Problem Sheet 1 - Part B Actuarial Science II - Oxford HT 2006

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1. Describe the main types of asset used by insurance companies for their funds. What are the advantages and disadvantages of each?

2. An investment project involves payments of £1,000,000 now and £500,000 two years from now. The proceeds from the project are £2,000,000, payable in exactly four years' time.

   Calculate the volatility and the convexity of this project, as a function of the underlying interest rate i (assumed constant).

   Calculate the yield and compare volatility and convexity at $i = 4\%$ and $i = 8\%$.

3. Consider a portfolio of n fixed income bonds (any positive cash flows) with present values $A_1, \ldots, A_n$. For $j = 1, \ldots, n$, let the discounted mean term and the convexity of the jth bond be denoted by τ_j and c_j. Let $p_j = A_j/A$ be the proportion of the jth security in the portfolio, $A = A_1 + \ldots + A_n$.

   Show that the discounted mean term and the convexity of the portfolio are

   $$\sum_{j=1}^{n} p_j \tau_j \qquad \text{and} \qquad \sum_{j=1}^{n} p_j c_j.$$

4. A fund has to provide an annuity of £50,000 p.a. payable yearly in arrears for the next 9 years followed by a final payment of £625,000 in 10 years' time.

   The fund has earmarked cash assets equal to the present value of the payments and the fund manager wants to invest these in two zero coupon bonds, A, repayable after 5 years, and B, repayable after 20 years.

   How much should the manager invest in A and B to have the same volatility in the assets and liabilities, assuming an effective rate of interest of 7% p.a.?

5. A fund will need to make payments of £10,000 at the end of each of the next five years. It wishes to immunise using two zero coupon bonds, one maturing in 5 years and one in 1 year. The rate of interest is 5% p.a.

   (a) Calculate the present value of the liabilities.

   (b) Find the discounted mean term (or *(Macaulay) duration*).

   (c) Calculate the nominal amounts of the zero-coupon bonds needed to equate the present value and duration of assets and liabilities.

   (d) Calculate the convexity of the assets, and comment on immunisation (Redington).

6. A government bond pays a coupon half-yearly in arrears of 10% per annum. It is to be redeemed at par in exactly ten years. The gross redemption yield from the bond is 6% per annum convertible half-yearly. Calculate the discounted mean term of the bond in years.

   Explain why the duration of the bond would be longer if the coupon rate were 8% per annum instead of 10% per annum.