Problem Sheet 3 - Part B Actuarial Science I - Oxford MT 2005

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1. Two business projects, each of which takes two years to complete, produce the following cashflows:

   Project A:

   - initial income of £2000;
   - after one year, expenditure of £3900;
   - after two years, income of £2000.

   Project B:

   - initial income of £360;
   - after one year, expenditure of £4000;
   - after two years, income of £4000.

   An investor considering the two projects has no spare cash, but can borrow or invest money at rate $i > 0$ for any desired term. For what range of i is each of the projects profitable? If both are profitable and the investor must choose between them, which is the more profitable (for various possible values of i)?

2. Buy now, pay later!! A luxury sofa is sold with the following payment options: either pay immediately and receive a 5% discount, or pay by monthly instalments (in arrears) for 15 months. To calculate the monthly payments, add a 5% finance charge to the purchase price, then divide the total amount into 15 equal payments. What annual effective rate of interest is being charged for paying by instalments?

3. A couple buying a house require a £150,000 mortgage. They know that they will have to sell the house (and redeem the mortgage) in three years' time. They have the choice of two mortgages with monthly payments and 25-year term.

   Mortgage A is at effective rate $i_A = 10.25\%$ per annum. This mortgage stipulates, however, that if you pay off the mortgage any time before the fifth anniversary, you will have to pay a penalty equal to 2.4% of the outstanding debt at the time of repayment.

   Mortgage B is at $i_B = 10.7\%$ but can be paid off at any time without penalty.

   Given that the buyers will have to repay the mortgage after three years and that they can save money at $j = 6\%$, which mortgage should they choose?

   Calculate the yields (assuming redemption after three years).

4. An annuity is payable continuously between time 0 and time n (where n is not necessarily an integer). The rate of payment of the annuity at time t ($0 \leq t \leq n$) is t per unit time.

   On the basis of an effective interest rate of 5% per annum, the value of the annuity at time 0 is equal to one-half of the total amount which will be paid. Estimate n.

5. Let c be the cash flow representing an investment project where all outflows precede all inflows. For what range of i is the discounted payback period $T_{+|i}$ under the constant i model well-defined? Show that it is an increasing function of i.

   $T_{+|0}$ is called the (undiscounted) *payback period*. Write down a formula for $T_{+|0}$ in the case of a mixed cash flow (i.e. one with both discrete and continuous components).

6. An investor has decided to purchase a leasehold property for £80,000, with a further payment of £5000 for repairs in one year's time. The income associated with letting the property will be £10,000 per annum, payable continuously for 20 years commencing in two years' time.

   (a) i. Given that the venture will be financed by bank loans on the basis of an effective annual interest rate of 7% and that the loans may be repaid continuously, find the discounted payback period for the project.

   ii. Given, further, that after the loans have been repaid the investor will deposit all the available income in an account which will earn interest at 6% per annum effective, find the accumulated amount of the account in 22 years' time.

   (b) Suppose that the bank loans may be repaid partially, but only at the end of each complete year, and that the investor may still deposit money at any time for any term at an annual rate of interest of 6% effective. Find

   i. the discounted payback period for the project, and

   ii. the accumulated amount in the investor's account in 22 years' time.

You should all read and comment on the following exercise. What is its purpose? If you are keen, work out a complete solution.

7. In return for an outlay of £1000 now and a further outlay of £600 after four years a business venture will provide a single receipt of £C in one year's time ($C > 0$).

   (a) Measuring time in years, write down the equation $NPV(i) = 0$ for the transaction.

   (b) Letting $\alpha = 400(20/3)^{1/2}5^{1/4}(\approx 1544.39)$, show that

   i. If $C \geq 1600$ the equation has one positive root;

   ii. If $\alpha < C < 1600$ the equation has two positive roots;

   iii. If $C = \alpha$ the equation has one positive root;

   iv. If $C < \alpha$ the equation has no positive roots.

   (c) Find all the positive roots when (i) $C = 1600$, (ii) $C = 1550$ and (iii) $C = \alpha$.

Course webpage: `http://www.stats.ox.ac.uk/~martin/BS4a.html`