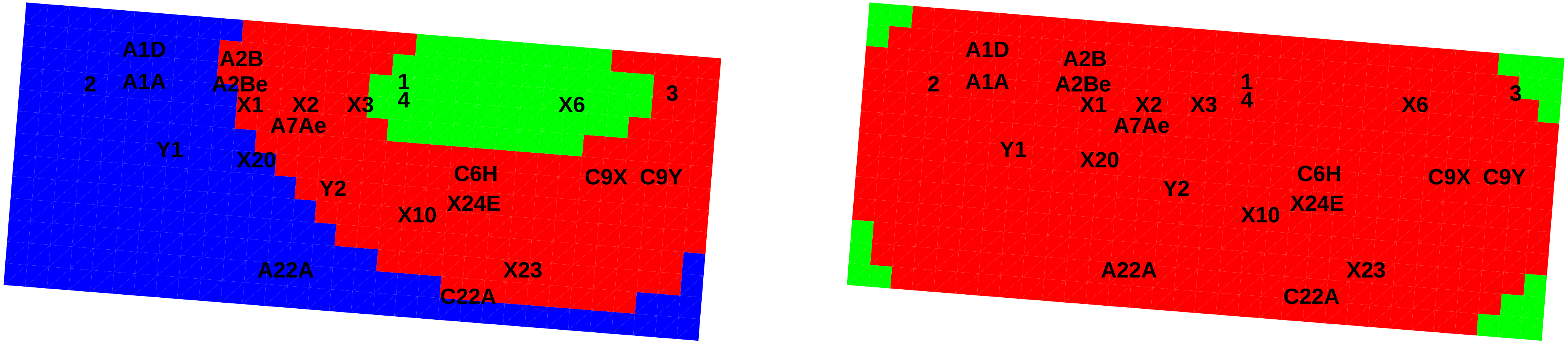


Mean field



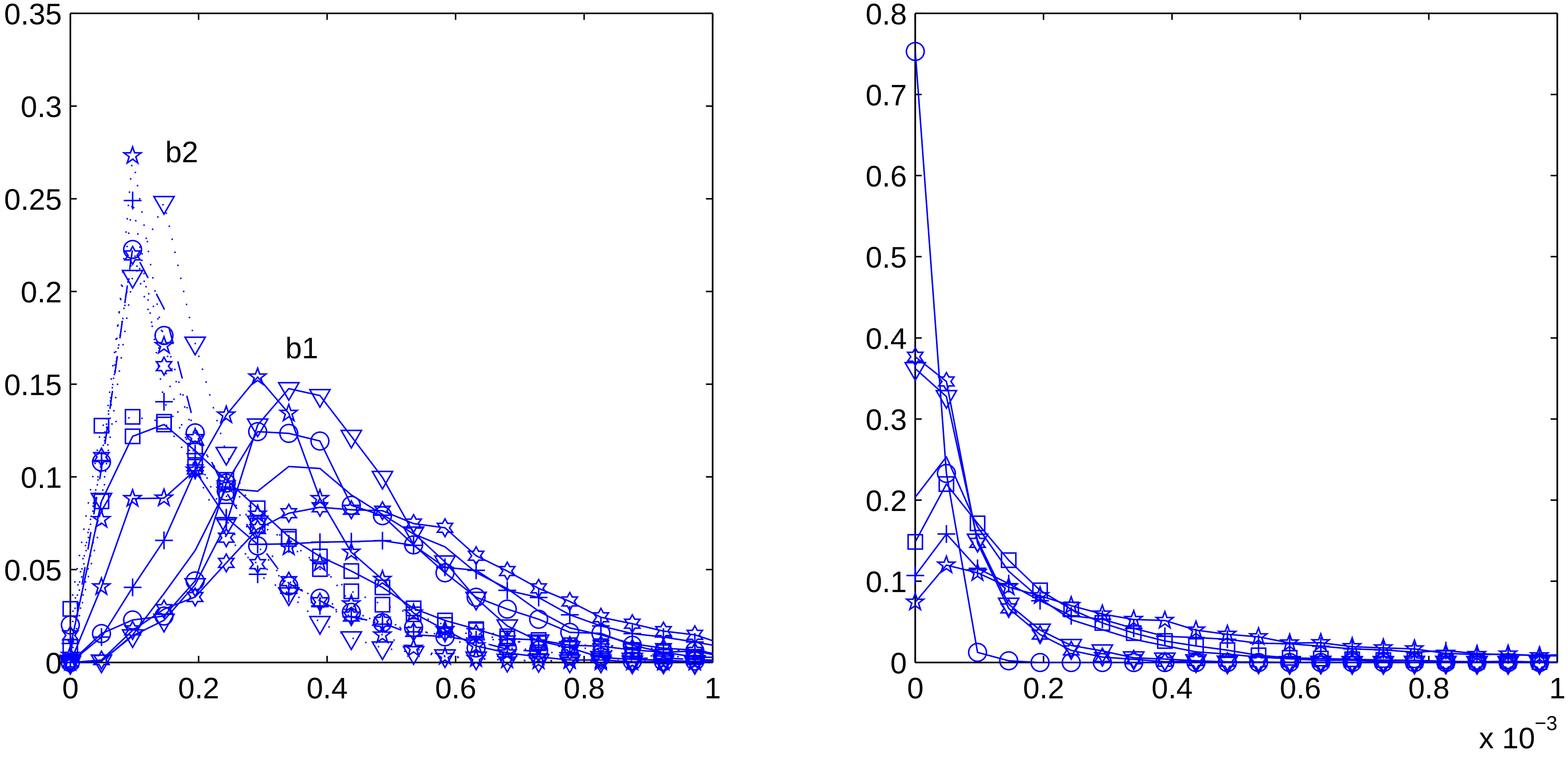
Marginal standard deviation


Smoothed calibrated dates (against 'saturated' model)
4
X6
1
3
X3
C9X
C6H
C9Y
X2
X24E
A7Ae
X1
A2B
X10
A2Be
Y2
X20
X23
Y1
A1D
A1A
C22A
A22A
2
psi_M
psi_0
3600
3400
3200
3000
2800
2600
2400
3200
3000
2800
2600
2400
2200
2000



Support for spread: (left) the posterior probability that $\psi_M - \phi_c < 150$ (respectively, $\psi_M - \phi_c > 150$) is greater than 0.8 for cells c in the green (blue) region (right) prior.

