Problem Sheet 1 - Part B Actuarial Science I - Oxford MT 2005

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0. This exercise is to test your intuition for risk. An insurer sells temporary life assurances to a group of 40 year olds providing £20,000 on death within the period. The probability of dying is 10% over the 20 year period. Ignoring interest, what (total) premium should the insurer charge?

   Would it make any difference if there was one potential buyer, 20, 2000 or 200,000?

1. Discuss the cash flows of a typical final-year undergraduate over 5 years. Aim for a concise presentation.

2. If the effective annual rate of interest is 9%, calculate the total accumulated value at 1 January 2006 of payments of £100 made on 1 January 2005, 1 April 2005, 1 July 2005 and 1 October 2005.

3. A man stipulates in his will that £50,000 from his estate is to be placed in a fund from which his three children are each to receive an equal amount when they reach age 21. When the man dies, the children are ages 19, 15 and 13. If this fund earns 6% interest per half-year (i.e. nominal 12% p.a. compounded semi-annually), how much does each receive? Is the distribution fair?

4. (i) Establish a table of relationships between (constant) δ, i, v and d.

   (ii) Show that $d = vi$ and interpret this.

   (iii) Show $\delta \approx i - i^2/2$ and $d \approx i - i^2$ for small i and $d \approx \delta - \delta^2/2$ for small δ.

5. Calculate the equivalent effective annual rate of interest of

   (a) a force of interest of 7.5% p.a.

   (b) a discount rate of 9% p.a.

   (c) a nominal rate of interest of 8% p.a. convertible half-yearly

   (d) a nominal rate of interest of 9% p.a. convertible monthly.

6. Under the terms of a savings scheme an investor who makes an initial investment of £4,000 may receive either

   - £2,000 after 2 years and a further £3,000 after 7 years
   - £4,400 at the end of 4 years

   Which of these options corresponds to a higher rate of interest on the investor's money?

7. Calculate the accumulated value of £1,000 after 2 years if $\delta(t) = 0.06(t + 1)$ for $0 \leq t \leq 2$.

8. *Stoodley's formula.* Suppose the force of interest is given by

$$\delta(t) = p + \frac{s}{1 + re^{st}}, \qquad t \in \mathbb{R}_+$$

where $p, r \in \mathbb{R}_+$ and $s \geq -p$.

Calculate the discount factor $v(t)$, $t \in \mathbb{R}_+$, and show that the model can be reparametrized such that $v(t) = \lambda v_1^t + (1 - \lambda) v_2^t$. Interpret this!

M.Sc. students and keen undergraduates should also try to solve the following exercises.

9. In valuing future payments an investor uses the formula

$$v(t) = \frac{\alpha(\alpha + 1)}{(\alpha + t)(\alpha + t + 1)}, \qquad t \in \mathbb{R}_+$$

where α is a given positive constant, for the value at time 0 of 1 due at time t (measured in years).

Show that the above formula implies that

(i) the force of interest per annum at time t will be

$$\delta(t) = \frac{2t + 2\alpha + 1}{(\alpha + t)(\alpha + t + 1)}$$

(ii) the effective rate of interest for the period r to $r + 1$ will be

$$i(r) = \frac{2}{r + \alpha}$$

(iii) the present value of a series of n payments, each of amount 1 (the rth payment being due at time r) is

$$a(n) = \frac{n\alpha}{n + \alpha + 1}.$$

(iv) Suppose now that $\alpha = 15$. Find the level annual premium, payable in advance for twelve years, which will provide an annuity of £1800 per annum, payable annually for ten years, the first annuity payment being made one year after payment of the final premium. What is the value at time 12 of the series of annuity payments, what at time 0?

10. (i) Show that the simple interest model $S_t = (1 + t\ i)C$ can be viewed to some extent as a time-dependent compound interest model $S_t = C \exp\{\int_0^t \delta(s) ds\}$. Calculate $\delta(\cdot)$ and interpret the result. What can be said about valuing cash flows?

(ii) Assume that for a bank account, interest is paid annually in arrears as simple interest at a fixed rate i (the same for negative and positive balances!). Given an initial capital of C, how much money can be withdrawn at time $t \in [0, 1]$ to achieve an empty account after the interest payment. If we define this as the accumulated value at time t, how does this relate to (i) and to the compound interest model at an effective interest rate i. Comments! What can be said about valuing cash flows?