

Problem Sheet 2 - Part B Actuarial Science II - Oxford HT 2006

Term structure of interest rates

1. Define what is meant by an n -year spot rate of interest. Suppose that the spot-yield curve is such that the t -year spot rate of interest is $y_t = 0.03e^{-0.05t} + 0.02$. Calculate the following quantities and explain their meaning.

- (a) $f_{2,4}$,
- (b) P_6 ,
- (c) $F_{3,5}$,
- (d) F_8 .

2. Consider a fixed-interest bond with coupon 6% pa, payable annually in arrears, with term to redemption of 3 years, redeemable at par. If the 1-year spot rate is 5% pa and the 2-year spot rate is 5.8% pa, what 3-year spot rate would give a price of the bond at $t = 0$ of £99.07 per £100 nominal?

3. The n -year spot rate of interest, y_n , is given by

$$y_n = 0.04 + \frac{n}{1000} \quad \text{for } n = 1, 2, 3.$$

Calculate the implied 1-year forward rates applicable at times $t = 1$ and $t = 2$.

4. The following n -year spot rates were observed at time $t = 0$.

1-year spot rate of interest 4%

2-year spot rate of interest 5%

3-year spot rate of interest 6%

4-year spot rate of interest 7%

5-year spot rate of interest 7.5%

6-year spot rate of interest 8%

- (a) Calculate the 2-year forward rate of interest at time $t = 3$.
 - (b) Using the above n -year spot rates calculate the 6-year par yield at time $t = 0$.
5. Three bonds paying annual coupons of 6% annually in arrears and redeemable at par will be redeemed in exactly one year, two years and three years respectively. The price of each of the bonds is £96 per £100 nominal.

- (a) Determine the gross redemption yield of the 3-year bond.
- (b) Determine the discount factors $v(1)$, $v(2)$ and $v(3)$ that the market is using to discount payments due in 1, 2 and 3 years respectively.
- (c) Calculate $f_{0,1}$, $f_{1,1}$ and $f_{2,1}$, where $f_{n,k}$ is the forward interest rate from time n to $n + k$.