Consider a contract which pays a sum of 1 at the end of each year as long as a life now aged x survives.

$$EPV = \sum_{k=1}^{\infty} v^k {}_kp_x \quad \text{which in actuarial terminology is} \quad a_x$$

If the sum is payable at the start of each year:

$$EPV = \sum_{k=0}^{\infty} v^k {}_kp_x = a_x + 1 \quad \text{which in actuarial terminology is} \quad \ddot{a}_x$$

Temporary annuity

This is similar to a life annuity but payments are limited to a specified term. Consider a contract which pays a sum of 1 at the end of each of the next n years as long as a life now aged x survives.

$$EPV = \sum_{k=1}^{n} v^k {}_kp_x \quad \text{which in actuarial terminology is} \quad a_{x\,\overline{n}|}$$

If the sum is payable at the start of each year:

$$EPV = \sum_{k=0}^{n-1} v^k {}_kp_x = a_{x\,\overline{n-1}|} + 1 \quad \text{which in actuarial terminology is} \quad \ddot{a}_{x\,\overline{n}|}$$

Equivalence

$$A_x = \sum_{k=0}^{\infty} v^{k+1} {}_kp_x\, q_{x+k} = \sum_{k=0}^{\infty} v^{k+1} {}_kp_x (1 - p_{x+k}) = \sum_{k=0}^{\infty} v^{k+1} ({}_kp_x - {}_{k+1}p_x)$$

$$= v\,\ddot{a}_x - (\ddot{a}_x - 1)$$

$$= 1 - d\, a_x$$

Continuous functions

Finally we consider cases where payment is made at the date of death (in the case of whole life assurance) or payment is made continuously in the case of a life annuity.

The actuarial terminology for the EPV of a whole life assurance of unit sum assured to a life aged x payable at the date of death is:

$$\bar{A}_x = \int_0^{\infty} v^t\, {}_tp_x\, \mu_{x+t}\, dt$$

Similarly for an annuity of 1 per annum payable continuously to a life aged x for as long as he lives:

$$\bar{a}_x = \int_0^{\infty} v^t\, {}_tp_x\, dt$$