Actuarial Science Hilary Term Lecture 4

Term structure of interest rates

Previous lectures have assumed that interest rates do not depend on the duration of the investment. However, consulting the financial pages shows that this is not the case. For instance, looking at the yields on UK Government stock at close of business on 2 dates in the past shows the following yields to redemption:

Stock	*Redemption Date*	*Redemption yield* % 31.01.03	*Redemption yield* % 03.02.2004
Treasury 10%	2003	3.63	
Treasury 4%	2004		3.90
Treasury 5%	2008	4.15	4.70
Treasury 8%	2013	4.26	4.86
Treasury 6%	2028	4.36	4.78

The table on the following page shows the yields at close of business on February 7 2006.

Discrete time spot rates

Consider a "zero coupon bond" of term n (i.e. an agreement to pay 1 at the end of n years with no coupon payable during the term). This is also called a "pure discount bond". Denote the price at issue of this bond to be P_n.

The yield (y_n) on this zero coupon bond is called the "n-year spot rate of interest" which clearly satisfies the equation:

$$P_n = \frac{1}{(1+y_n)^n} \implies (1+y_n) = P_n^{-1/n}$$

As demonstrated above, rates of interest normally depend on the term of the investment and so normally $y_s \neq y_t$ when $s \neq t$.

Each fixed interest investment can be considered as a string of zero coupon bonds.

Consider an n year bond, with coupon D and redemption price R. This is equivalent to n zero coupon bonds with maturity value D payable at durations 1,2,…,n-1 together with a zero coupon bond with maturity value R+D payable at duration n.

Price A is given by:

$$A = D.(P_1 + P_2 \ldots..+ P_n) + R.\ P_n$$

Define $v_{y_t} = (1+y_t)^{-1}$

Then $A = D.(v_{y_1} + v_{y_2}^2 + \ldots..+ v_{y_n}^n) + R.\ v_{y_n}^n$