Problem Sheet 6 - Part B Actuarial Science I - Oxford MT 2005

---

1. Given a curtate lifetime $K = [T]$ and a constant-δ interest model, consider the following insurance products:

   (i) pure endowment with term n;

   (ii) whole life assurance;

   (iii) term assurance with term n;

   (iv) endowment assurance with term n (under this product, there is a payment of one unit at the end of the year of death or at end of term whichever is earlier).

   (a) Write each one as a random cash-flow of the form $C = \big((t_k, c_k B_k)\big)_{k=1,2,\ldots}$, where the B_k are Bernoulli random variables defined in terms of K.

   (b) Find expressions for the net premiums and variances of these products.

   (c) Relate the products, premiums and variances of (iv) and (i) and (iii).

   (d) Comment on (a) and (b) for the corresponding products where payment is made at death rather than at the end of the year of death (and n is not necessarily an integer).

2. A cash-flow is payable continuously at a rate of $\rho(t)$ per annum at time t provided a life who is aged x at time 0 is still alive. T_x is a random variable which models the residual lifetime in years of a life aged x.

   (a) Write down an expression, in terms of T_x, for the (random) present value at time 0 of this cash-flow, at a constant force of interest δ p.a., and show that the expected present value at time 0 of the cash flow is equal to

   $$\int_0^\infty e^{-\delta s}\rho(s)P(T_x > s)ds.$$

   (b) An annuity is payable continuously during the lifetime of a life now aged 30, but for at most 10 years. The rate of payment at all times t during the first 5 years is £5,000 p.a., and thereafter £10,000 p.a. The force of mortality to which this life is subject is assumed to be 0.01 p.a. at all ages between 30 and 35, and 0.02 p.a. between 35 and 40. Find the expected present value of this annuity at a force of interest of 0.05 p.a.

   (c) If the mortality and interest assumptions are as in (b), find the expected present value of the benefits of a term assurance, issued to the life in (ii), which pays £40,000 immediately on death within 10 years.

For the questions below, use the A 1967-70 Mortality table whenever table data is needed (all three columns, assuming medical checks were successful).

3. Find the present value of £5000 due in 5 years' time at $i = 4\%$ if

   a) the payment is certain to be made;

   b) the payment is contingent upon a life aged 35 now surviving to age 40.

4. How large a pure endowment, payable at age 65 can a life aged 60 buy with £1000 cash if $i = 7\%$?

5. a) Show that $\ddot{a}_x = 1 + (1+i)^{-1} p_x \ddot{a}_{x+1}$

   b) Show that $\ddot{a}_{x:\overline{n}|} = a_{x:\overline{n}|} + 1 - A_{x:\overline{n}|}^{\;\;1}$

   c) Show that $A_x = (1+i)^{-1}\ddot{a}_x - a_x$

6. Find the net single premium for a 5-year temporary life annuity issued to a life aged 65 if $i = 8\%$ for the first 3 years and $i = 6\%$ for the next 2 years.

7. Find the net annual premium for a £40000, 4-year term assurance policy issued to a life aged 26 if $i = 10\%$.

8. A deferred (temporary) life annuity is a deferred perpetuity (annuity-certain) restricted to a lifetime. For a life aged x, deferred period of m years (and a term of n years, from $m+1$ to $m+n$), the notation for the single premium is ${}_{m|}a_x$ (respectively ${}_{m|}a_{x:\overline{n}|}$).

   a) Give expressions in terms of the life table probabilities q_k.

   b) Show that ${}_{m|}a_x = A_{x:\overline{m}|}^{\;\;1} a_{x+m}$.

9. Describe the benefit which has the present value random variable function given by Z below; T denotes the future lifetime of a life aged x.

$$Z = \begin{cases} \bar{a}_{\overline{n}|} & T \leq n \\ \bar{a}_{\overline{T}|} & T > n \end{cases}$$

10. A special deferred annuity provides as benefits for a life aged 60:

    - on survival to age 65 an annuity of £2,000 p.a. payable in advance for two years certain and for life thereafter

    - on death between ages 63 and 65, £5,000 payable at the end of the year of death

    - on death between ages 60 and 63, £10,000 payable at the end of the year of death.

    Annual premiums are payable in advance until age 65 or earlier death. Determine the level annual premium based on an effective interest rate of 4% p.a. For the life annuity part, you may approximate all one-year death probabilities for age greater than 65 by the constant 0.05 (which leads close to the correct numerical answer).

Course webpage: `http://www.stats.ox.ac.uk/~martin/BS4a.html`