Actuarial Science Hilary Term Lecture 6

Back to Life Contingencies

Recap from the end of last term:

Future lifetime of an individual aged x is a random variable T_x continuously distributed on the interval $[0,\omega-x]$ where ω is the "limiting age" – normally taken as 120. When x=0 this is written as T.

$F_x(t)$ is the distribution function of T_x

and $S_x(t) = P[T_x > t] = 1 - F_x(t)$ is the survival function of T_x

Actuarial notation is ${}_tq_x = F_x(t)$

and ${}_tp_x = 1 - {}_tq_x = S_x(t)$

Normally work in units of one year so that t=1 and the t is then dropped:

$q_x = {}_1q_x$ and $p_x = {}_1p_x$

Force of mortality is written as μ_x and defined as:

$$\mu_x = \lim_{h \to 0} \frac{1}{h} * P[T < x+h \mid T > x]$$

From definitions above $P[T < x+h \mid T > x] = F_x(h) = {}_hq_x$

For small h we can ignore the limit and ${}_hq_x \simeq h.\ \mu_x$

$$S_x(t) = P[T_x > t]$$

$$= P[T > x+t \mid T > x]$$

$$= \frac{P[T > x+t\,]}{P[T > x]}$$

$$= \frac{S(x+t)}{S(x)}$$

or $${}_tp_x = \frac{{}_{x+t}p_0}{{}_xp_0}$$

$$= \frac{{}_{x+s}p_0}{{}_xp_0} \cdot \frac{{}_{x+s+t}p_0}{{}_{x+s}p_0}$$

$$= {}_sp_x * {}_tp_{x+s}$$

Probability density function $f_x(t) = \frac{d}{dt} F_x(t)$