For an endowment assurance:

$$P_{x\overline{n|}} = A_{x\overline{n|}} / \ddot{a}_{x\overline{n|}}$$

A slightly different type of policy is a whole life policy with premiums only payable for a limited term (say n years). In this case we have:

$${}_nP_x = A_x / \ddot{a}_{x\overline{n|}}$$

The same notation is similarly used for continuous functions. Thus for a whole life assurance payable immediately on death, with premiums payable continuously:

$$\bar{P}_x = \bar{A}_x / \bar{a}_x$$

Reserves

For most assurance policies (but not all) the costs of paying benefits increases as the policy proceeds. For instance, in a whole life policy the chance of dying in a given year (q_x) increases with age and thus the expected payments increase year by year. At the beginning of the contract, the premium payable at the start of the year is more than sufficient to cover the expected benefit payments during the year. At later stages, the premium received each year is less than the expected benefit payments during the year. Therefore, the insurance company needs to build up reserves.

We start by looking at the "prospective policy value" for an in force life insurance contract. "In force" means that the policy has started and has not been completed either by death or the expiry of the stated term of the contract (or by surrender of the contract).

The prospective policy value is defined as:

> Expected present value of future outgo less expected present value of future income.

The notation for policy values is V.

For instance ${}_tV_x$ represents the policy value for a whole life issued to a life aged x at the end of t years. It will be seen that:

$${}_tV_x = A_{x+t} - P_x\ddot{a}_{x+t}$$

but $P_x = A_x / \ddot{a}_x$

and $A_{x+t} = 1 - d\,\ddot{a}_{x+t}$

$${}_tV_x = (1 - d\,\ddot{a}_{x+t}) - (1 - d\,\ddot{a}_x)\ \ddot{a}_{x+t} / \ddot{a}_x = 1 - \ddot{a}_{x+t} / \ddot{a}_x$$

Recursive Calculation

To prove: $({}_tV_x + P_x)(1+i) = q_{x+t} + p_{x+t} * {}_{t+1}V_x$