Speed, and prior sensitivity



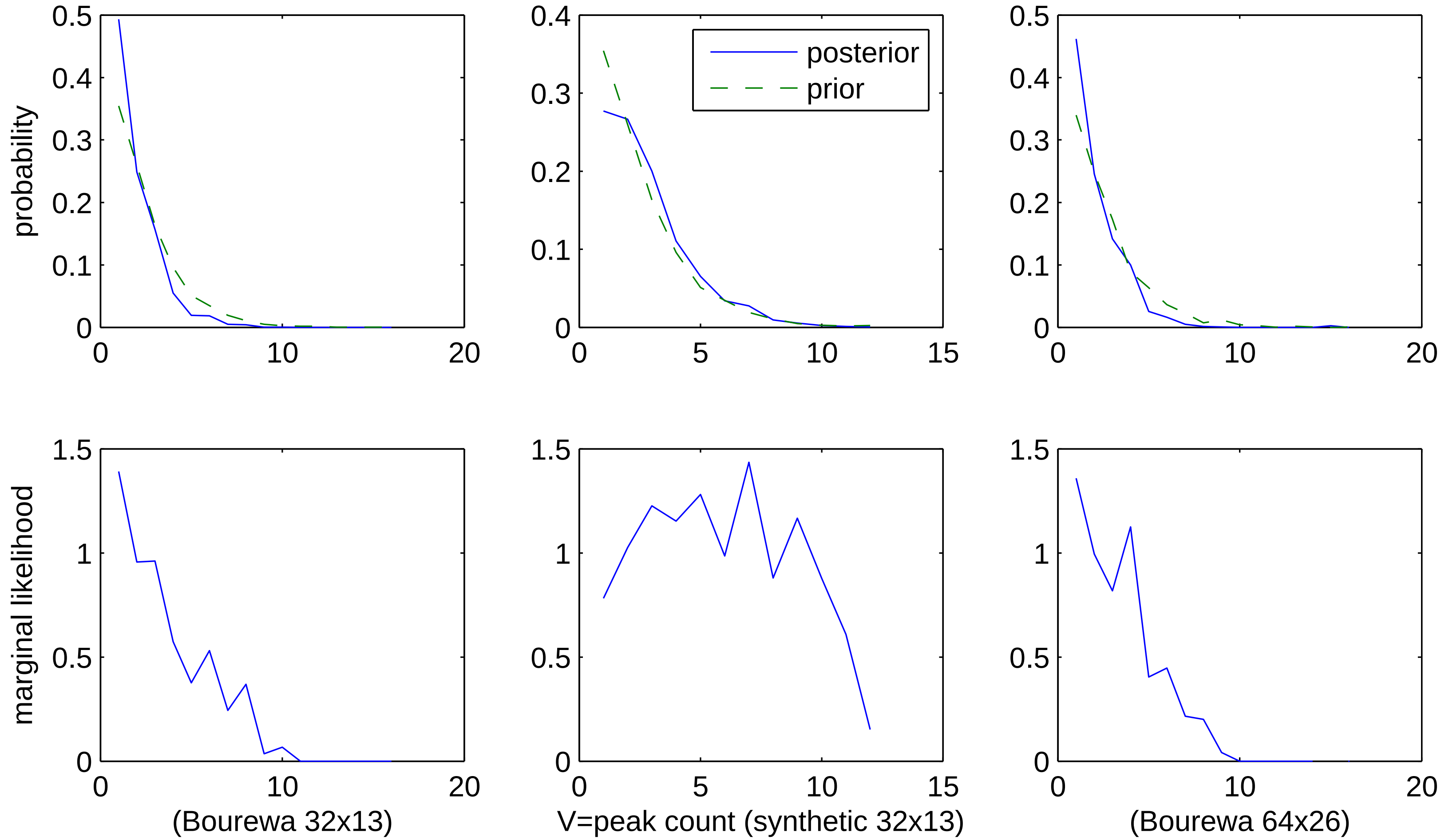
x-axis units metres/year. y-axis frequency.

(Left) the posterior β_1 (solid, 'b1') and β_2 (dashed, 'b2')

A and B hyperprior parameters equal $(A, B) = (10, 1), (20, 2), (5, 0.5), (20, 0.5), (5, 2)$ (no symbol,$+$,triangle,5-star,6-star), $(A, B) = (10, 1)$ on 64×25 (circle), prior $(A, B) = (10, 1)$ (square). (right) α.

Count the number of arrival-events: peak at cell c if $\phi_c > \phi_{c'}\ \forall c' \in \mathcal{N}(c)$

$$V(\phi) = \#\ \text{peaks}$$



(left column) Bourewa data (middle column) Synthetic data, (right column) Bourewa data 64×25.

The paper on which this talk is based at
`http://www.stats.ox.ac.uk/~nicholls/`
adds multiphase fitting and link to supplementary material.

Zhu, J., J. Rasmussen, J. Møller, B. Aukema, and K. Raffa (2008). Spatial-temporal modeling of forest gabs generated by colonization from below- and above-ground bark beetle species. *J. Am. Statist. Ass. 103*, 162–177.

Ibáñez, M. V. and A. Simó (2007). A geostatistical spatiotemporal modelling with change points. Fifth workshop on Bayesian inference in stochastic processes, Valencia

Majumdar, A., A. Gelfand, and S. Banerjee (2005). Spatio-temporal change-point modeling. *Journal of Statistical Planning and Inference 130*, 149–166.

Møller, J. (1999). Topics in Voronoi and Johnson-Mehl tessellations. In W. K. O.E. Barndorff-Nielsen and M. van Lieshout (Eds.), *Stochastic Geometry: Likelihood and Computations*, Monographs on Statistics and Applied Probability. Chapman and Hall/CRC.

Durrett, R. and T. M. Liggett (1981). The shape of the limit set in Richardson's growth model. *The Annals of Probability 9*, 186–193.

Richardson, D. (1973). Random growth on a tessalation. *Proc. Cambridge Philos. Soc. 74*, 515–528.