

Problem Sheet 4 - Part B Actuarial Science I - Oxford MT 2005

1. *Hardy's Formula.* Consider a fund F , and incomplete information as follows: the amounts of the fund $F_0 \in \mathbb{R}_+$ just before time 0 and $F_1 \in \mathbb{R}_+$ just after time 1 and the (net) total new money $N \in \mathbb{R}$ invested during $[0, 1]$. Denote $I = F_1 - F_0 - N$, the total interest and capital gain earned by the fund during $[0, 1]$.

- (a) Assume that N is received in two equal instalments at the beginning and end of the year. Show that the money-weighted rate of return is given by the formula

$$i = \frac{2I}{F_1 + F_0 - I}.$$

What is the time-weighted rate of return?

- (b) Assume that N is received in one instalment at $t = 1/2$. Find an expression for the money-weighted rate of return. What about the time-weighted rate of return?
- (c) Assume that N is received continuously at constant rate in $[0, 1]$. Give the yield equation for the money-weighted rate of return i . Show that for small i , the formula in part (a) is still a good approximation for the money-weighted rate of return.
2. (i) A fund had the following revenue account for 1998

	£million
Value of fund at 1 January 1998	30
Add: new investments received during the year	18
Deduct: withdrawals and other payments	30
Value of fund at 31 December 1998	21

Assuming that the new investments received and the withdrawals and other payments are spread evenly over the year, use Hardy's Formula from the previous question to calculate an approximate effective annual rate of interest for 1998.

- (ii) A second set of accounts was constructed to show the dates at which the new investments were received, the dates at which the withdrawals and other payments actually occurred, and the value of the fund at various times during the year.

	£million
Value of fund at 1 January 1998	30
Value of fund at 31 March	36
Value of fund at 15 May	35
New investments at 16 May	18
Value of fund at 30 June	51
Value of fund at 30 September	45
Withdrawals on 1 October	30
Value of fund at 31 December 1998	21

- (a) Show that the annual yield (i.e. the annual money weighted rate of return) on the fund for the year ending 31 December 1998 is approximately 8.87%.
- (b) Calculate the annual time-weighted rate of return on the fund for the year ending 31 December 1998.
- (iii) Explain the differences between your answers for (i), (ii)(a) and (ii)(b).
3. Calculate the time-dependent force of inflation between two successive index values if intermediate values are computed by (a) linear (b) exponential interpolation.
4. A loan of £25,000 was issued and was repaid at par after three years. Interest was paid on the loan at the rate of 8% per annum, payable annually in arrears. The value of the Retail Price Index at various times was as follows:

At the date the loan was made	205.0
One year later	215.6
Two years later	223.5
Three years later	231.5

Estimate the real rate of return of the loan.

5. In a particular accumulation fund income is retained and used to increase the value of the fund unit. The 'middle price' of the unit on 1 April in each of the years 1979 to 1985 is given in the following table:

Year	1 April	1979	1980	1981	1982	1983	1984	1985
Middle price of unit in £		1.86	2.11	2.55	2.49	2.88	3.18	3.52

- (a) On the basis of the above prices and ignoring taxation and expenses:
- (i) Find the time-weighted rate of return over the period 1 April 1979 to 1 April 1985,
- (ii) Show that the yield obtained by an investor who purchased 200 units on 1 April in each year from 1979 to 1984 inclusive, and who sold his holding on 1 April 1985, is approximately 10.60%.
- (iii) Show that the yield obtained by a person who invested £500 in the fund on 1 April each year from 1979 to 1984 inclusive, and who sold back his holding to the fund managers on 1 April 1985, is approximately 10.67% (You should assume that investors may purchase fractional parts of units.)
- (b) Suppose that, in order to allow for expenses, the fund's managers sell units 2% above the published middle price and buy back units 2% below the middle price. On this basis find revised answers to (ii) and (iii) of (a).

Course webpage: <http://www.stats.ox.ac.uk/~martin/BS4a.html>