Discrete time forward rates

Discrete time forward rate $f_{t,r}$ is the annual interest rate agreed at time 0 for an investment made at time t (>0) for a duration of r years.

In other words if an investor agrees at time 0 to invest 1 at time t for r years, the maturity amount (at time t+r) will be:

$$(1+ f_{t,r})^r$$

There are connections between forward rates, spot rates and zero coupon bonds.

Take an investment of 1 for t years and also agree at commencement that the accumulation at time t will be reinvested for a further r years.

Then final amount at time t+r will be:

$$(1+ y_t)^t . (1+ f_{t,r})^r$$

But 1 invested for t+r years accumulates to:

$$(1+ y_{t+r})^{t+r}$$

Also, using the zero coupon bond price, 1 invested for t+r years accumulates to:

$$(P_{t+r})^{-1}$$

Therefore $(1+ y_t)^t . (1+ f_{t,r})^r = (1+ y_{t+r})^{t+r} = (P_{t+r})^{-1}$

And $(1+ f_{t,r})^r = \dfrac{(1+ y_{t+r})^{t+r}}{(1+ y_t)^t} = \dfrac{P_t}{P_{t+r}}$

"One period forward rate" at time t (agreed at time 0) is denoted f_t and defined as $f_{t,1}$

So an amount of 1 invested at time 0 for t years at the spot rate y_t will be equivalent to the same 1 invested at time 0 for t one year forward rates. In other words:

$$(1+ y_t)^t = (1 + f_0) (1 + f_1) (1 + f_2)\ldots\ldots. (1 + f_{t-1})$$

and, of course, $f_0 = y_1$

Continuous time spot rates

P_t is the price of a zero coupon bind of term t. The t year "spot force of interest" is Y_t which is defined as satisfying the equation:

$P_t = e^{-Y_t t}$ $\qquad\qquad$ $Y_t = -\dfrac{1}{t} . \log P_t$